Harnessing geometric and material nonlinearities to design tunable phononic crystals and acoustic metamaterials

Katia Bertoldi, Pai Wang, Filippo Casadei, Sahab Babaee

School of Engineering and Applied Sciences, Harvard University

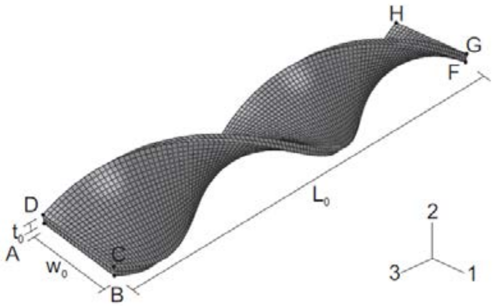
Atlanta, April 4, 2014

Bertoldi group

We combine theoretical, computational and experimental methods to gain deeper insight into the non-linear behavior of materials and structures.

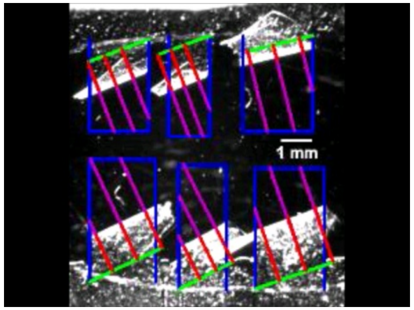
Non-linear response of materials

Dielectrics



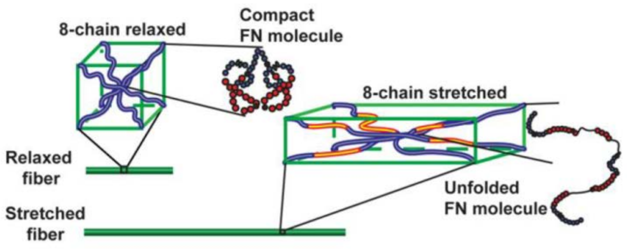
Henann, Chester, Bertoldi , JMPS 2013

Bio-hybrid materials



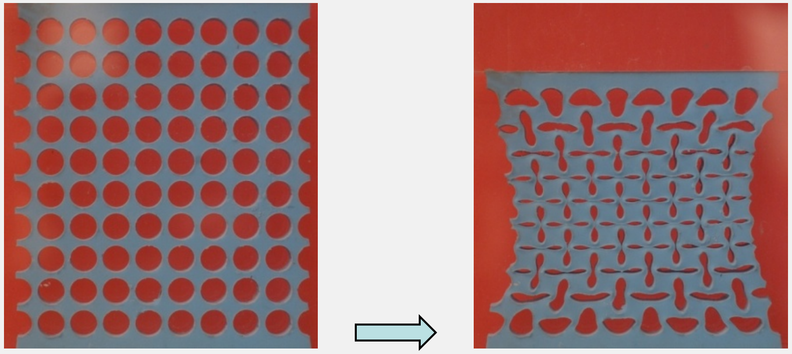
Shim, Grosberg, Nawroth, Parker, Bertoldi , J Biomechanics, 2012

