Wave Propagation in Geometrically Reconfigurable Magneto-Elastic Meta-Structures

Marshall Schaeffer, <u>Massimo Ruzzene</u>
D. Guggenheim School of Aerospace Engineering
G. Woodruff School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA

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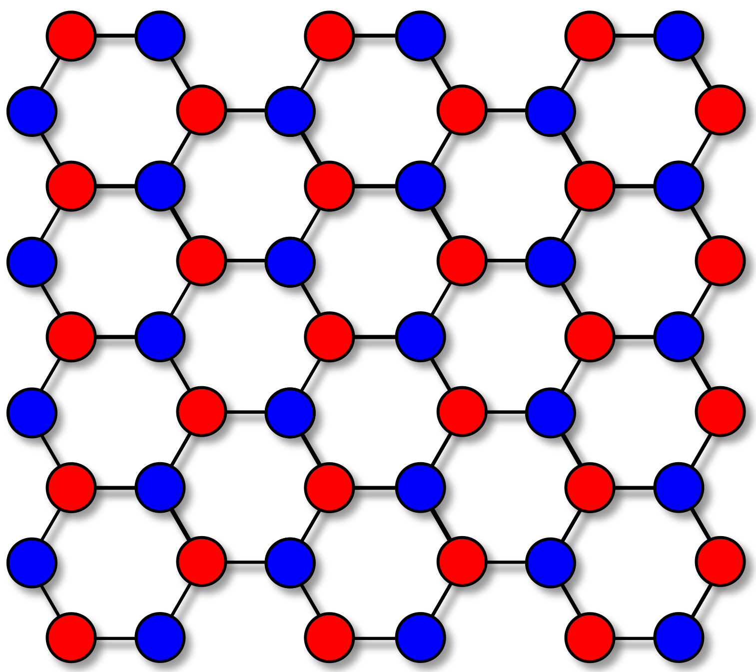
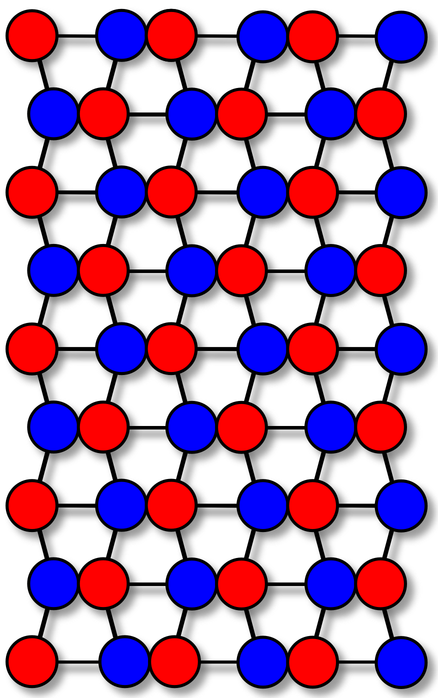
- Objectives:
  - Investigate shape and topology adaptation of periodic lattices
  - Investigate effects on:
    - Overall geometry and shape
    - Mechanical properties
    - Wave propagation characteristics

- Approach
  - Exploit bi-stable interactions at the unit cell level to achieve large overall effect at structural level
  - Bi-stability is achieved through magneto-elastic interactions
  - Study is conducted through:
    - Numerical simulations
    - Determination of equilibrium configurations and
    - Bloch analysis of linearized systems

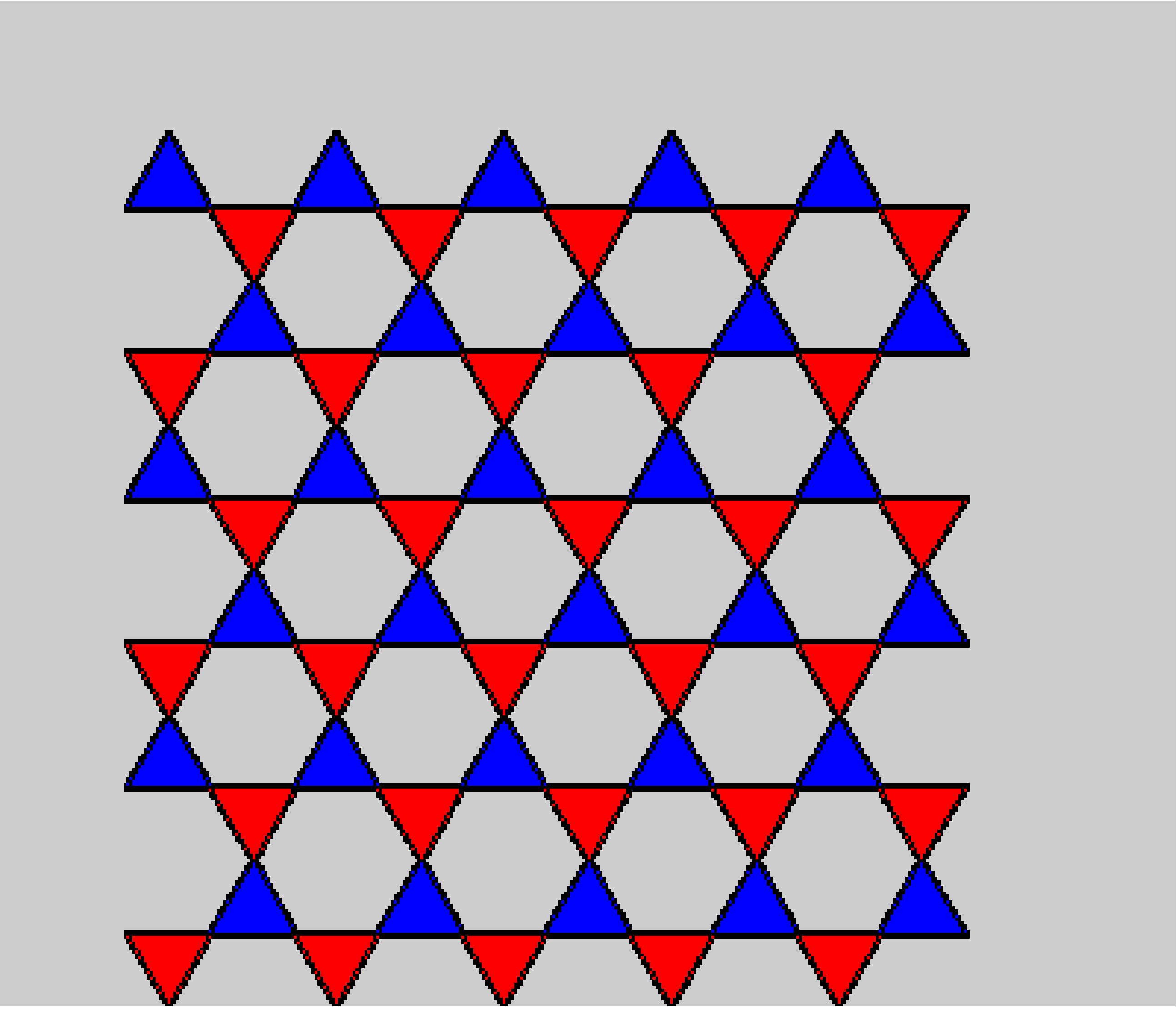
Hexagonal Lattice



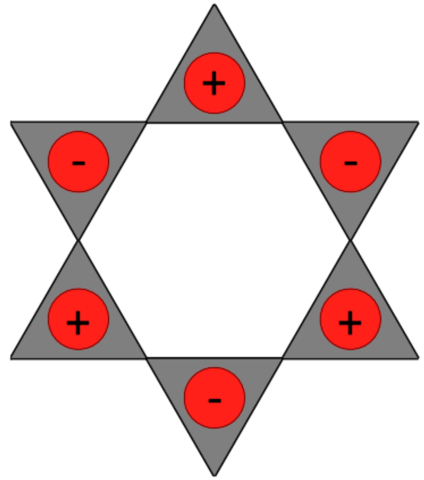
Re-entrant Lattice

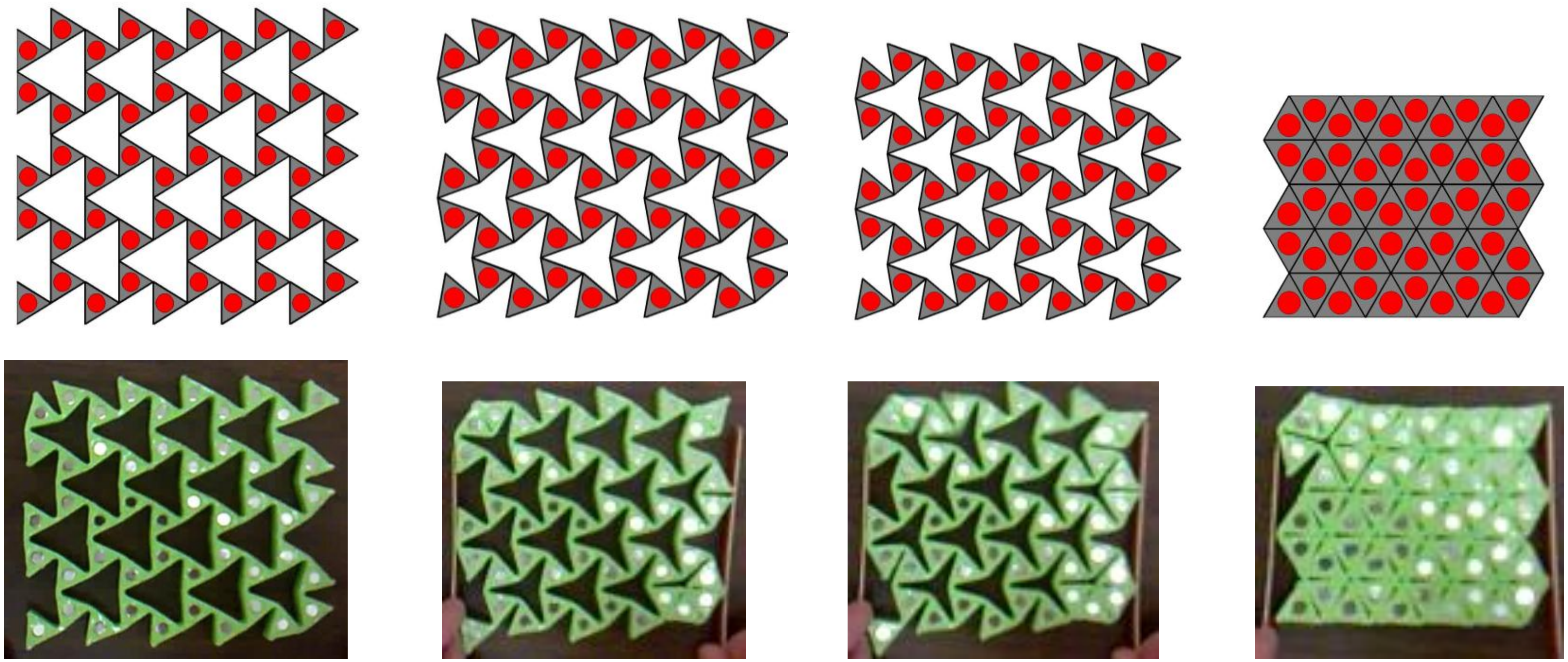


Configurations: Kagome Lattices

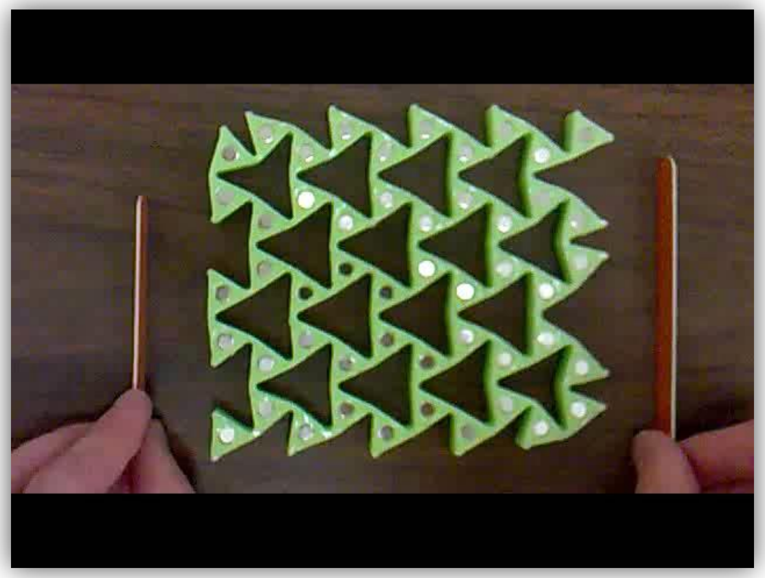


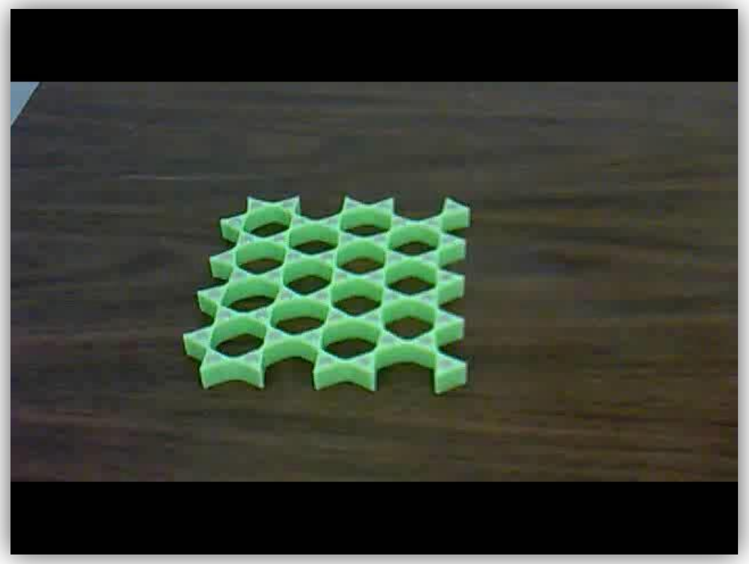
Magnetic bi-stable lattices





Magnetic bi-stable lattices



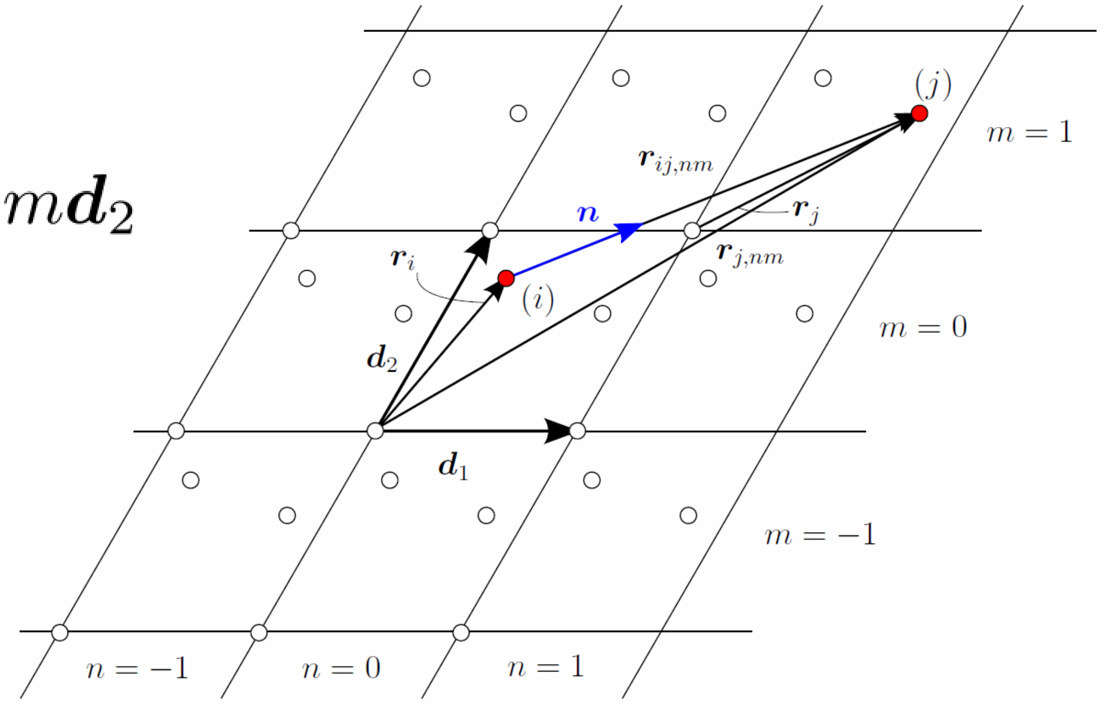


Periodic Lattice

- Particle Position

$$\boldsymbol{r}_{i,nm} = \boldsymbol{r}_i + n\boldsymbol{d}_1 + m\boldsymbol{d}_2$$

$$\boldsymbol{r}_i = x_i\boldsymbol{i} + y_i\boldsymbol{j}$$

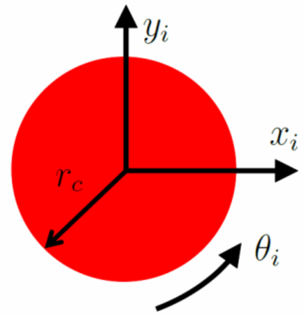


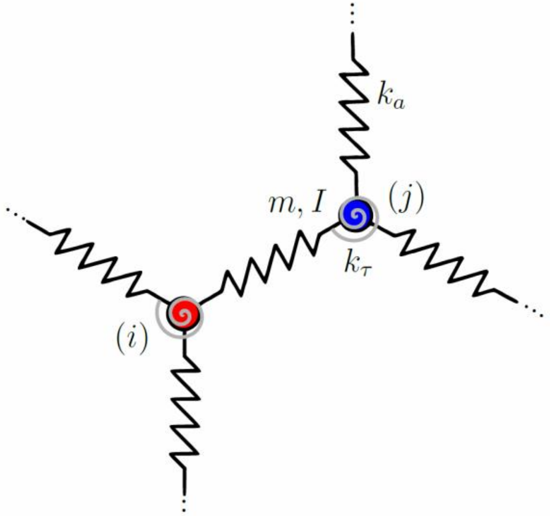
- Relative Position

$$\boldsymbol{r}_{ij} = \boldsymbol{r}_j - \boldsymbol{r}_i$$

- Generalized DOFs

$$\boldsymbol{q}_i = [x_i,\ y_i,\ \theta_i]^T$$





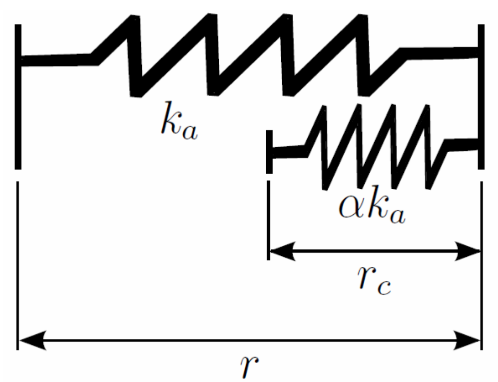
- Kinetic energy:

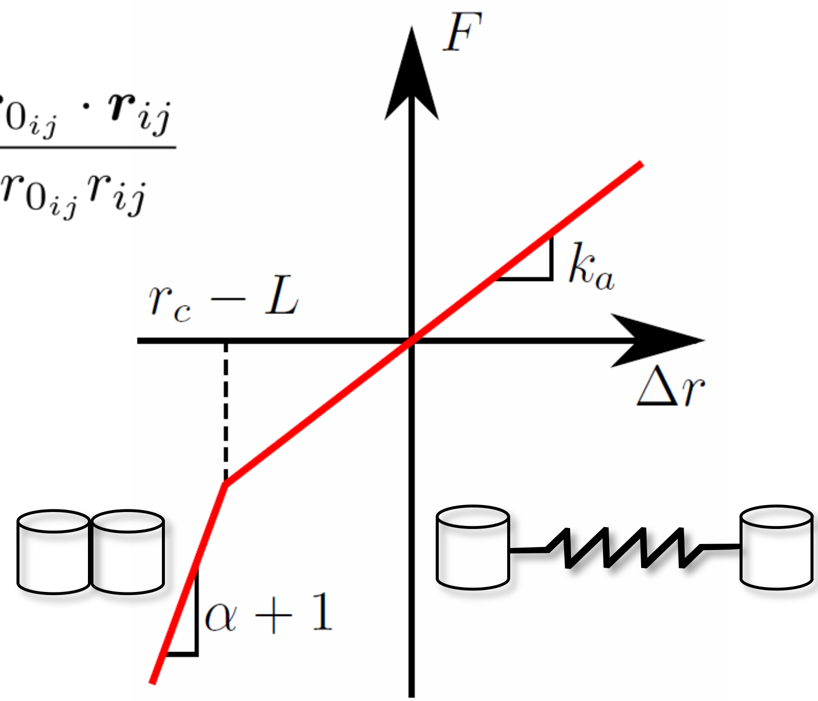
$$T_i = \frac{1}{2}m(\dot{x}_i^2 + \dot{y}_i^2) + \frac{1}{2}I\dot{\theta}_i^2$$

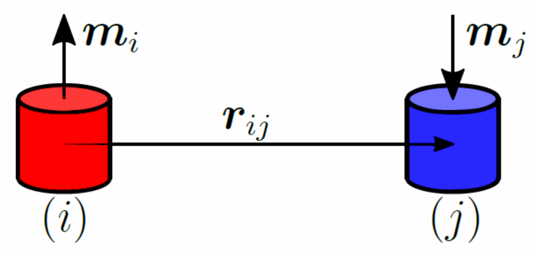
- Potential energy

$$U_{ij}^{(e)} = \frac{1}{2}k_a(r_{ij} - r_{0_{ij}})^2 + \frac{1}{2}\alpha k_a \left\langle r_c - r_{ij}\right\rangle^2 + \frac{1}{2}k_\tau(\theta_i - \theta_b)^2 + \frac{1}{2}k_\tau(\theta_j - \theta_b)^2$$

$$\langle \bullet \rangle = \begin{cases} 0 & \bullet < 0 \\ \bullet & \bullet \geq 0 \end{cases} \qquad \cos\theta_b = \frac{\boldsymbol{r}_{0_{ij}} \cdot \boldsymbol{r}_{ij}}{r_{0_{ij}} r_{ij}}$$







- Magnetic Work

$$W_{ij}^{(m)} = -\int \boldsymbol{f}_{ij}^{(m)} \cdot d\boldsymbol{r}_{ij}$$

- Magnetic Force(*)

$$\boldsymbol{f}_{ij}^{(m)} = -\frac{\mu_0}{4\pi}\boldsymbol{\nabla}\left(\frac{\boldsymbol{m}_i \cdot \boldsymbol{m}_j}{r_{ij}^3} - 3\frac{(\boldsymbol{m}_i \cdot \boldsymbol{r}_{ij})(\boldsymbol{m}_j \cdot \boldsymbol{r}_{ij})}{r_{ij}^5}\right)$$

$$\boldsymbol{f}_{ij}^{(m)} = \frac{3\mu_0}{4\pi}\frac{m_i m_j}{r_{ij}^4}\boldsymbol{n} = \frac{f^{(m)}}{r_{ij}^4}\boldsymbol{n}$$

- Magnetic Potential

$$U_{ij}^{(m)} = -\int_{\infty}^{r_{ij}} \frac{f^{(m)}}{r_{ij}^4}\boldsymbol{n} \cdot d\boldsymbol{r}_{ij} = \frac{1}{3}\frac{f^{(m)}}{r_{ij}^3}$$

- Radius of influence

$$r_\infty = \left(\frac{f^{(m)}}{3\varepsilon_e^{(m)}}\right)^{1/3}$$

An Analytic Solution for the Force Between Two Magnetic Dipoles, Yung, et al., *1998*

- Dissipative Work

$$W_{ij}^{(d)} = -\int \boldsymbol{f}_{ij}^{(d)} \cdot d\boldsymbol{r}_{ij} - \int \phi_i^{(d)} d\theta_i - \int \phi_j^{(d)} d\theta_j$$

- Linear Damping

$$\boldsymbol{f}_{ij}^{(d)} = \begin{cases} c_a \dot{r}_{ij} \hat{\boldsymbol{n}} & r_{ij} > r_c \\ \beta c_a \dot{r}_{ij} \hat{\boldsymbol{n}} & r_{ij} \leq r_c \end{cases}$$

- Angular Damping

$$\phi_i^{(d)} = -c_\tau (\dot{\theta}_i - \dot{\theta}_b)$$

- Total Energy

$$\mathcal{E} = \sum_{e=1}^{E} \mathcal{L}_e + W_e^{(d)}$$

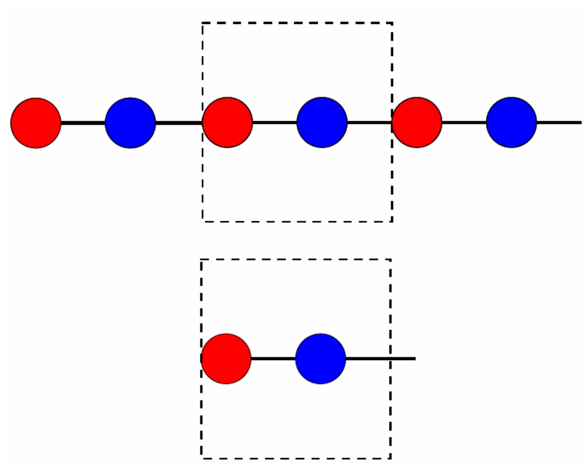
- EOM for one particle

$$\boldsymbol{M}_i \ddot{\boldsymbol{q}}_i + \sum_{e_i}^{E_i} \left[\boldsymbol{C}_{e_i}(\boldsymbol{q}_{e_i}) \dot{\boldsymbol{q}}_{e_i} + \boldsymbol{K}_{e_i}(\boldsymbol{q}_{e_i}) \boldsymbol{q}_{e_i} \right] = \boldsymbol{f}_i^{(m)}(\boldsymbol{q}_i, \ldots, \boldsymbol{q}_{m_i}, \ldots, \boldsymbol{q}_{M_i})$$

- Equation of motion for a particle is extended to a unit cell, and to a lattice

- Lattice Potential Energy

$$\mathcal{E} = \sum_{i=1}^{E} U_i^{(e)} + U_i^{(m)}$$

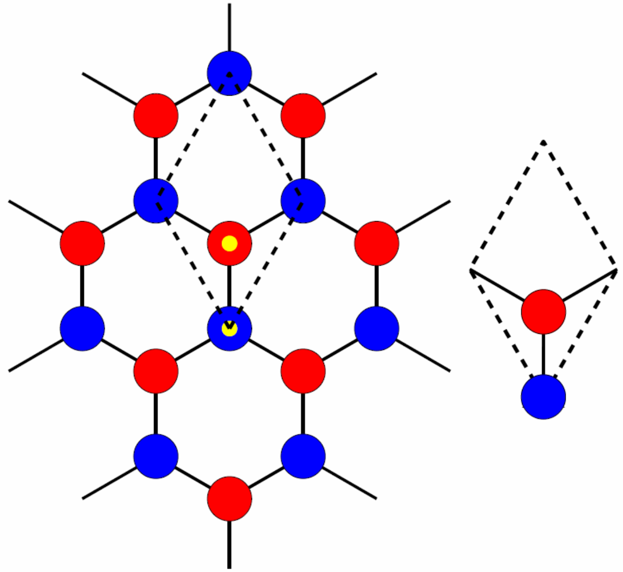


- Unit cell DOFs

$$\boldsymbol{q} = [\boldsymbol{q}_e, \ldots, \boldsymbol{q}_E]^T$$

- Periodic Lattice conditions are enforced on a specified unit cell size

$$\boldsymbol{u} = [\boldsymbol{q}, \boldsymbol{d}_1, \boldsymbol{d}_2]$$

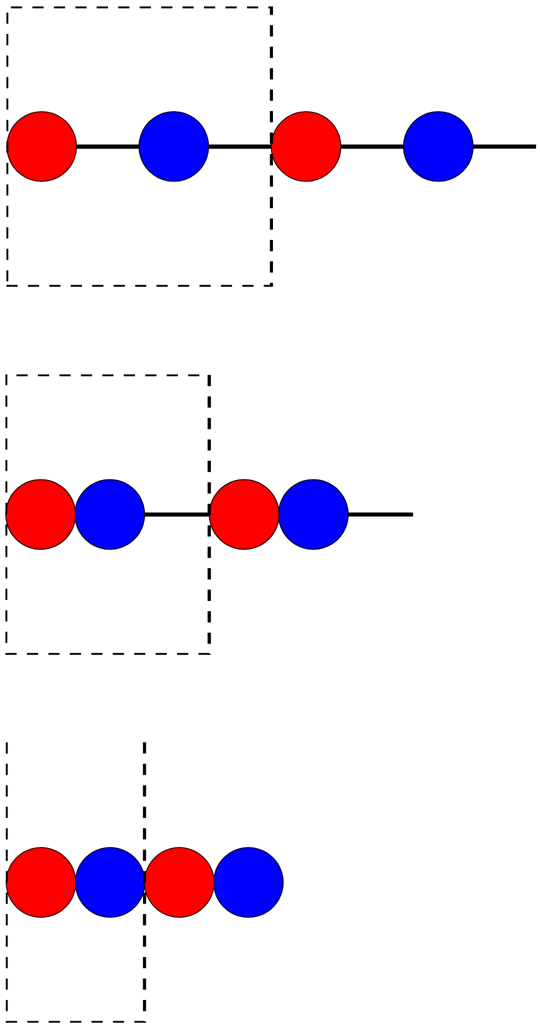


- Optimization Problem

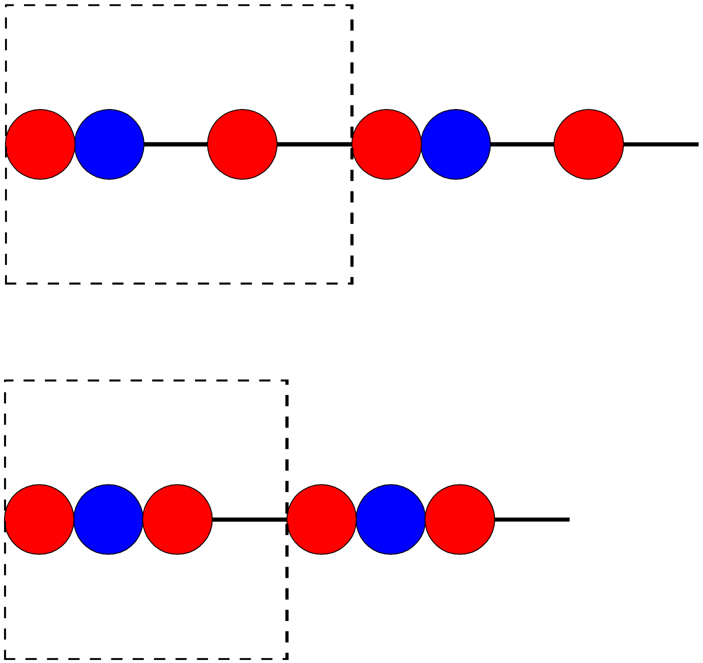
$$\min_{\boldsymbol{u}} \quad \mathcal{E}(\boldsymbol{u})$$