Fibronectin nanotextiles

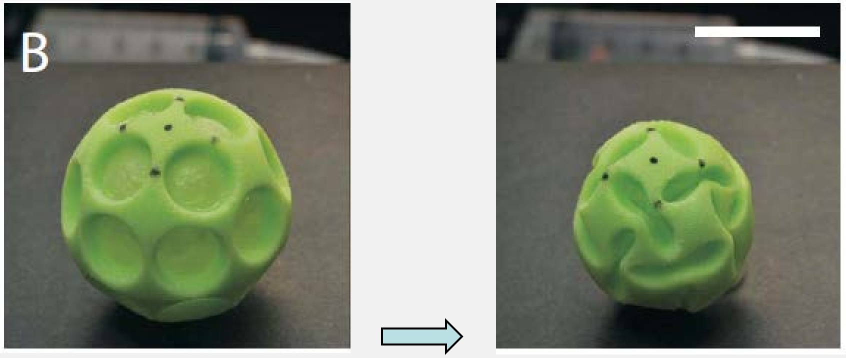


Deravi, Su, Paten, Ruberti, Bertoldi, Parker. NanoLetters, 2012

Non-linear response of structures



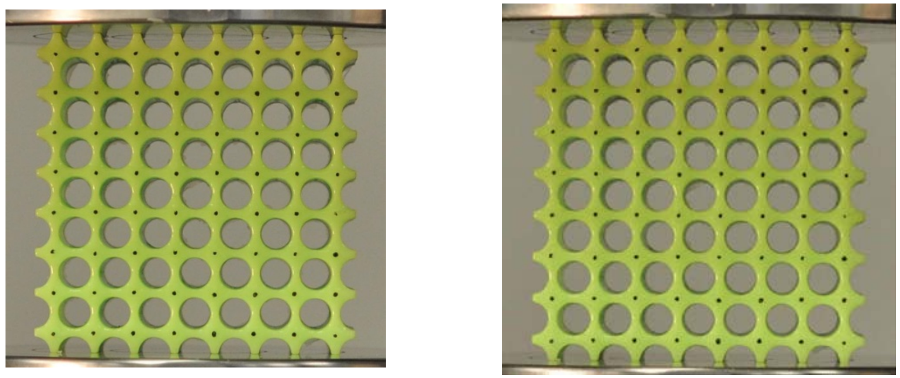
Deformation



Deformation

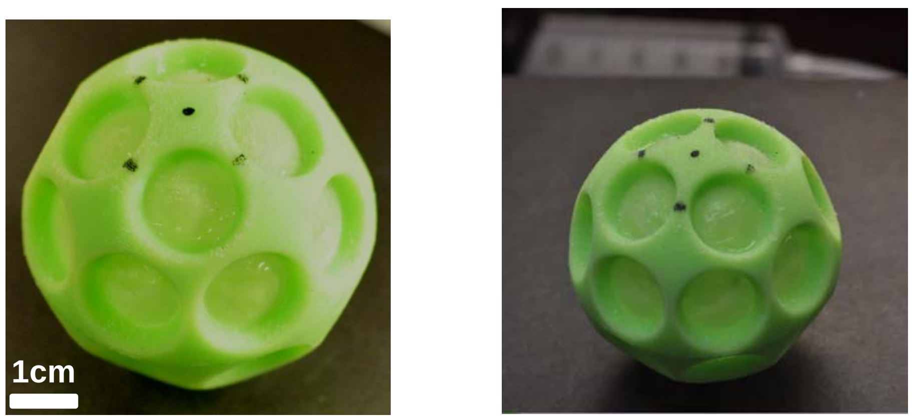
Adaptive Materials

Periodic structures with deliberately designed patterns



Mullin et al., Phys. Rev. Letters, 2007

Dramatic geometric rearrangements induced by instabilities



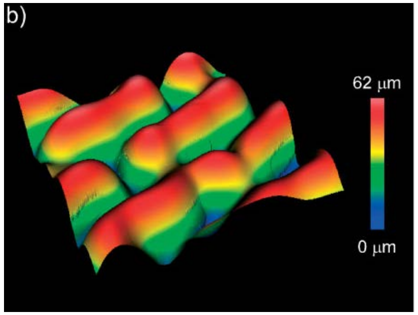
Shim et al., PNAS, 2012

….. can we exploit deformation and instabilities to tune and control the propagation of elastic waves?

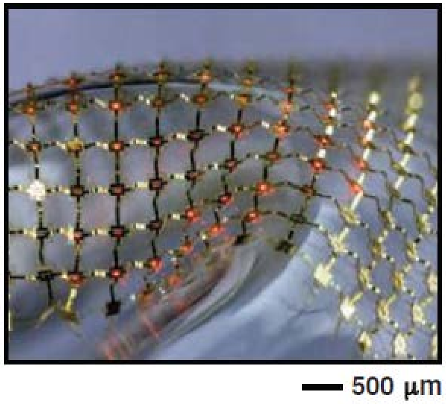
Large deformation & Instabilities

……traditionally we want to avoid them

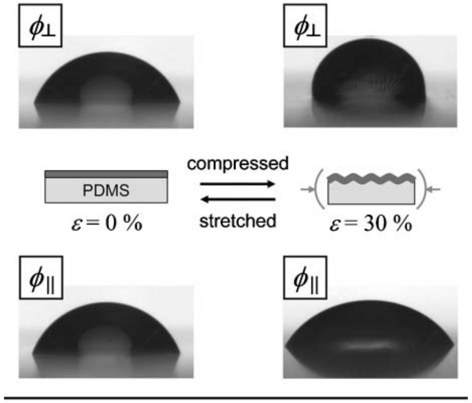
….but they can be exploited to
•control adhesion
•facilitate flexible electronics
•fabricate micro-fluidic structures
•control surface wettability