Equilibrium Configurations

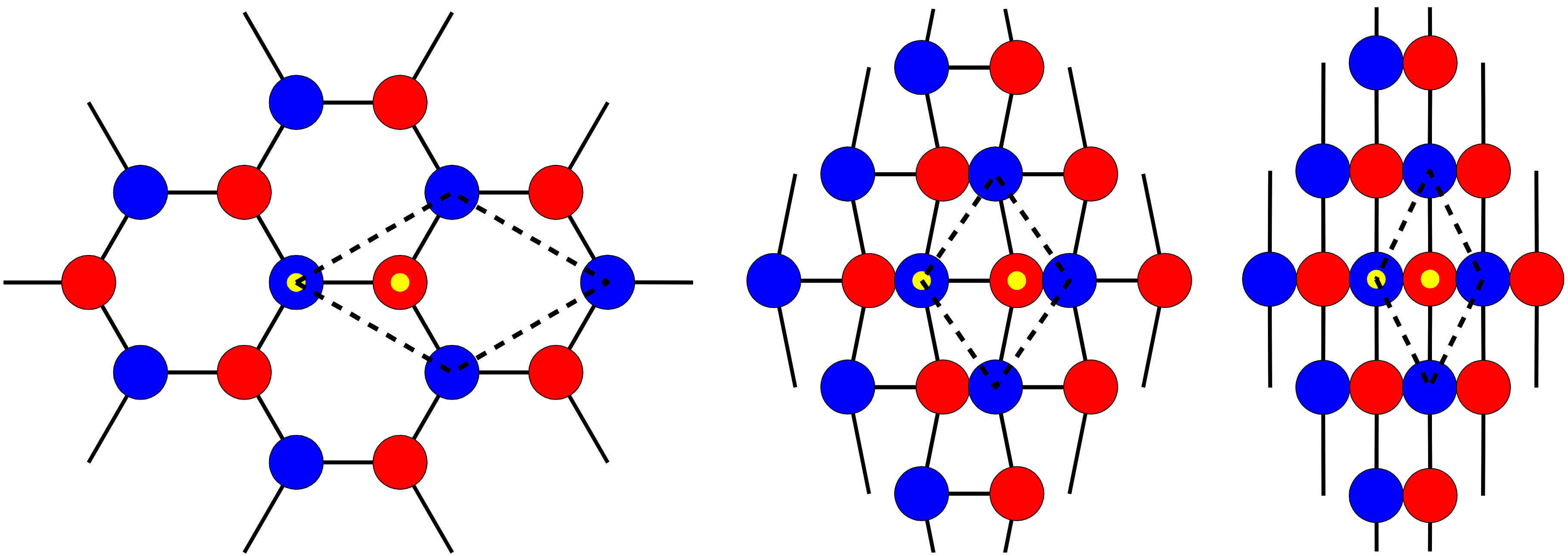
2 Particle Cells



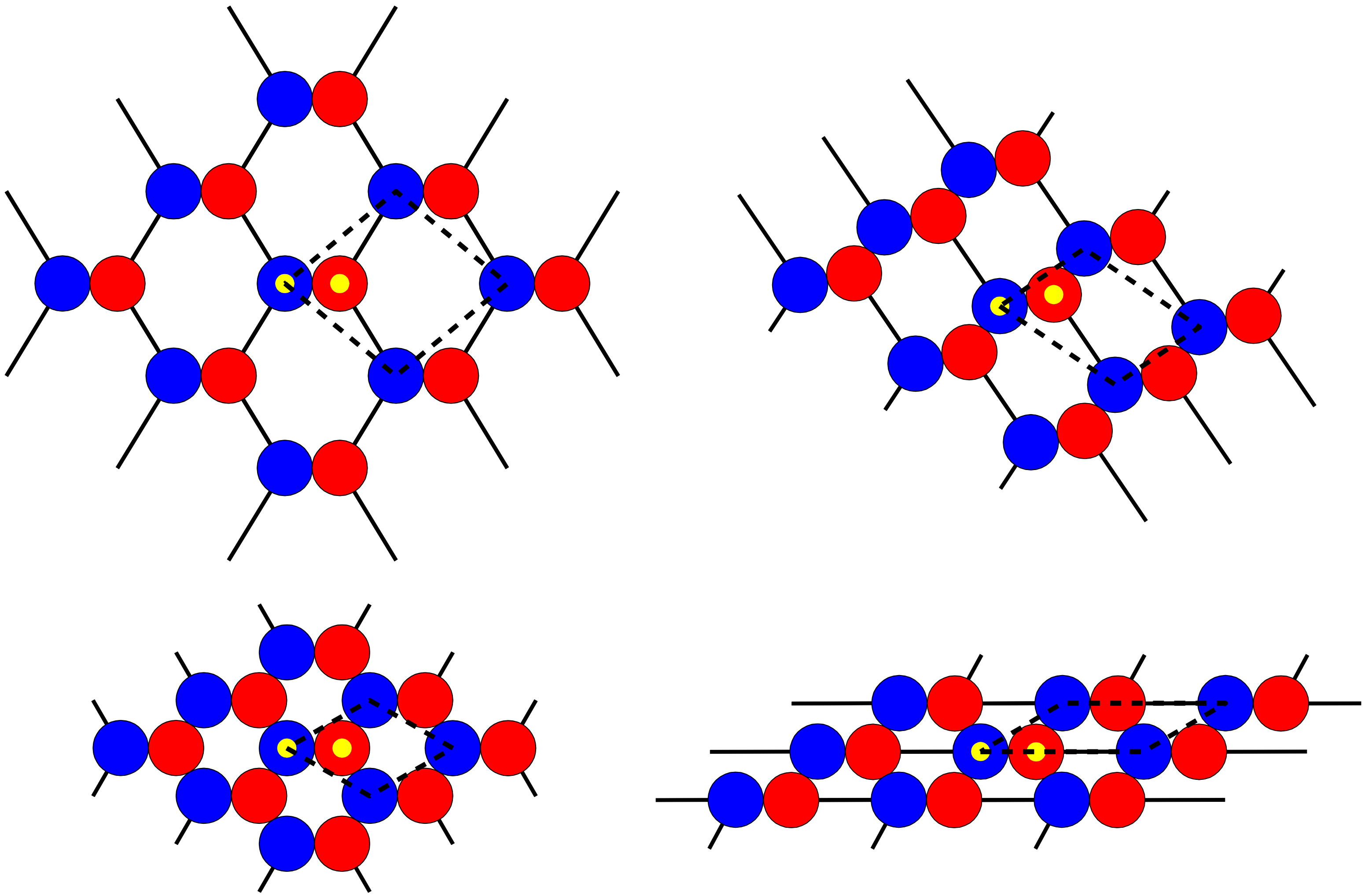
3 Particle Cells



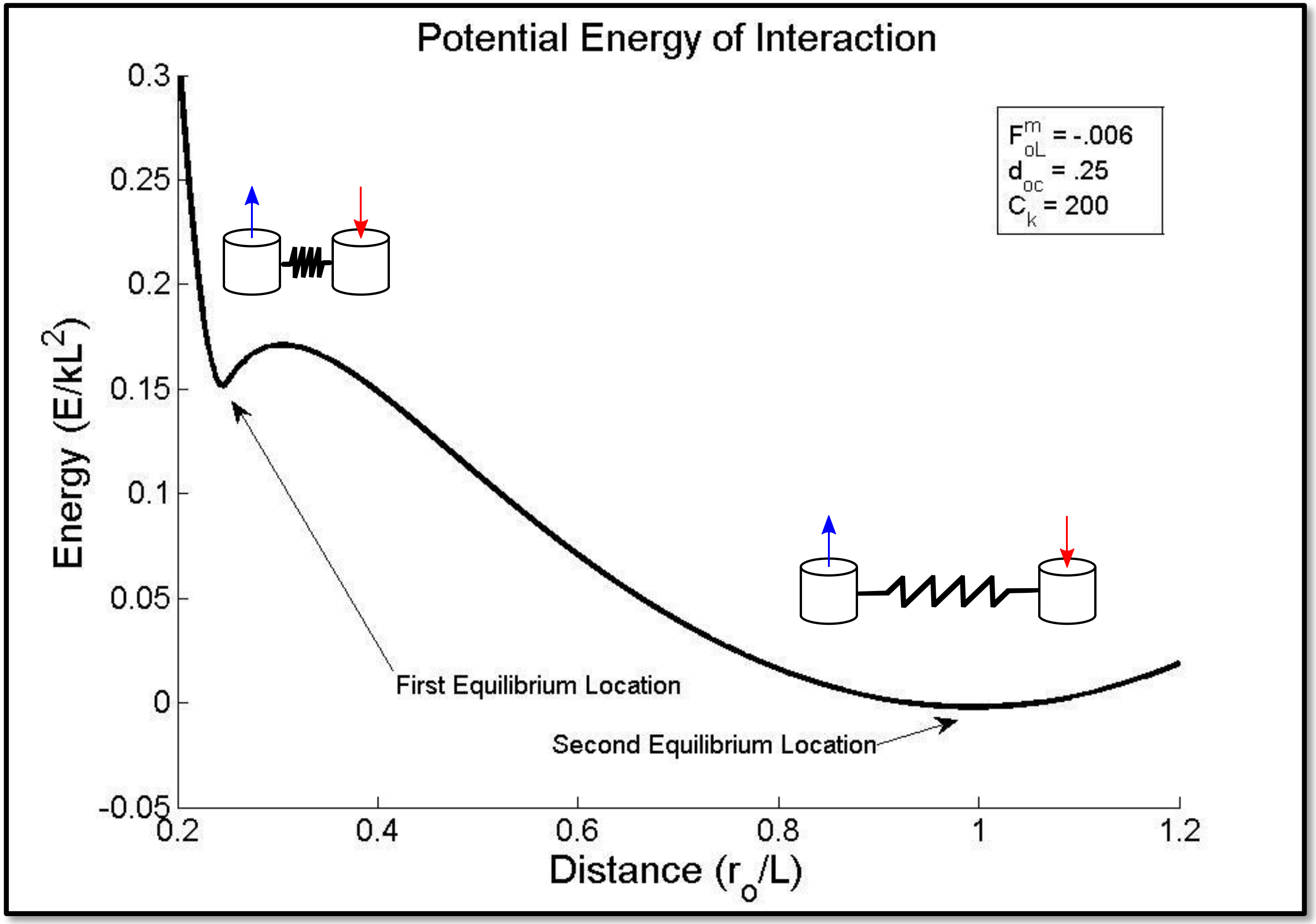
2D Equilibrium Configuration Examples: 2 Particle Cells



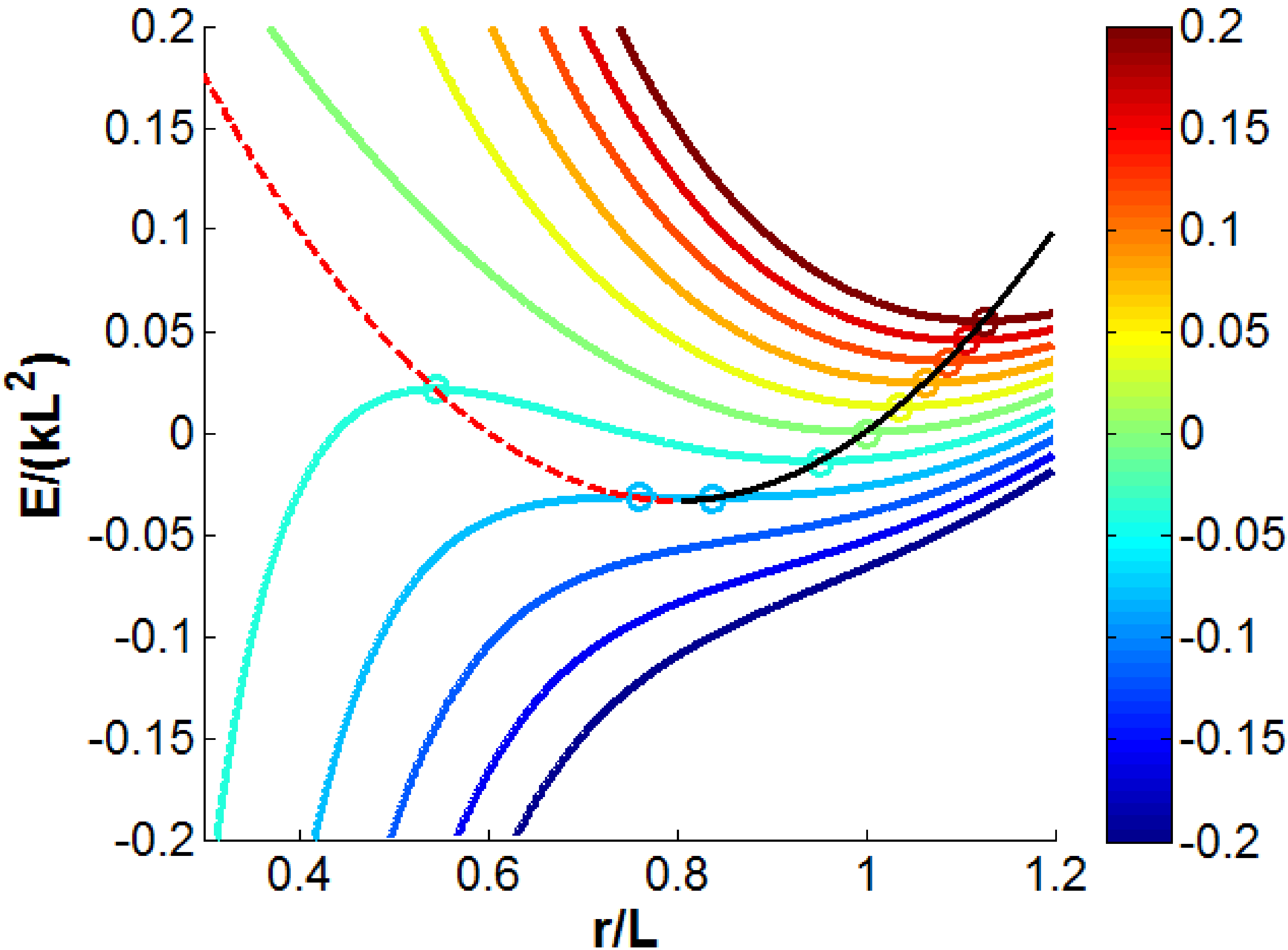
2D Equilibrium Configuration Examples: 2 Particle Cells



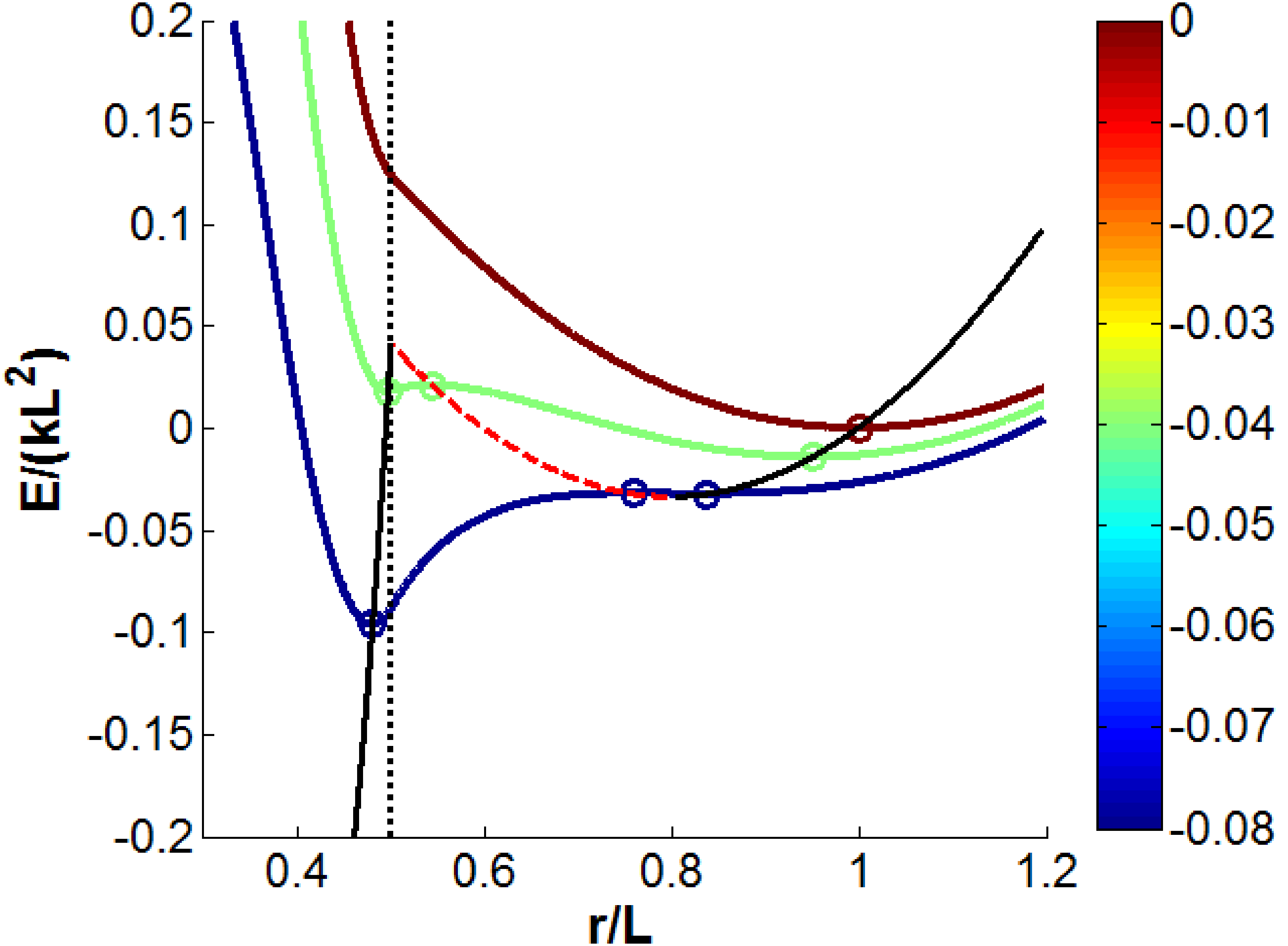
Energy profile of particle pair



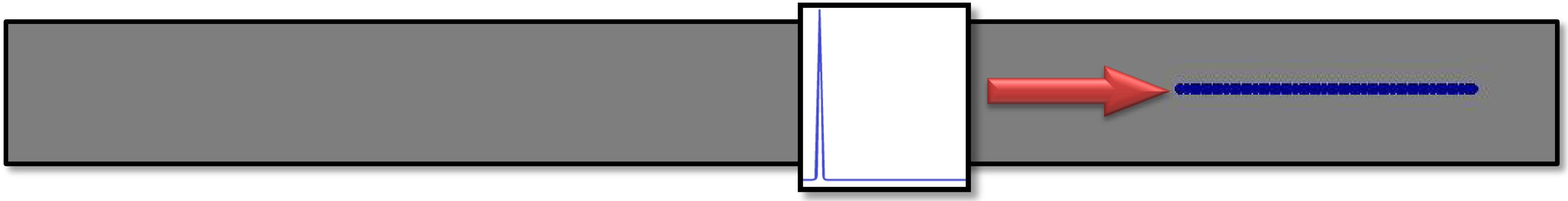
Energy profile of particle pair, no contact

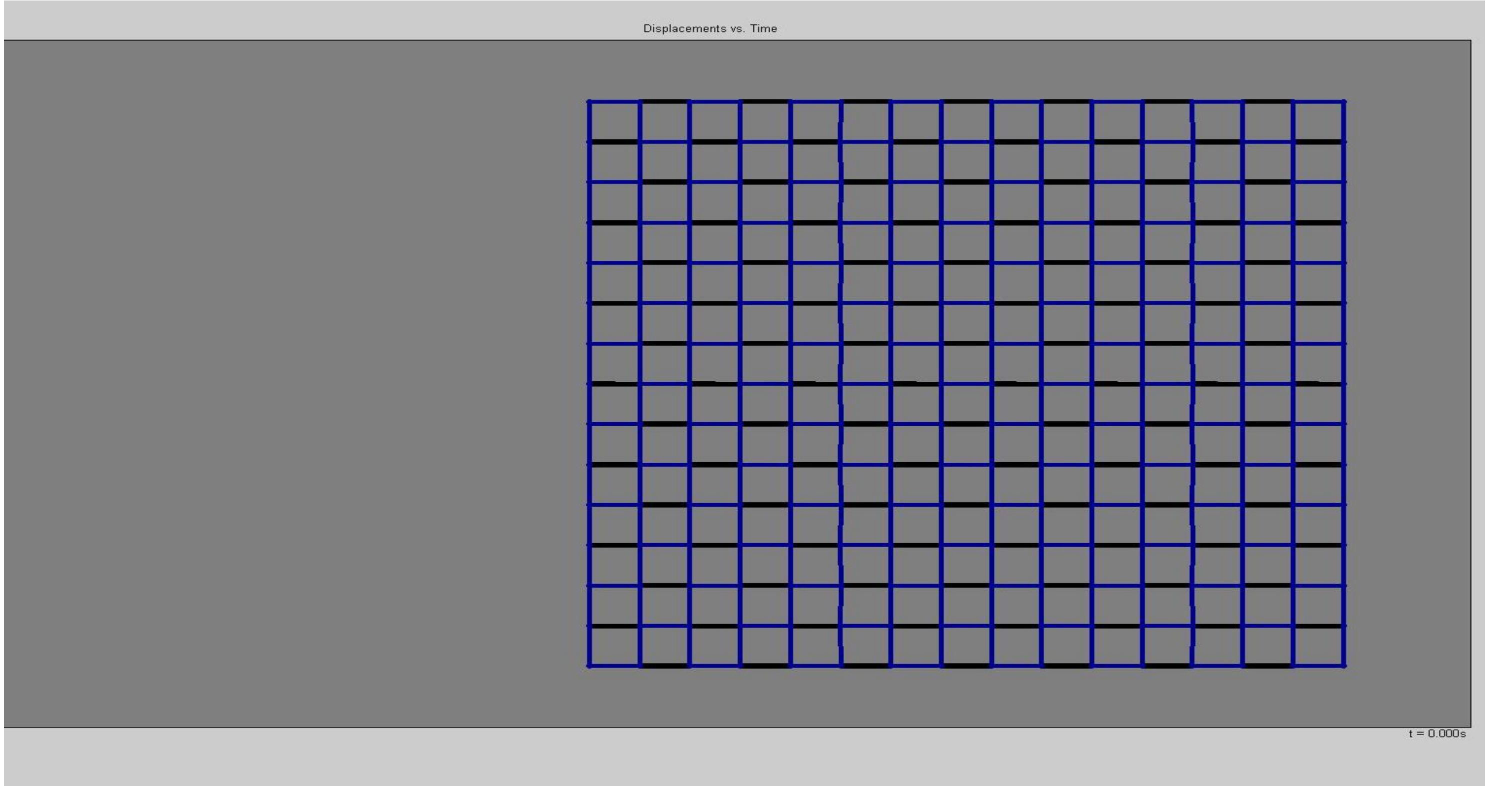


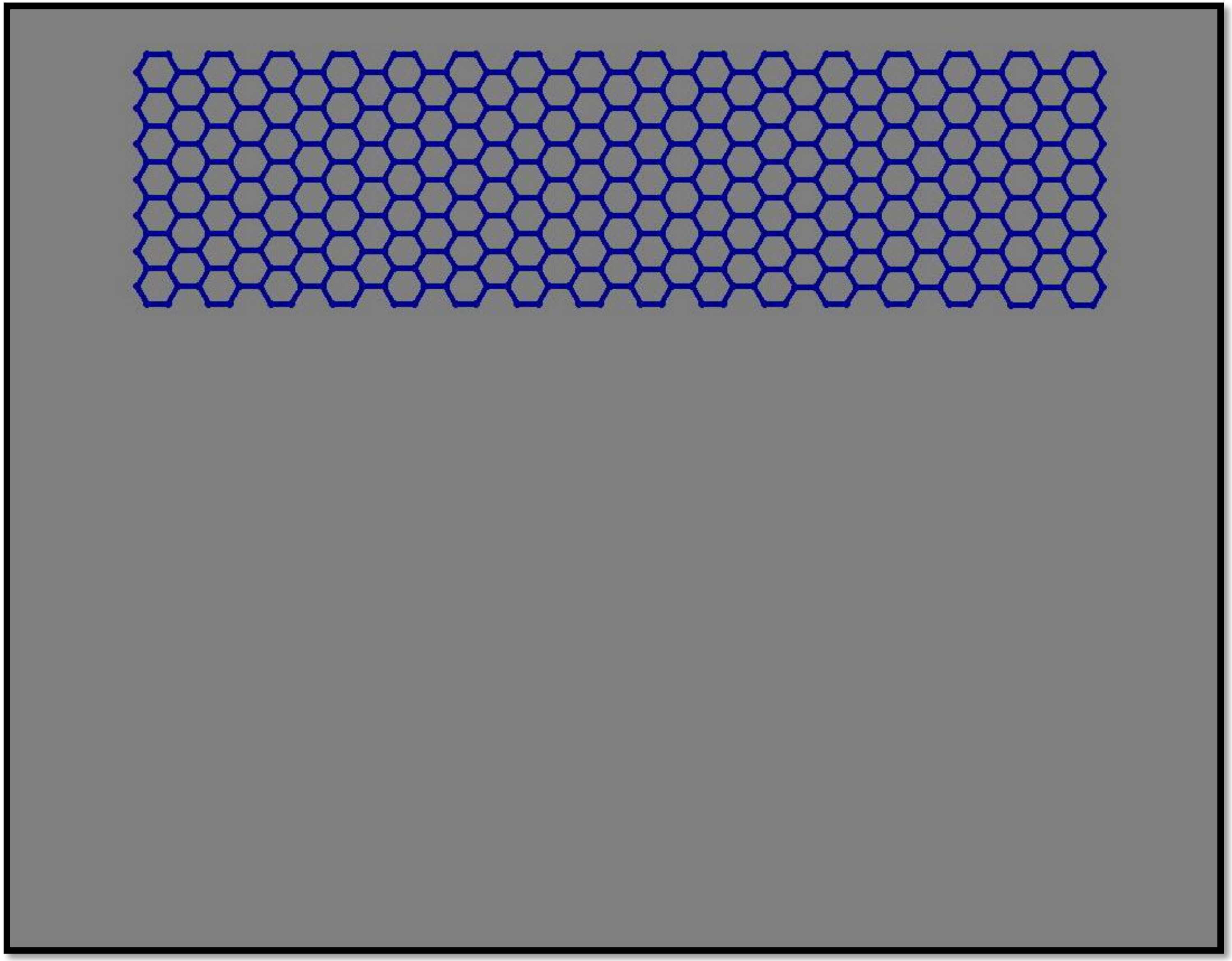
Energy profile of particle pair,
with contact



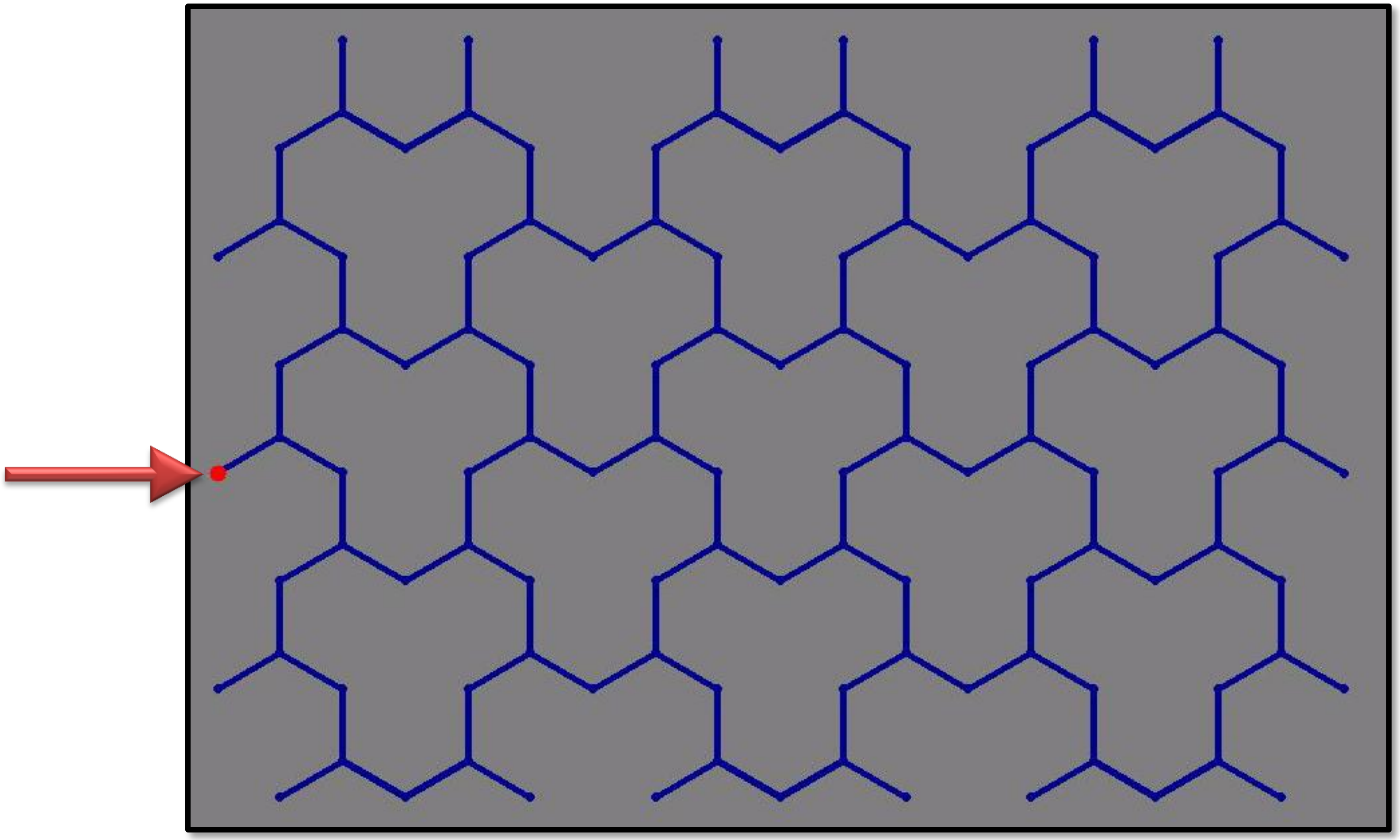
Expansion through bistability



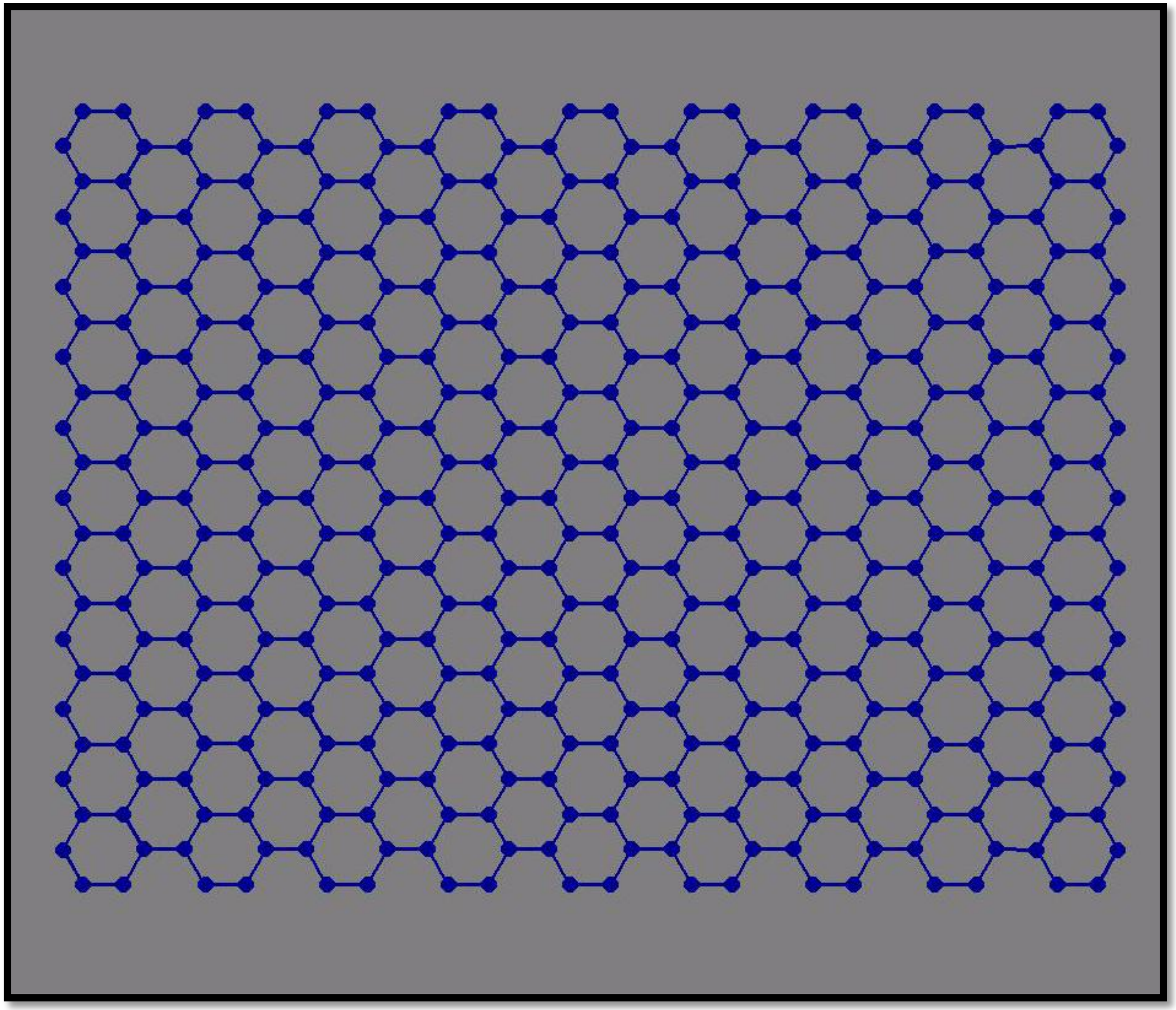




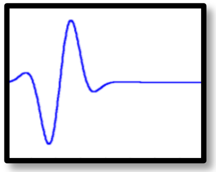
Attractive Magnetic Forces

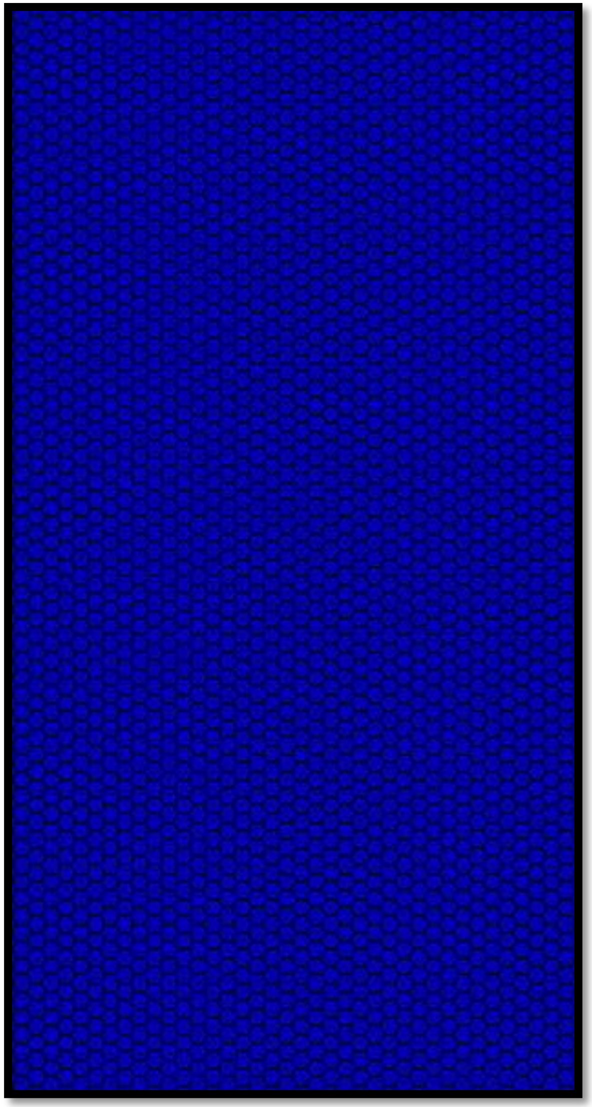


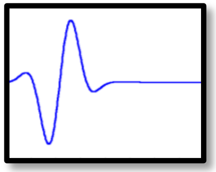
2D System Reconfiguration

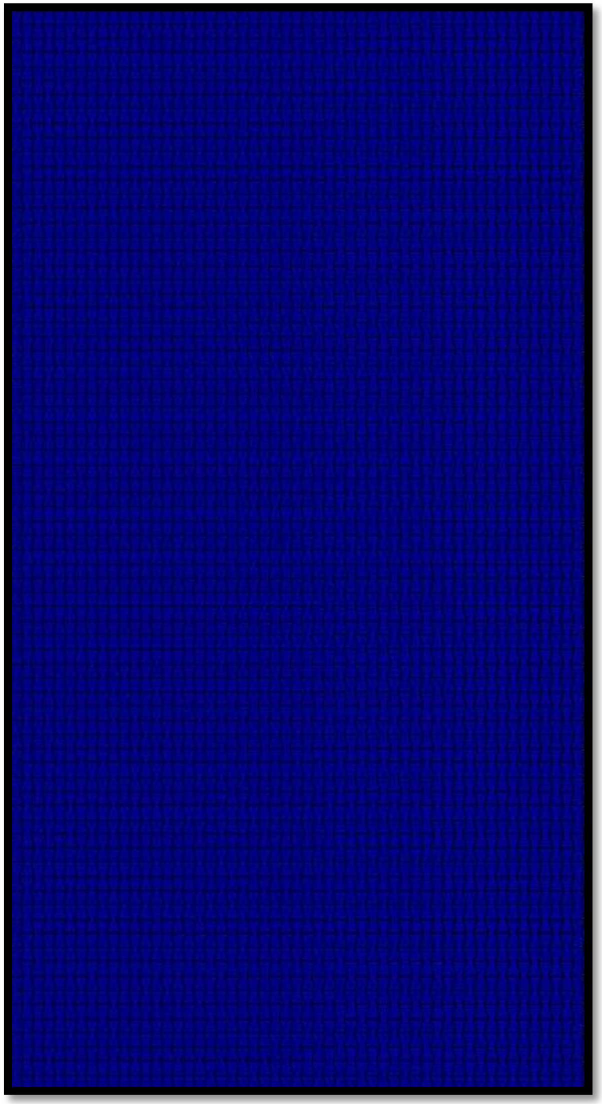


- Hexagonal Lattice
- Re-entrant Lattice

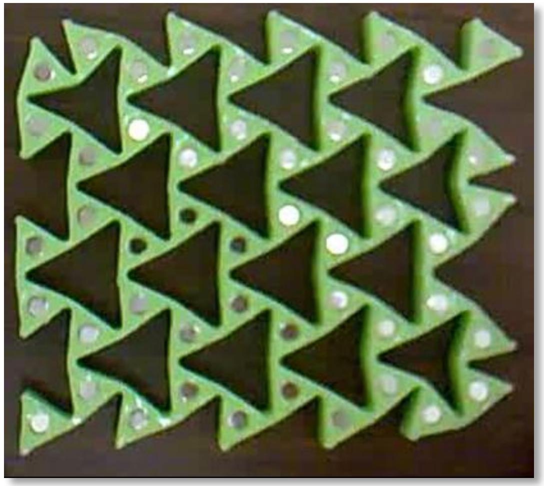


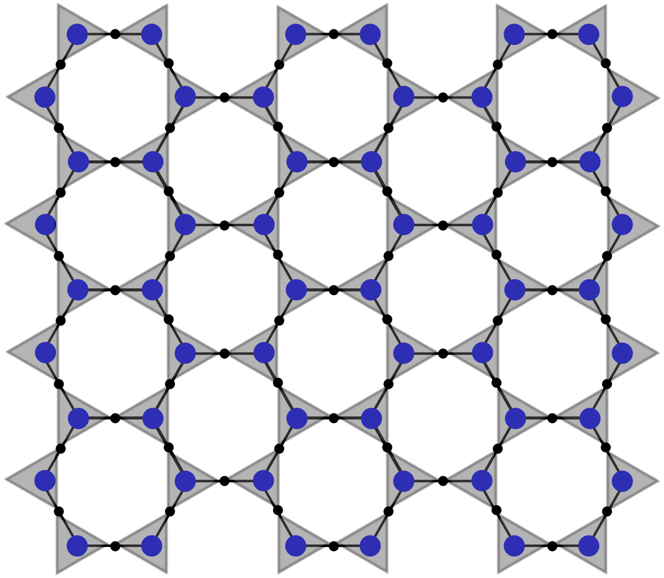




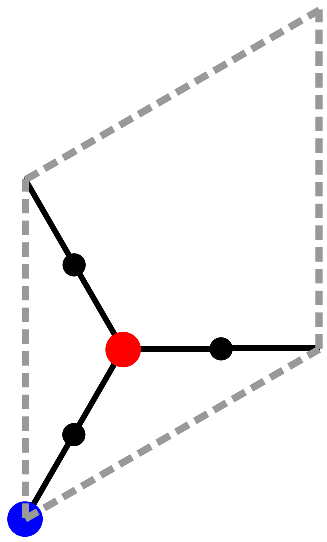


- Lumped parameter model

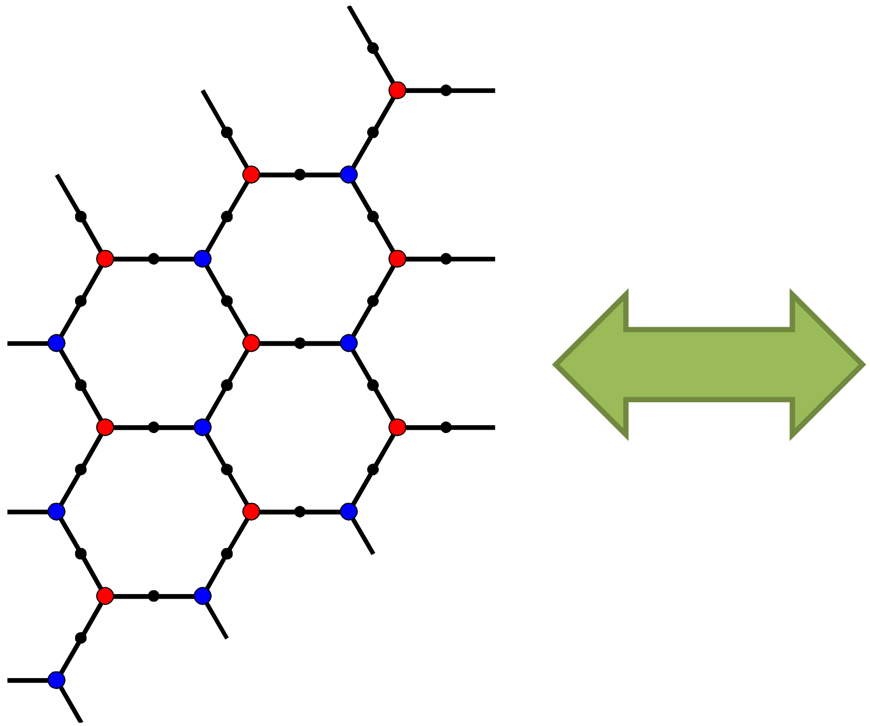




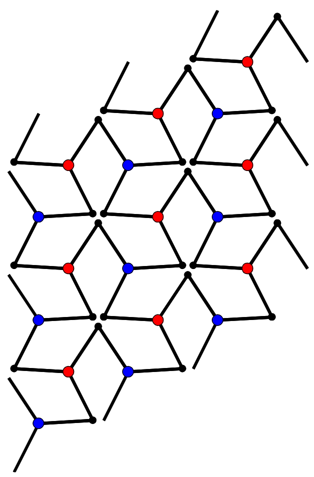
Unit Cell



Open



Closed



- Linearized magnetic forces

$$\bar{\boldsymbol{f}}^{(m)} = \boldsymbol{f}_0^{(m)} - 4\frac{f^{(m)}}{r_0^5}(r - r_0)\hat{\boldsymbol{n}} \qquad \theta_b = \frac{y_j' - y_i'}{r_{ij}}$$

- EOM for a Unit Cell

$$\boldsymbol{M}\ddot{\boldsymbol{q}}_{n,m} + \sum_{n=-N}^{N}\sum_{m=-M}^{M}\left(\boldsymbol{K}_{n,m}^{(e)} + \boldsymbol{K}_{n,m}^{(m)}\right)\boldsymbol{q}_{n,m} = \boldsymbol{0}$$