Chan et al., Adv. Mat, 2008



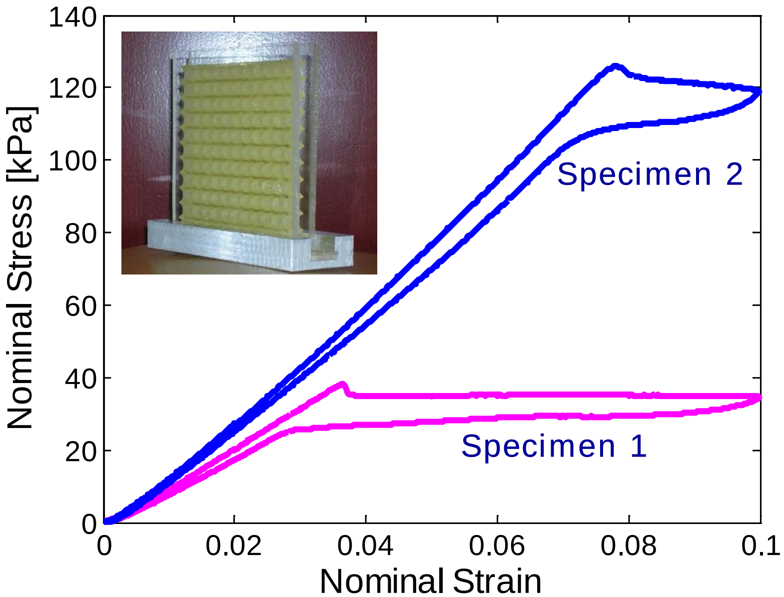
Rogers et al., Science, 2010

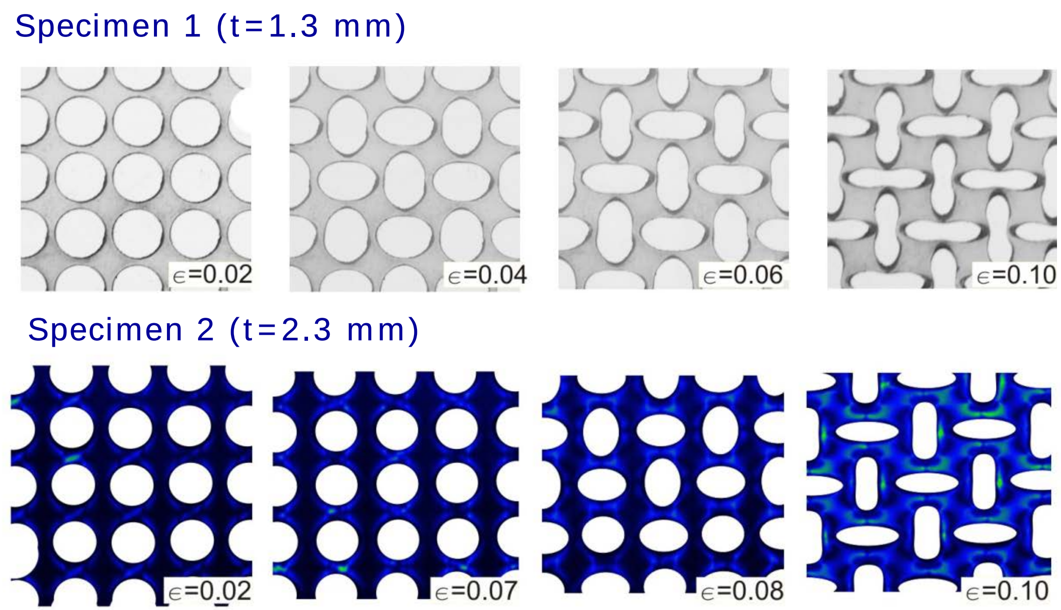


Chung et al., Soft Matter, 2007

Highly non-linear response

Mullin et al, PRL, 2007 Bertoldi et al., JMPS, 2008



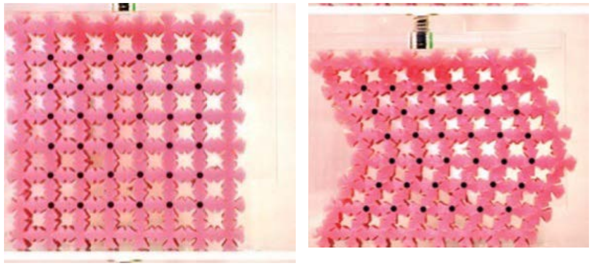


- Initial linear elastic behavior with a sudden departure from linearity to a plateau stress
- Completely homogeneous pattern transformation ➡ corresponds to the plateau region
- The transformed pattern is accentuated with continuing deformation
- The critical triggering stress level scales consistently with ligament buckling

Analysis of instabilities: Global and Local modes

Geymonat et al., 1993   Bertoldi et al. , JMPS, 2008

•Macroscopic (global) instabilities with wavelengths much larger than the characteristic size of the microstructure. They can be computed from the loss of strong ellipticity of the corresponding homogenized properties (Geymonat et al., 1993).

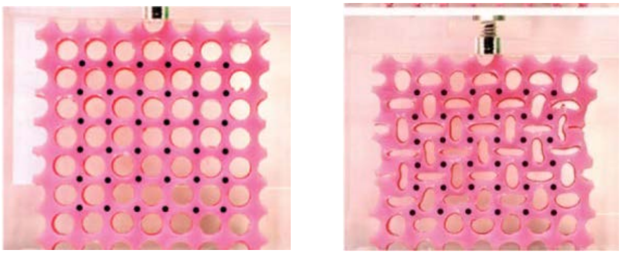


•Microscopic (local) instabilities with finite wavelengths. They alter the periodicity of the solid, but are investigated of on the primitive cell through a Bloch wave analysis.

Instability→wave of vanishing frequency

Bloch wave analysis provides:

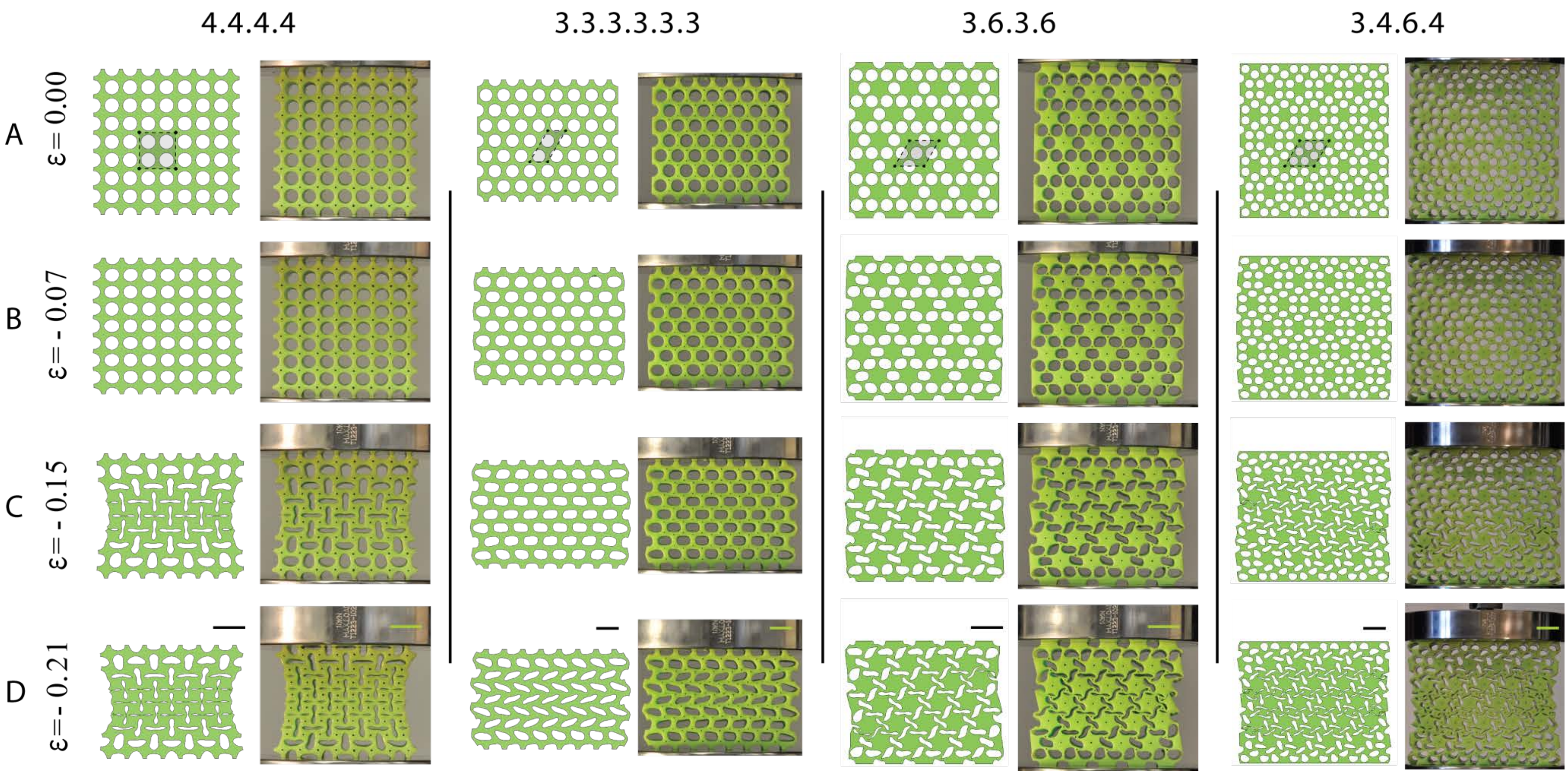
- point along the loading path where instability occurs
- the new periodicity of the structure (p_1, p_2).

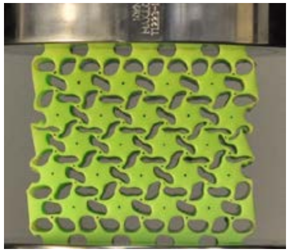


We are interested in triggering microscopic (local) instabilities

Arrangement of Holes

Shim J, Shan S, Kosmrlj A, Kang SH, Chen ER, Weaver JC, Bertoldi K. Soft Matter, 2013

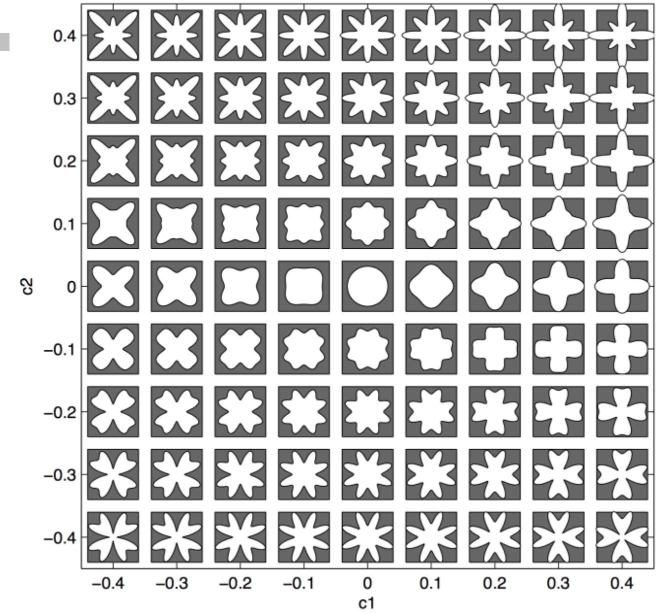




Two buckled structures have a chiral pattern !