- Plane wave solution

$$\boldsymbol{q}_{n,m} = \boldsymbol{q}_0 e^{i(\omega t + \boldsymbol{\kappa}\cdot(n\boldsymbol{d}_1 + m\boldsymbol{d}_2))}$$

- Linear EVP

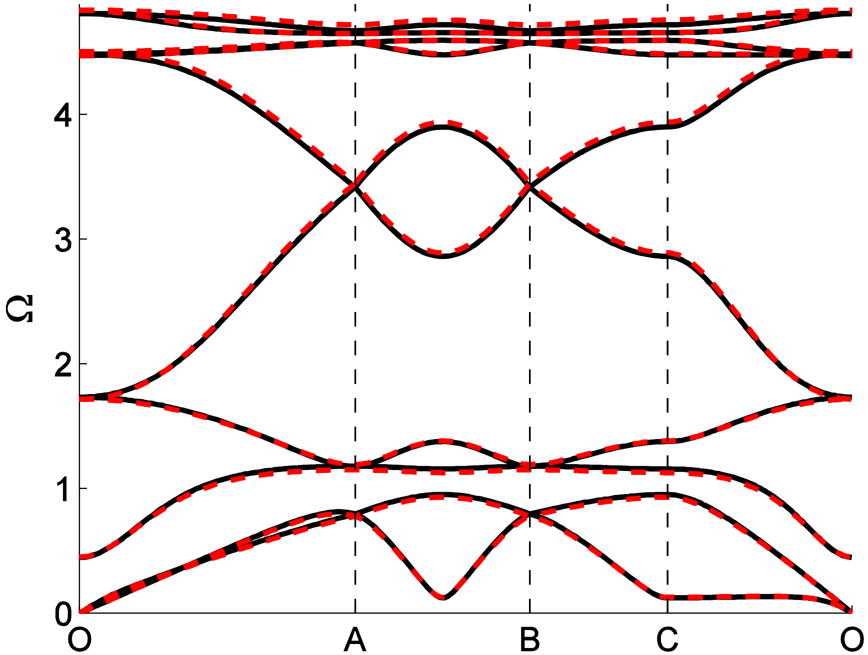
$$\left[-\omega^2\boldsymbol{M} + \sum_{n,m}\left(\boldsymbol{K}_{n,m}e^{i\boldsymbol{\kappa}\cdot(n\boldsymbol{d}_1 + m\boldsymbol{d}_2)}\right)\right]\boldsymbol{q}_0 = \boldsymbol{0}$$

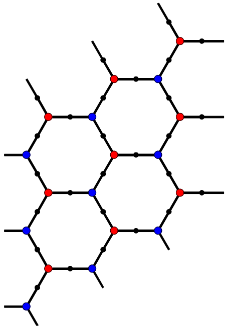
Non-local interactions

$$\left(\boldsymbol{K}(\boldsymbol{\kappa}) - \omega^2\boldsymbol{M}\right)\boldsymbol{q}_0 = \boldsymbol{0} \qquad \boldsymbol{K}(\boldsymbol{\kappa}) = \sum_{n,m}\left(\boldsymbol{K}_{n,m}e^{i\boldsymbol{\kappa}\cdot(n\boldsymbol{d}_1 + m\boldsymbol{d}_2)}\right)$$

Effect of Reconfiguration on Band Structure

Open Configuration

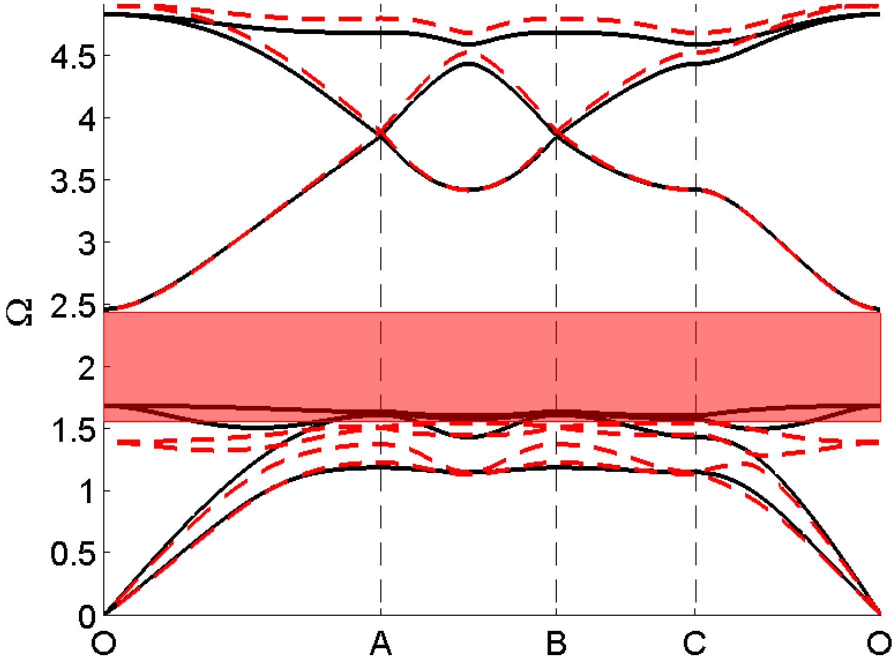


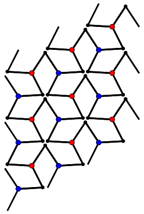


Black $f^{(m)} = 0$

Red (--) $f^{(m)} = 0.2$

Closed Configuration





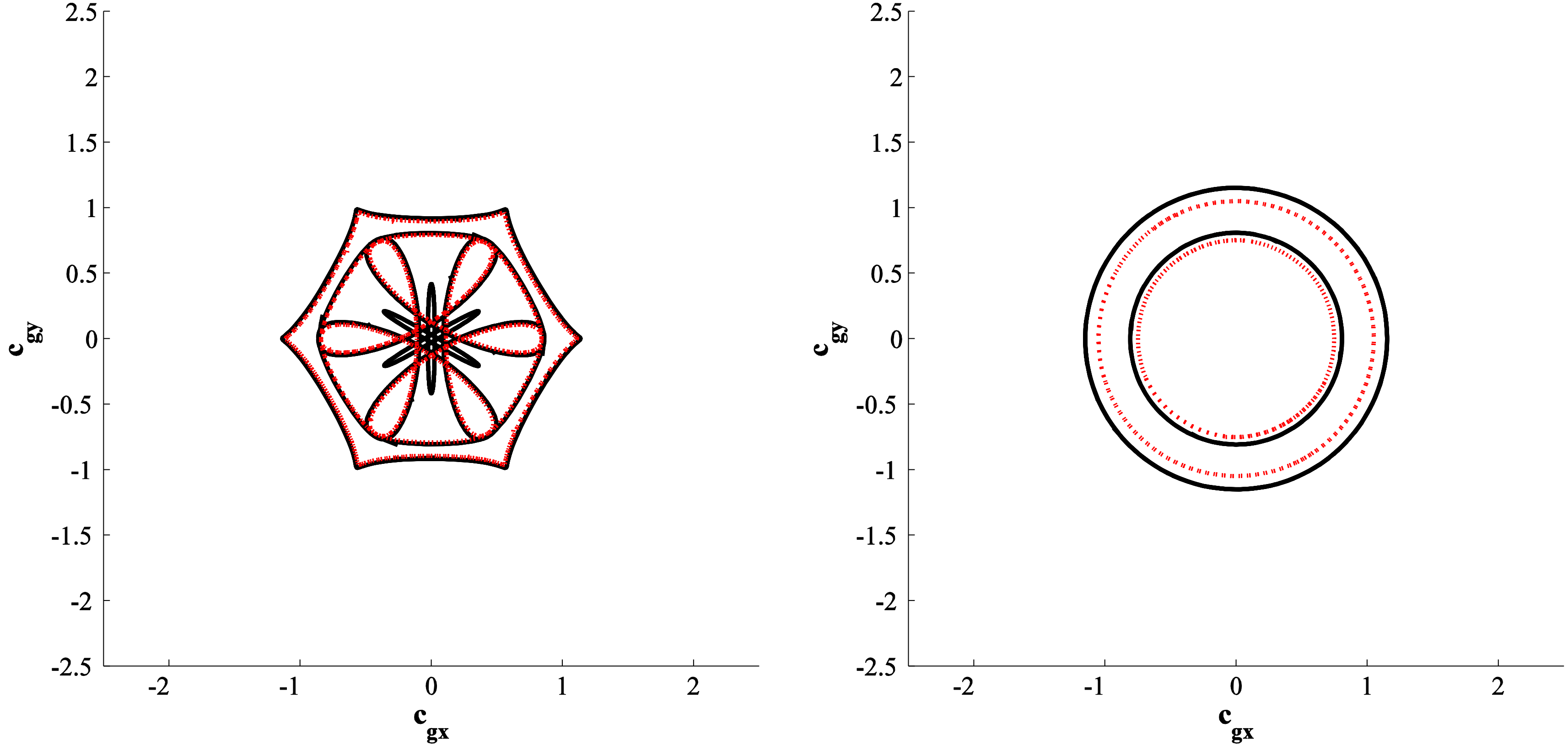
Black $f^{(m)} = 0.025$

Red (--) $f^{(m)} = 0.125$

Effect of Reconfiguration on Group Velocity

$$\Omega = 0.127$$

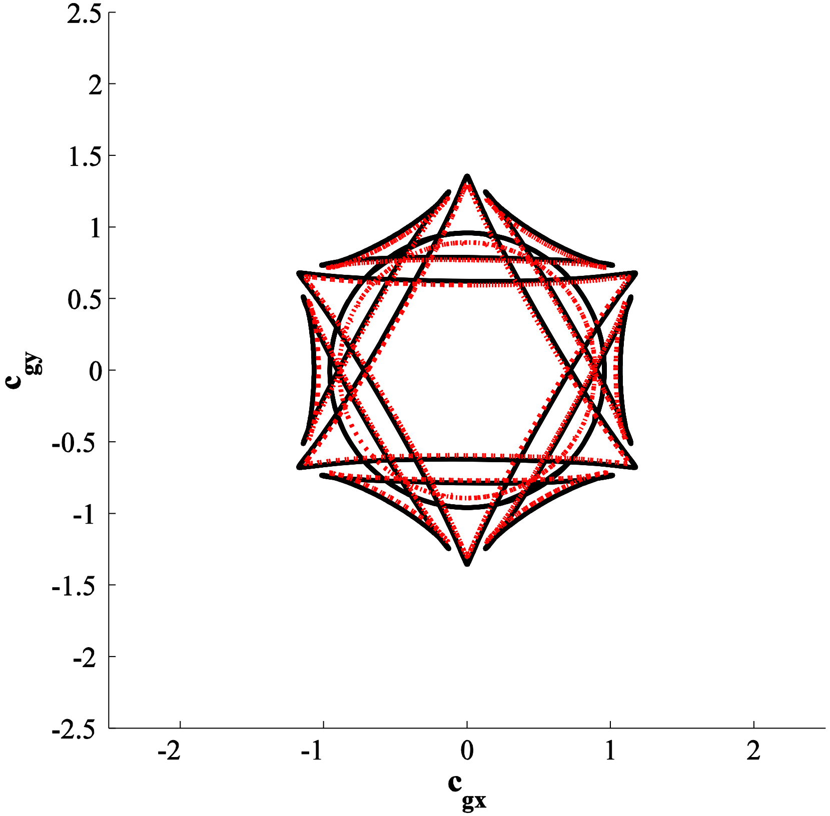
Open **Closed**



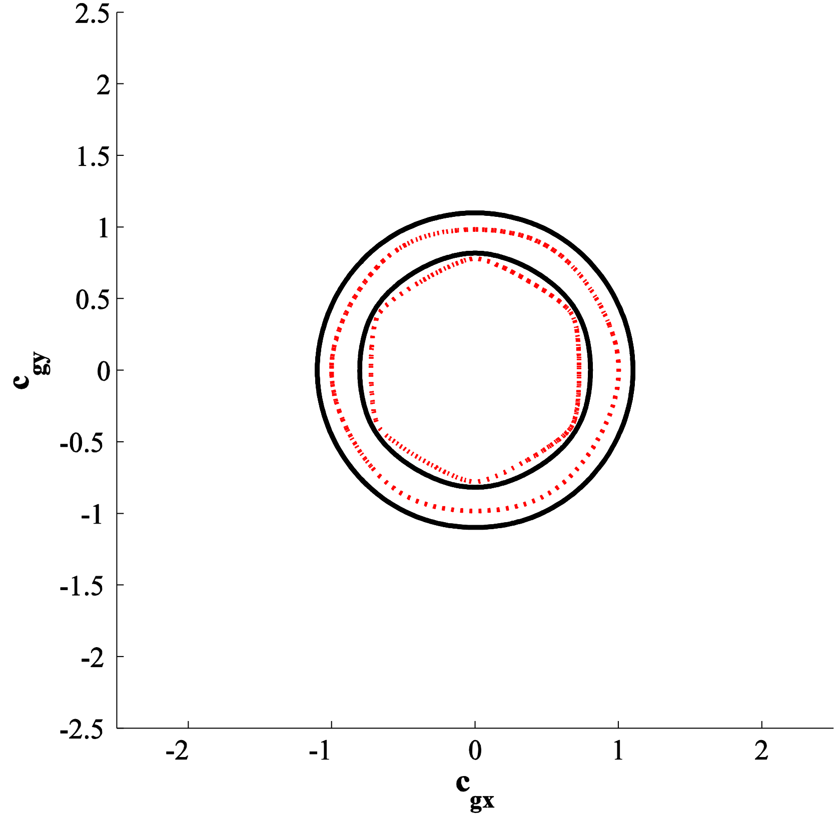
Effect of Reconfiguration on Group Velocity

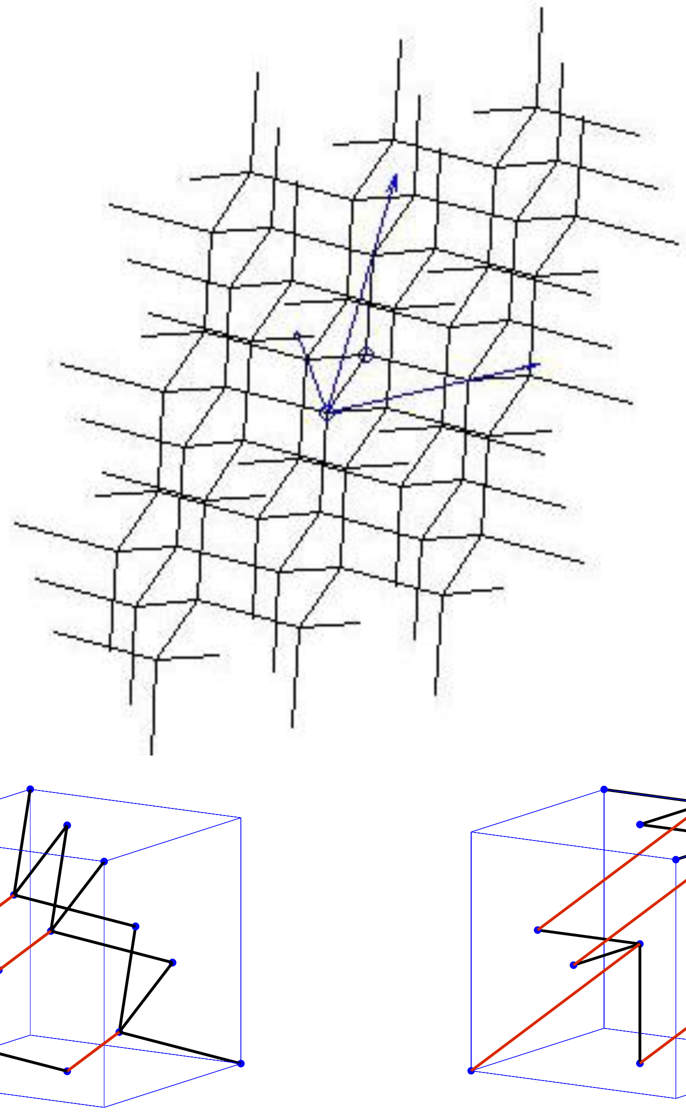
$$\Omega = 0.5$$

Open



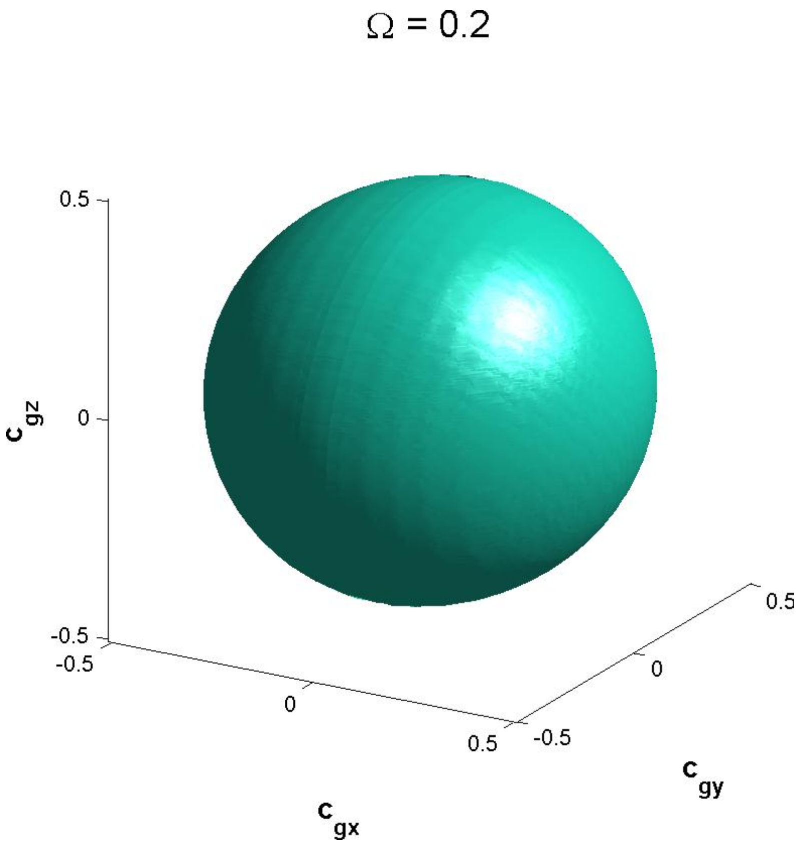
Closed



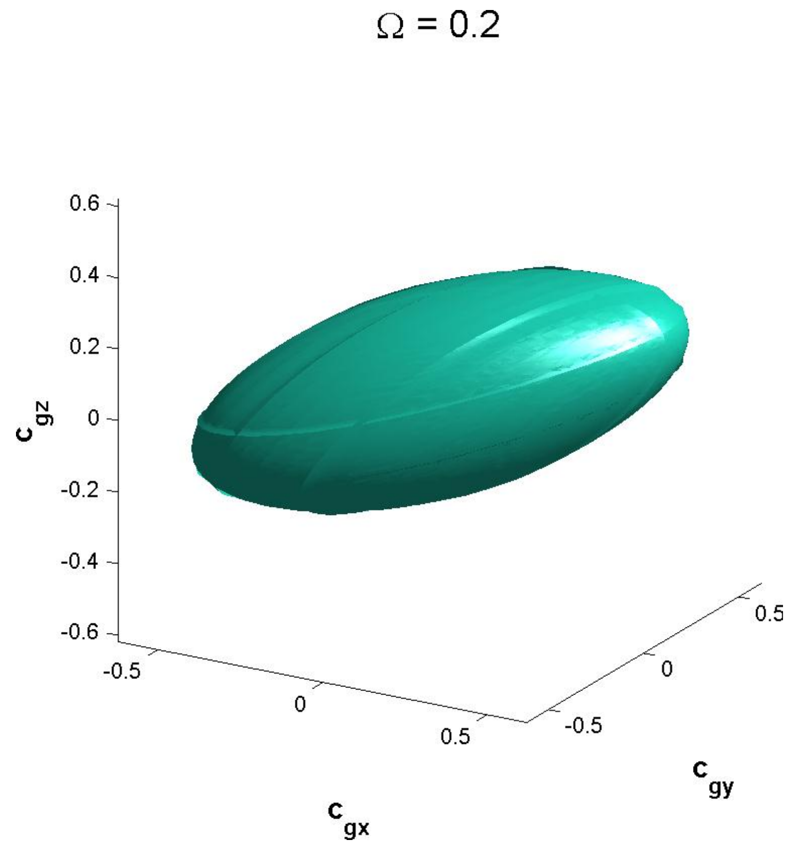


Krödel, S., Delpero, T., Bergamini, A., Ermanni, P., & Kochmann, D. M. (2013). 3D Auxetic Microlattices with Independently Controllable Acoustic Band Gaps and Quasi-Static Elastic Moduli. Advanced Engineering Materials, 15(9999).

Open



Re-entrant



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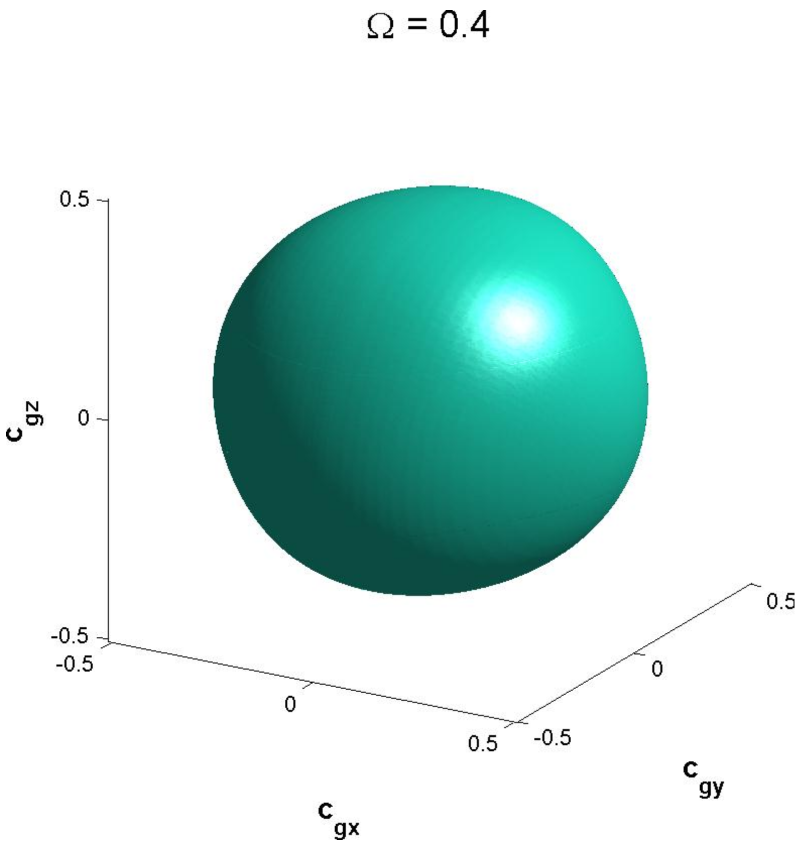
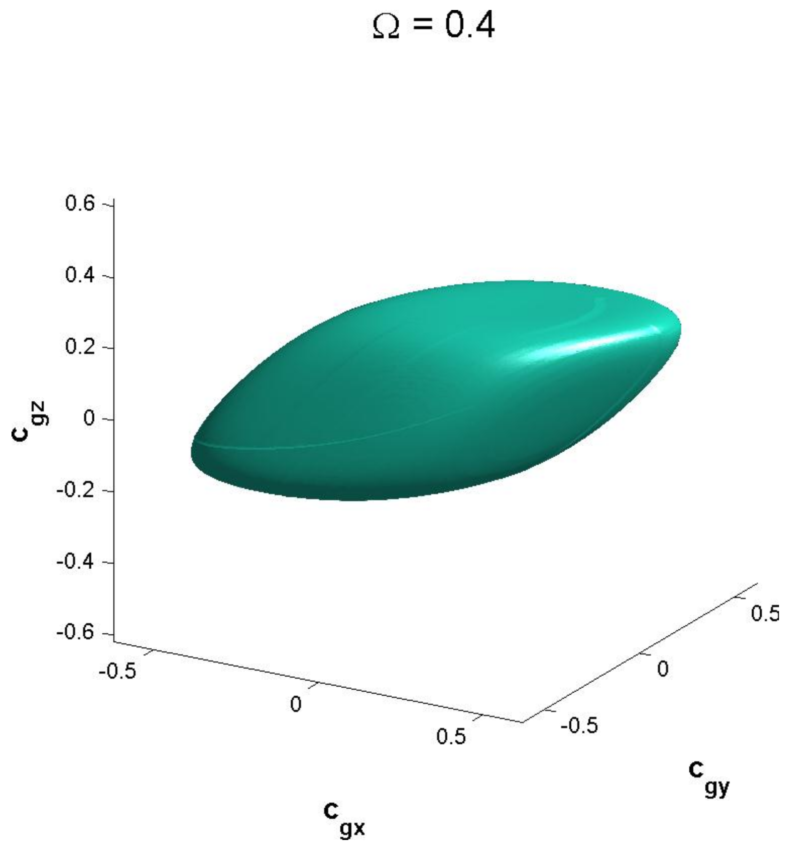


Re-entrant



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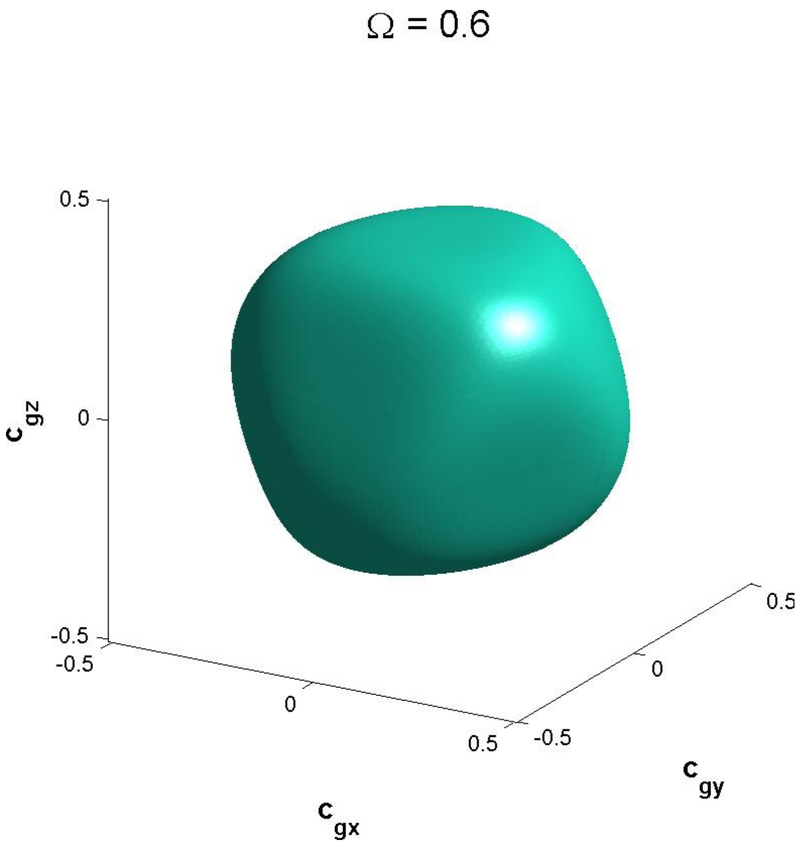
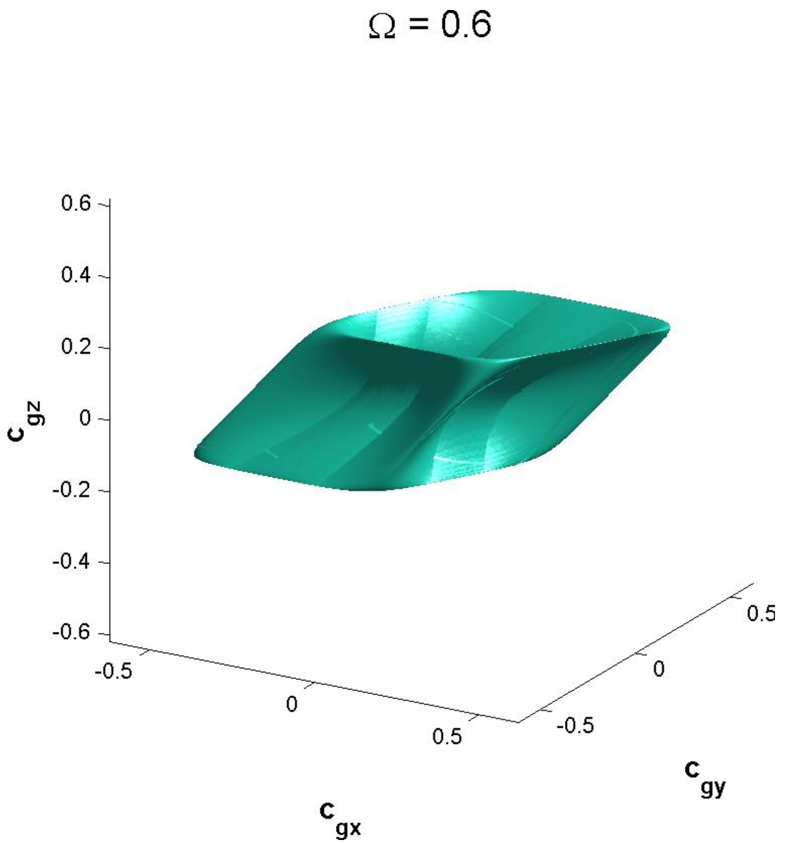
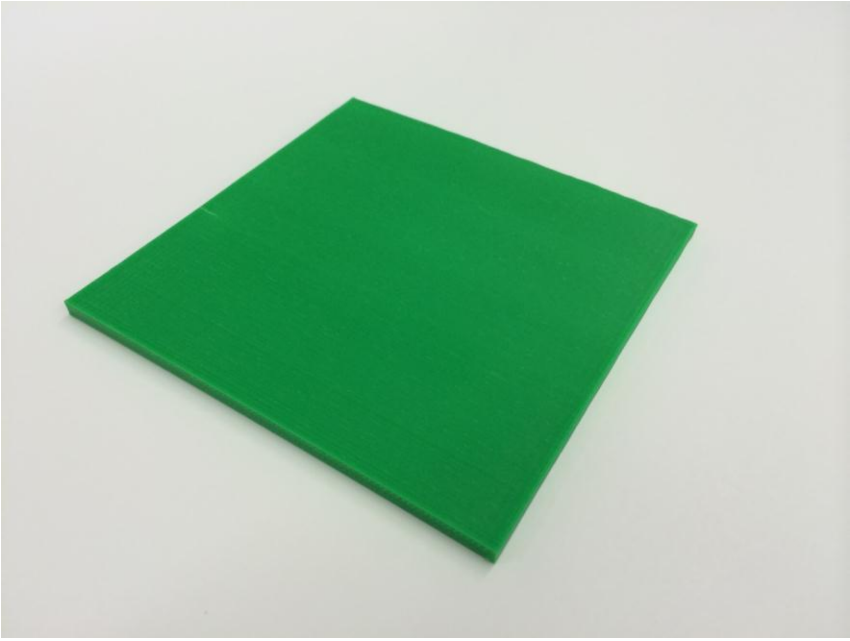


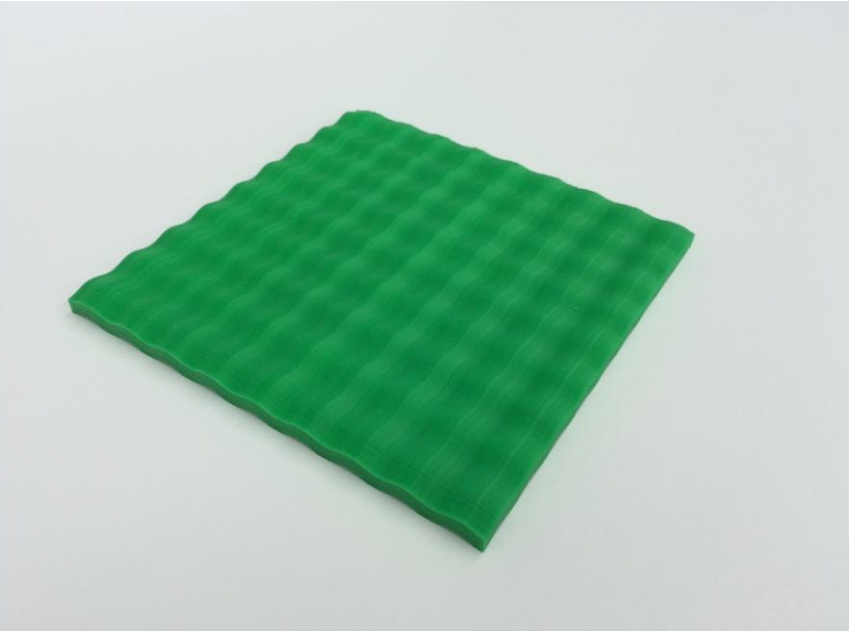
Re-entrant



Flat plate

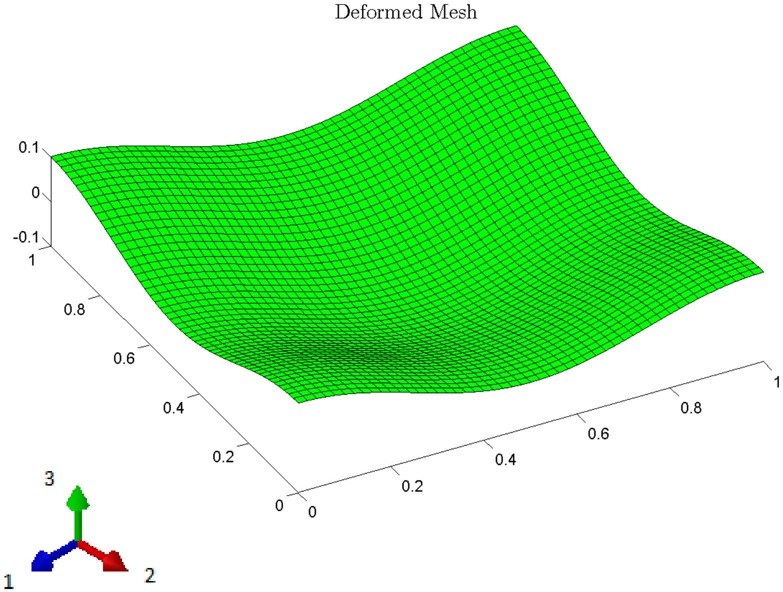
Curved plate

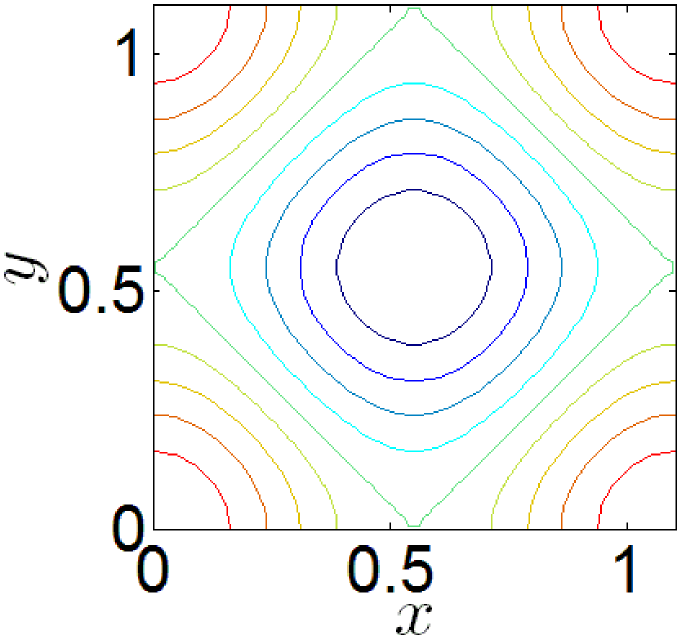


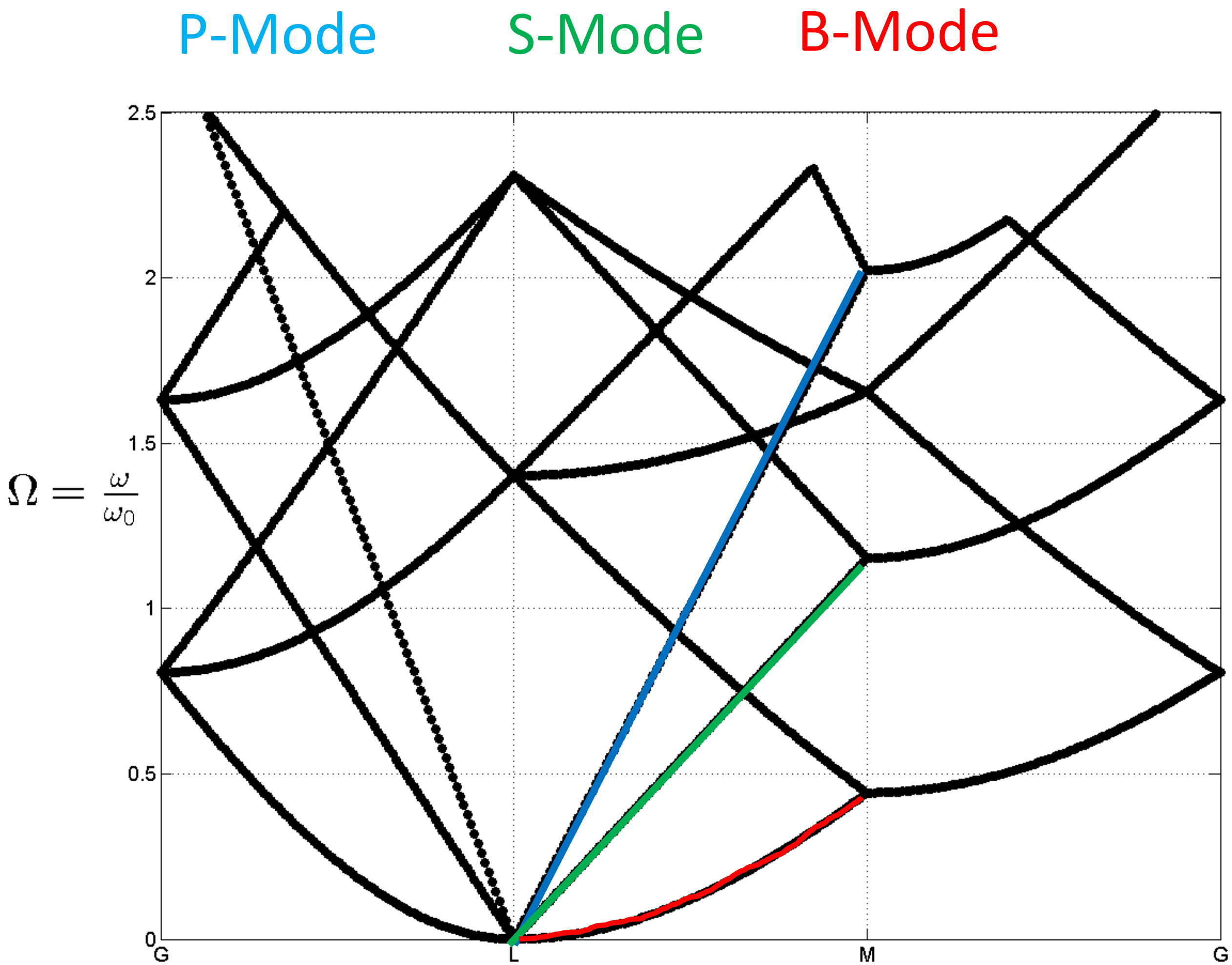


Plate

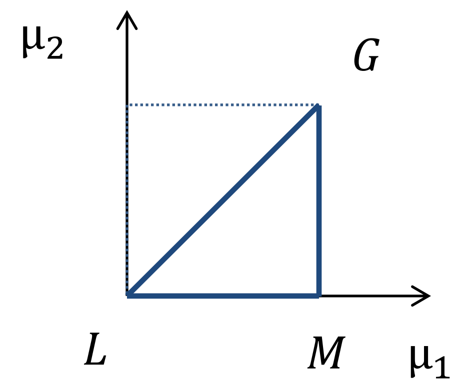
$$|x_{3,max}| = \eta L$$

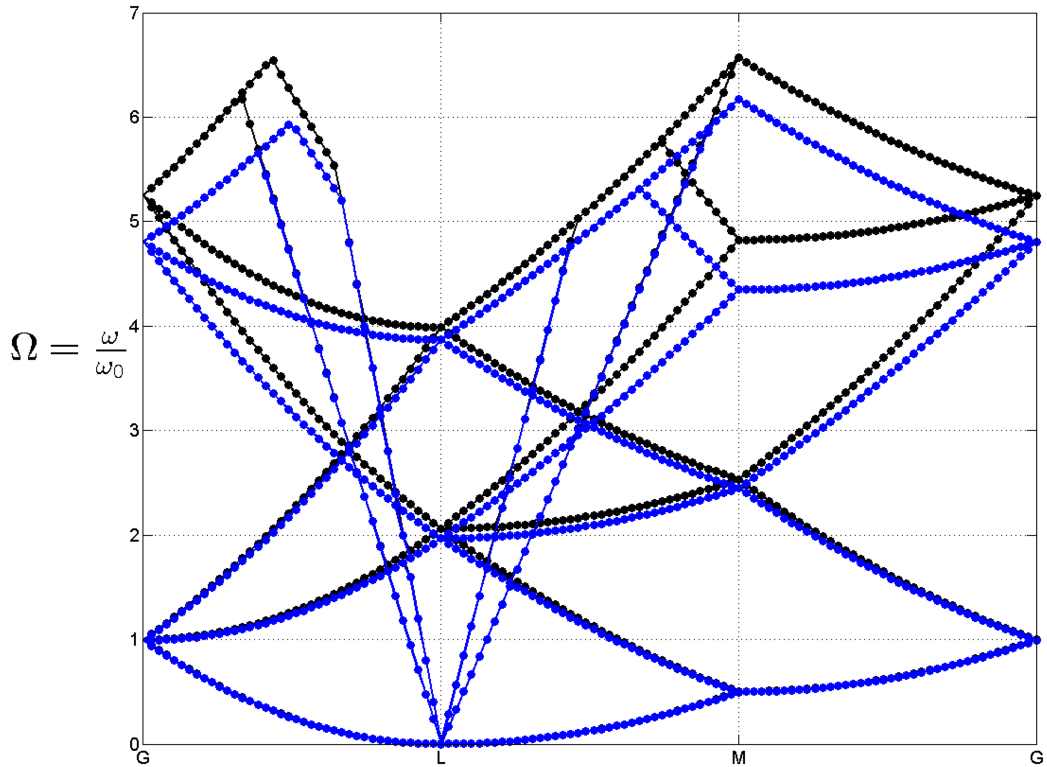




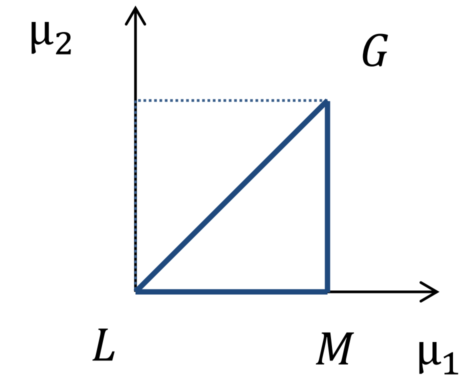

P-Mode
S-Mode
B-Mode
2.5
2
1.5
1
0.5
0
$\Omega = \frac{\omega}{\omega_0}$
G
L
M
G

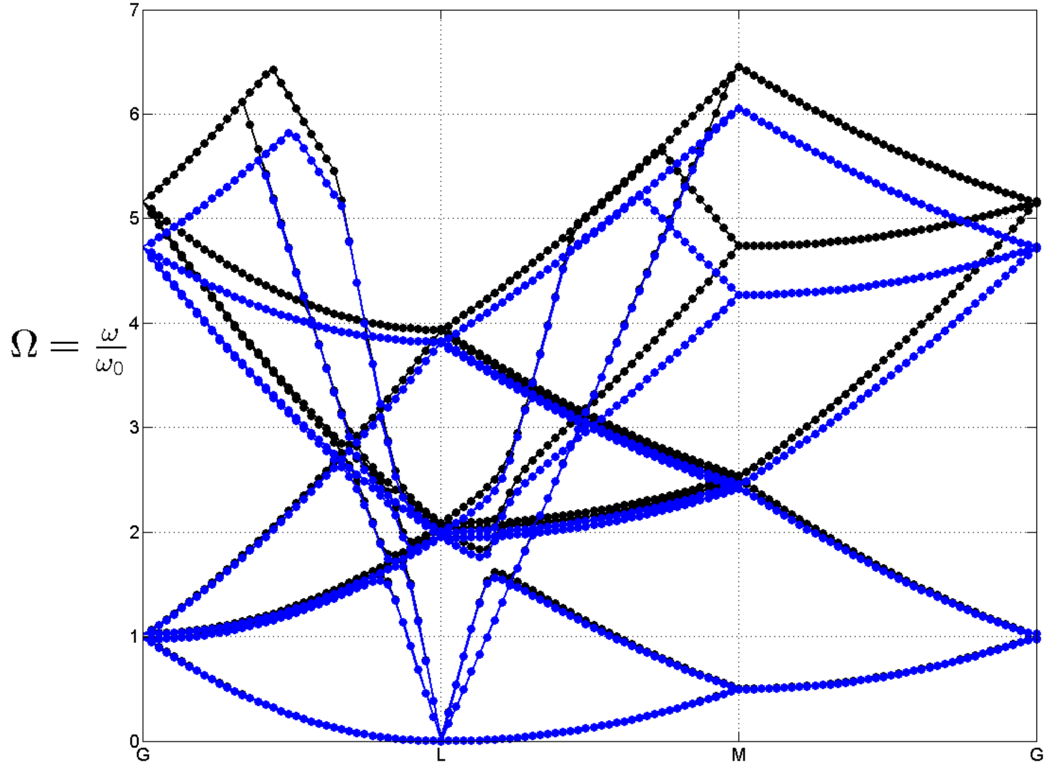
- Band diagram, flat plate
  - Black line: 100 elements
  - Blue line: 2500 elements



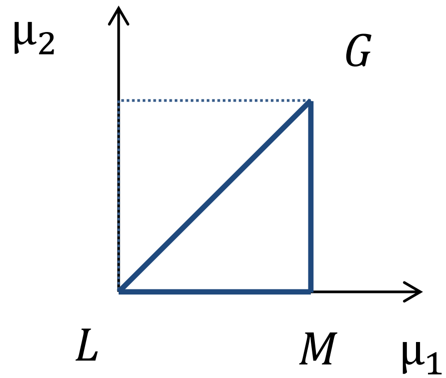


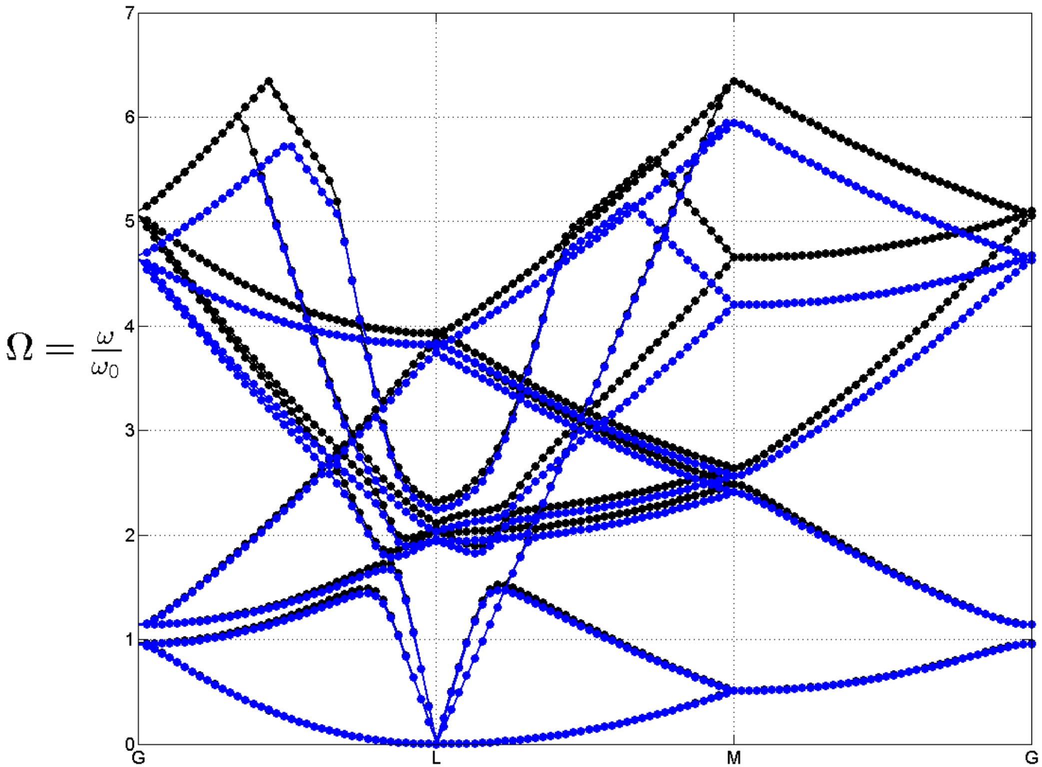
- Band diagram, $\alpha = 0°, \eta = 0.01$
  - Black line: 100 elements
  - Blue line: 2500 elements



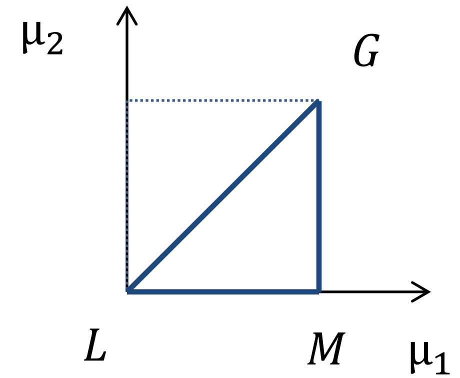


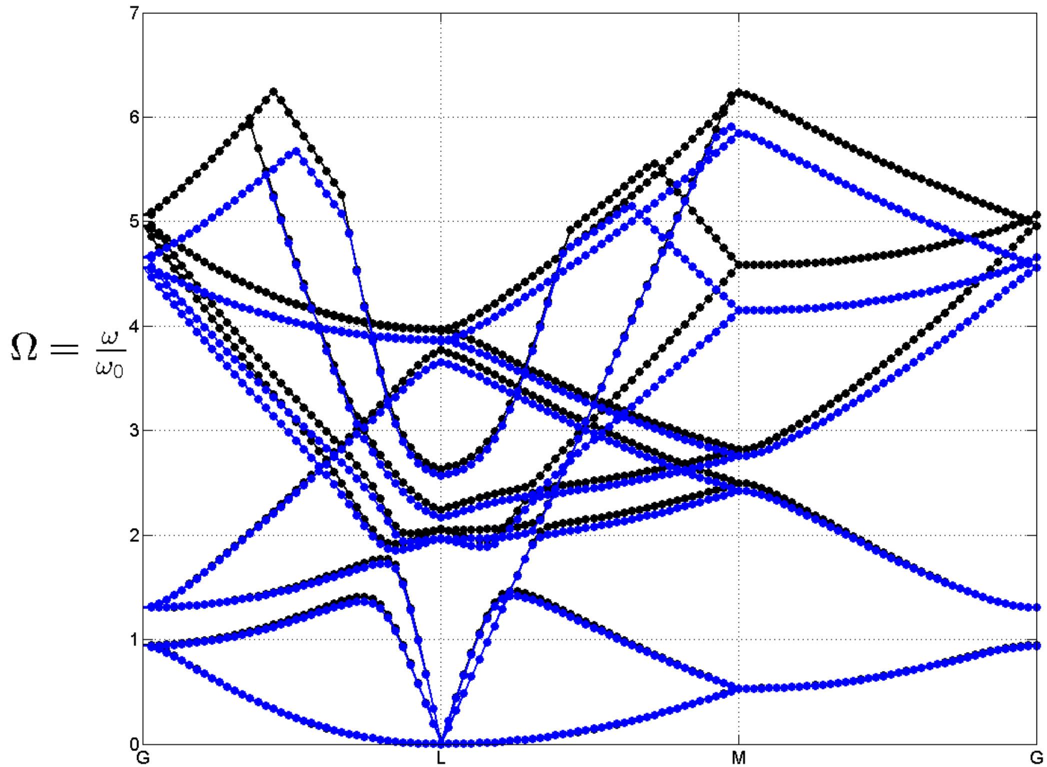
- Band diagram, $\alpha = 0°, \eta = 0.02$
  - Black line: 100 elements
  - Blue line: 2500 elements



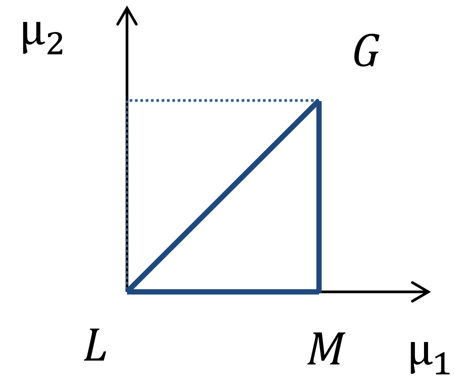


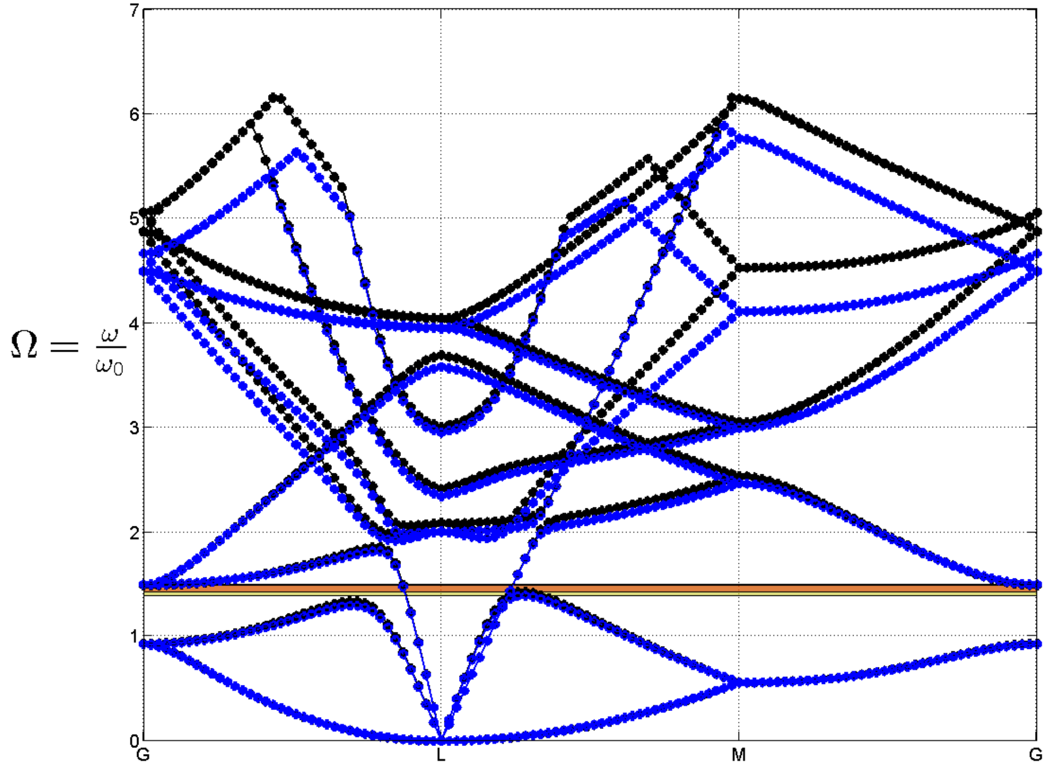
- Band diagram, $\alpha = 0°, \eta = 0.03$
  - Black line: 100 elements
  - Blue line: 2500 elements



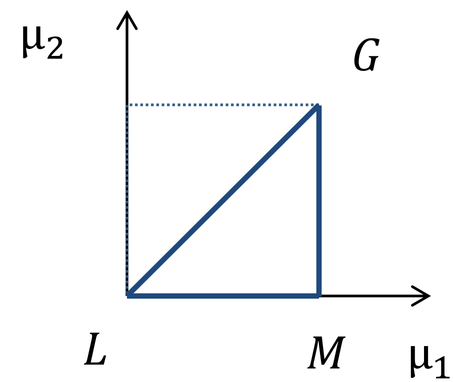


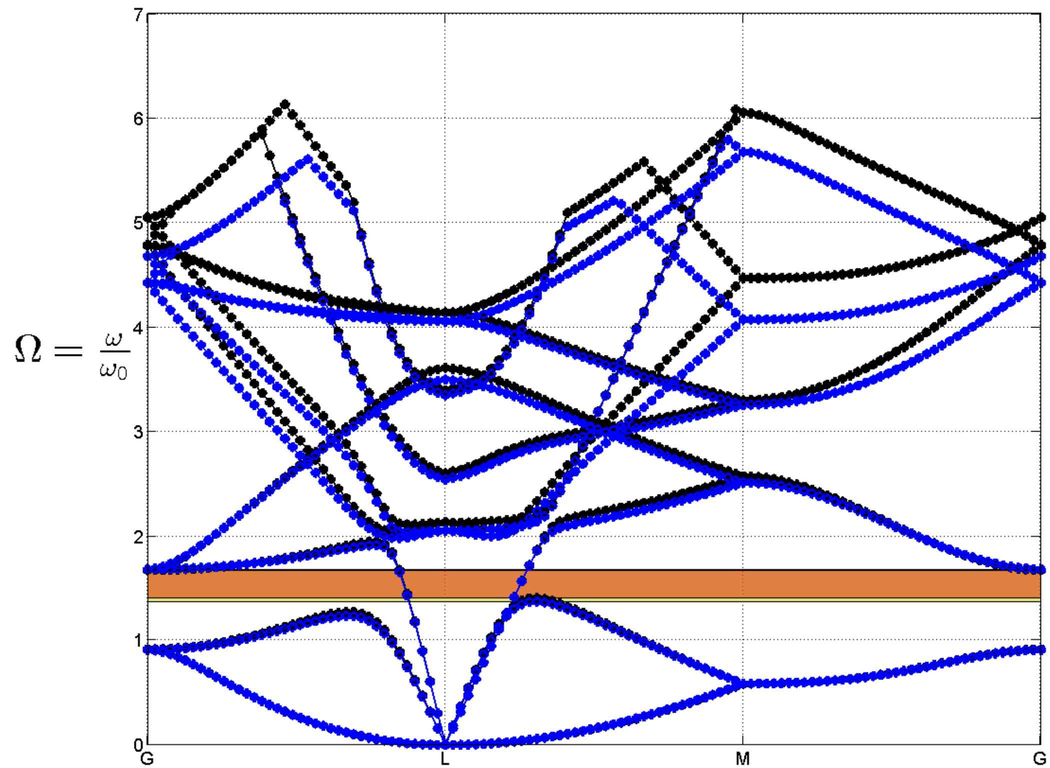
- Band diagram, $\alpha = 0°, \eta = 0.04$
  - Black line: 100 elements
  - Blue line: 2500 elements



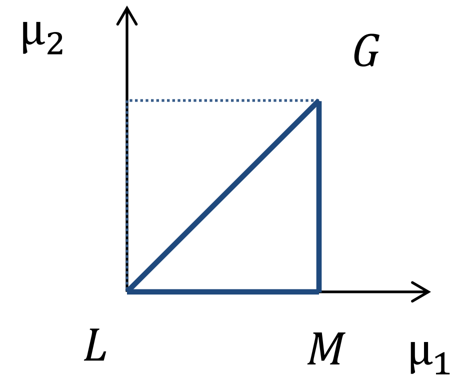


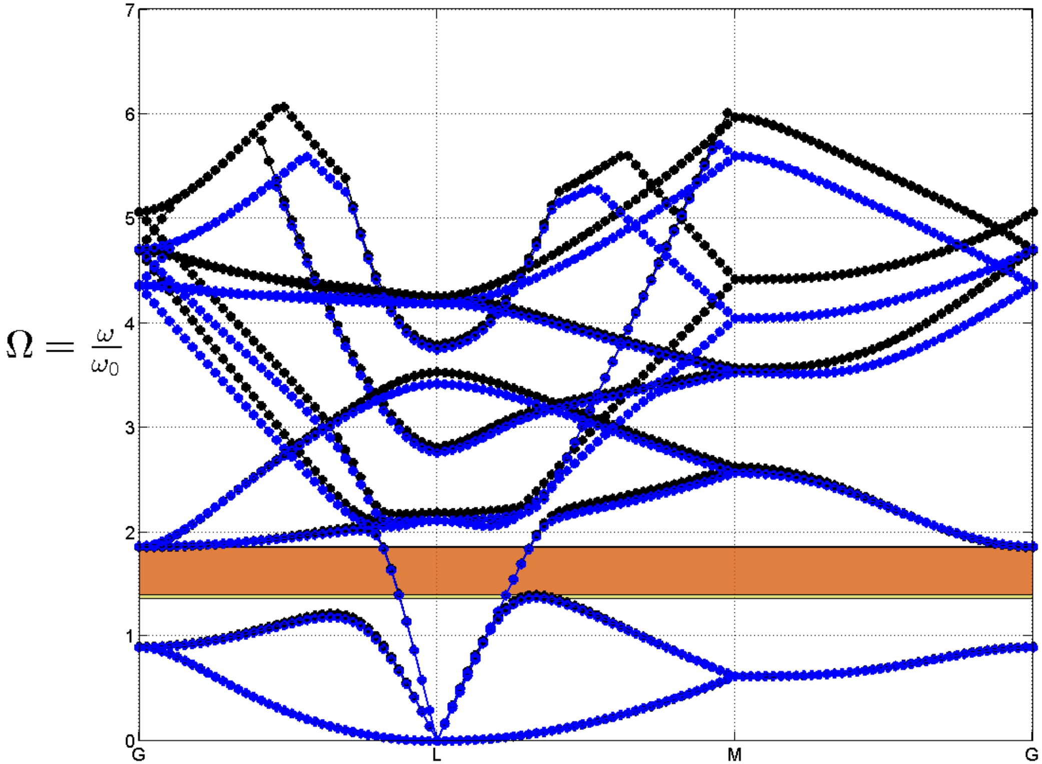
- Band diagram, $\alpha = 0°, \eta = 0.05$
  - Black line: 100 elements
  - Blue line: 2500 elements





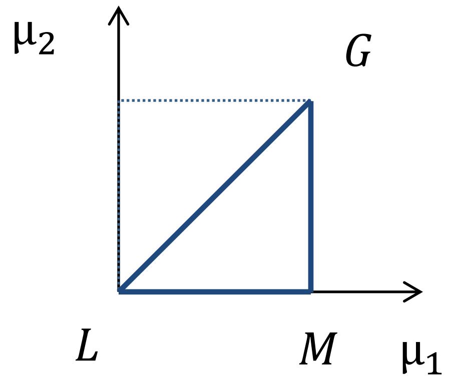
- Band diagram, $\alpha = 0°, \eta = 0.06$
  - Black line: 100 elements
  - Blue line: 2500 elements

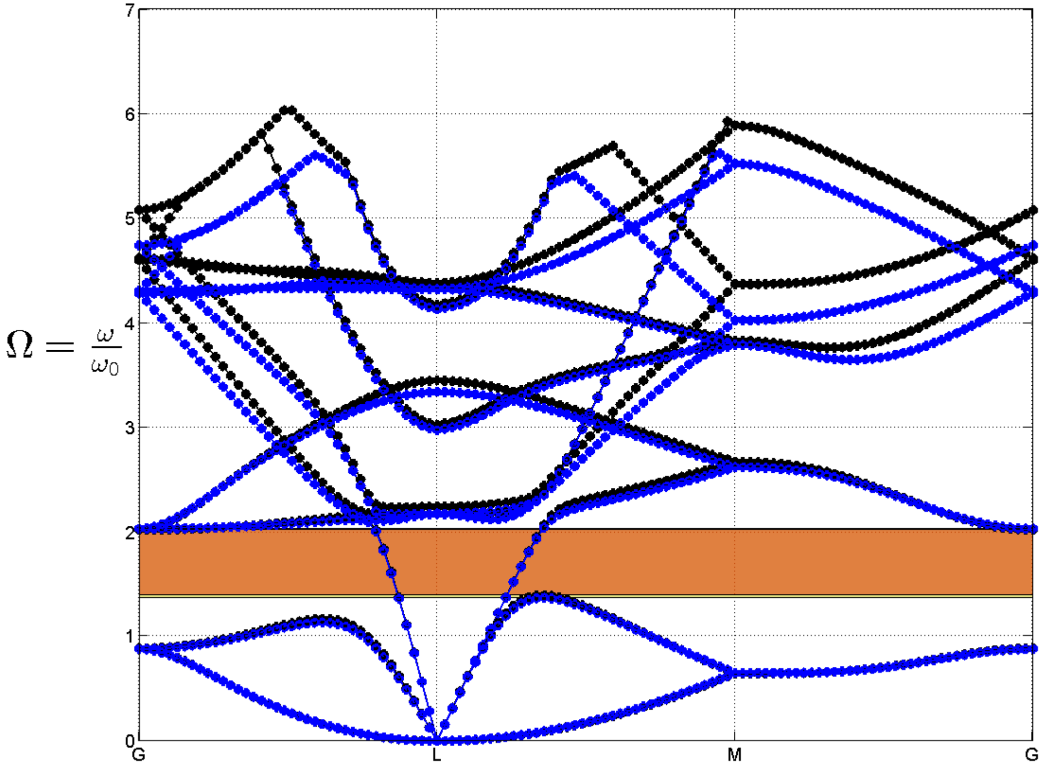




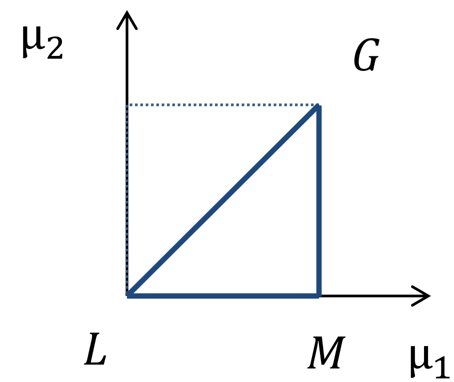
Plate

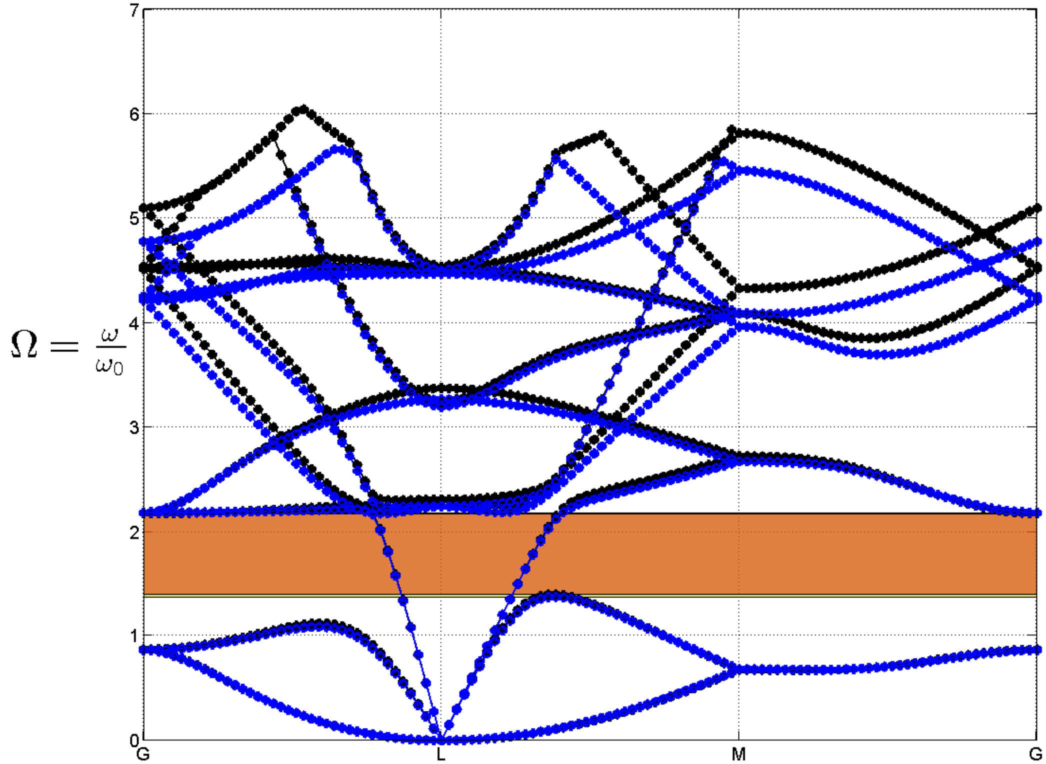
- Band diagram, $\alpha = 0°, \eta = 0.07$
  - Black line: 100 elements
  - Blue line: 2500 elements



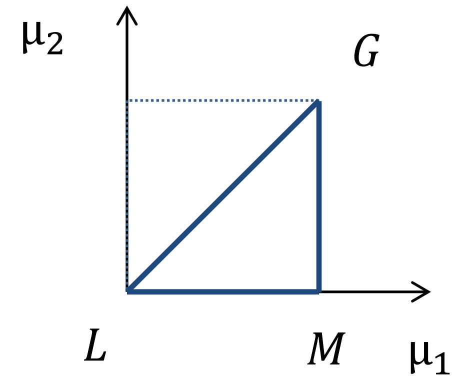


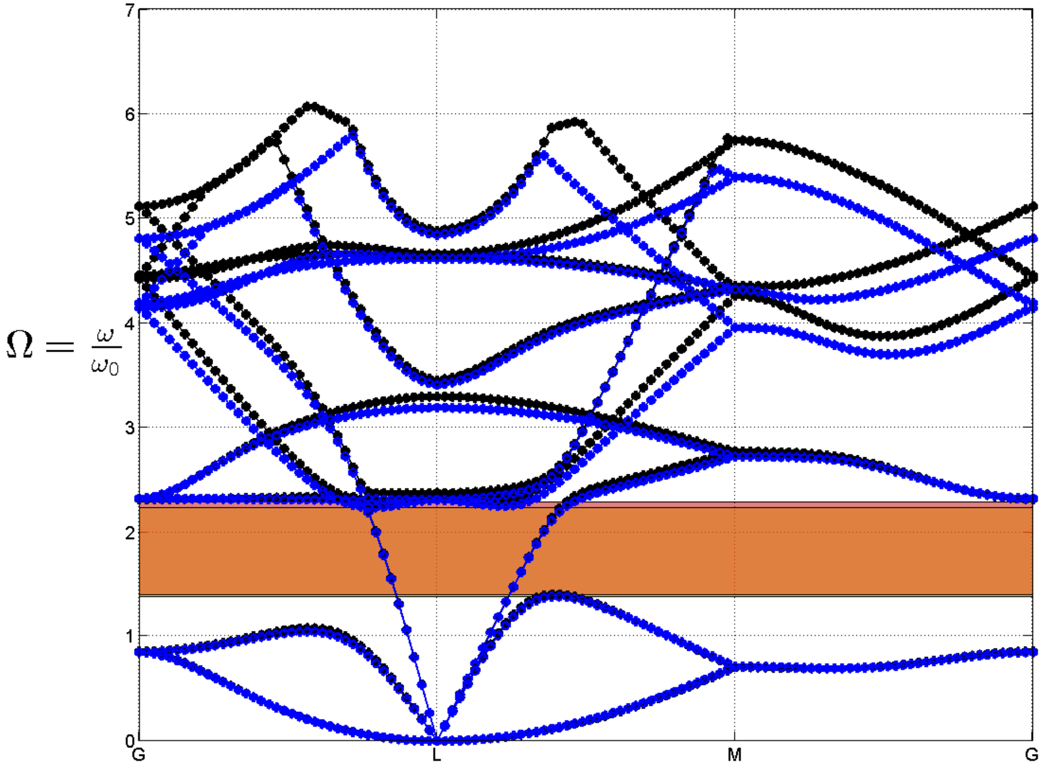
- Band diagram, $\alpha = 0°, \eta = 0.08$
  - Black line: 100 elements
  - Blue line: 2500 elements



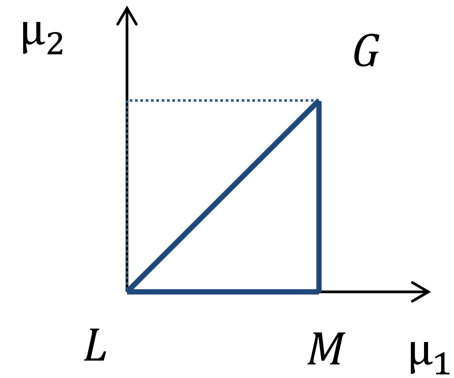


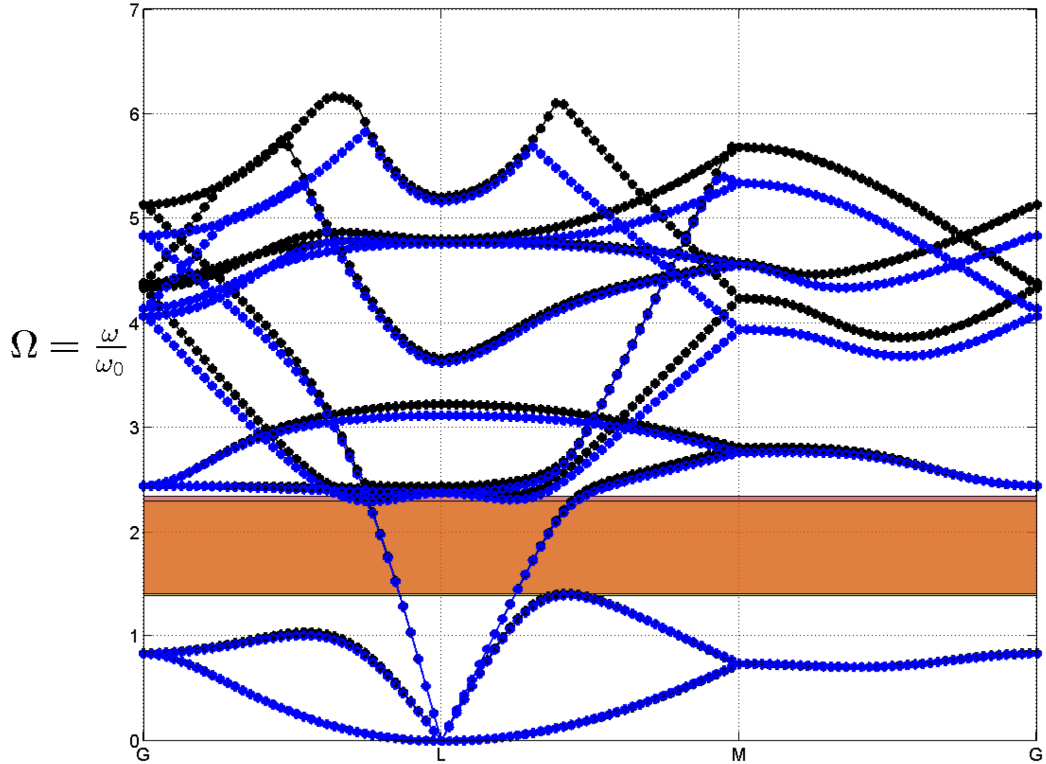
- Band diagram, $\alpha = 0°, \eta = 0.09$
  - Black line: 100 elements
  - Blue line: 2500 elements





- Band diagram, $\alpha = 0°, \eta = 0.10$
  - Black line: 100 elements
  - Blue line: 2500 elements





Summary

- Adaptive wave properties are highly desirable in a variety of applications
- Adaptivity can be achieved from controlled topological changes associated with:
  - large deformations and
  - multifield interactions
- Magneto-elastic lattices provide a good framework for these studies
- Can a “material” concept be derived from such configurations?

Thank you