Pore shape

Overvelde JTB, Shan S., Bertoldi K, Advanced Materials, 2012
Overvelde JTB, Bertoldi K, JMPS,2014



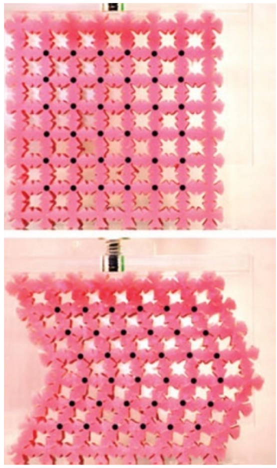
Holes with four-fold symmetry. Fourier series expansion to describe their contour as

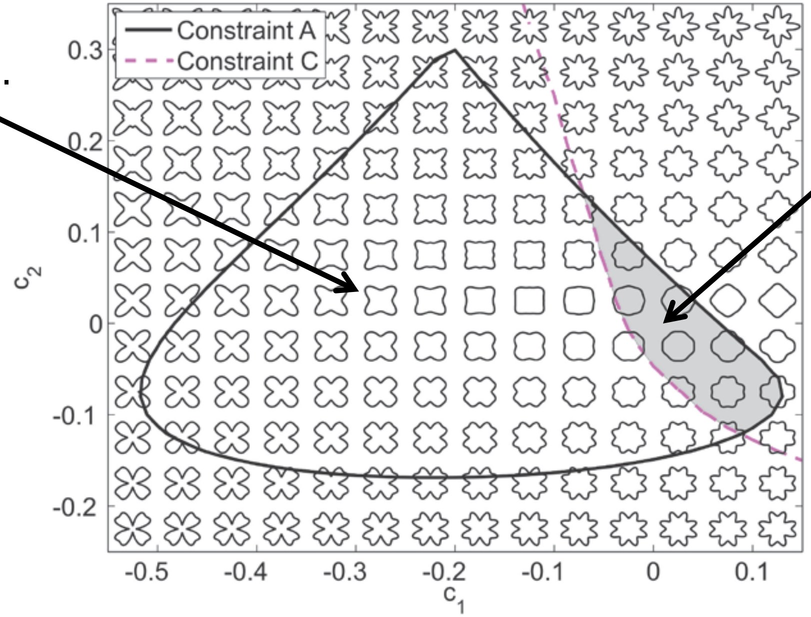
$$x_1(\theta) = r(\theta)\cos(\theta)$$

$$y_1(\theta) = r(\theta)\sin(\theta)$$

$$r(\theta) = r_0\left[1 + c_1\cos(4\theta) + c_2\cos(8\theta)\right],$$

Here **macroscopic** instabilities are critical.





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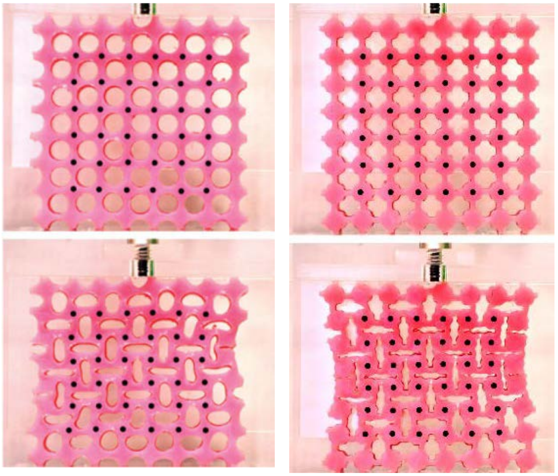
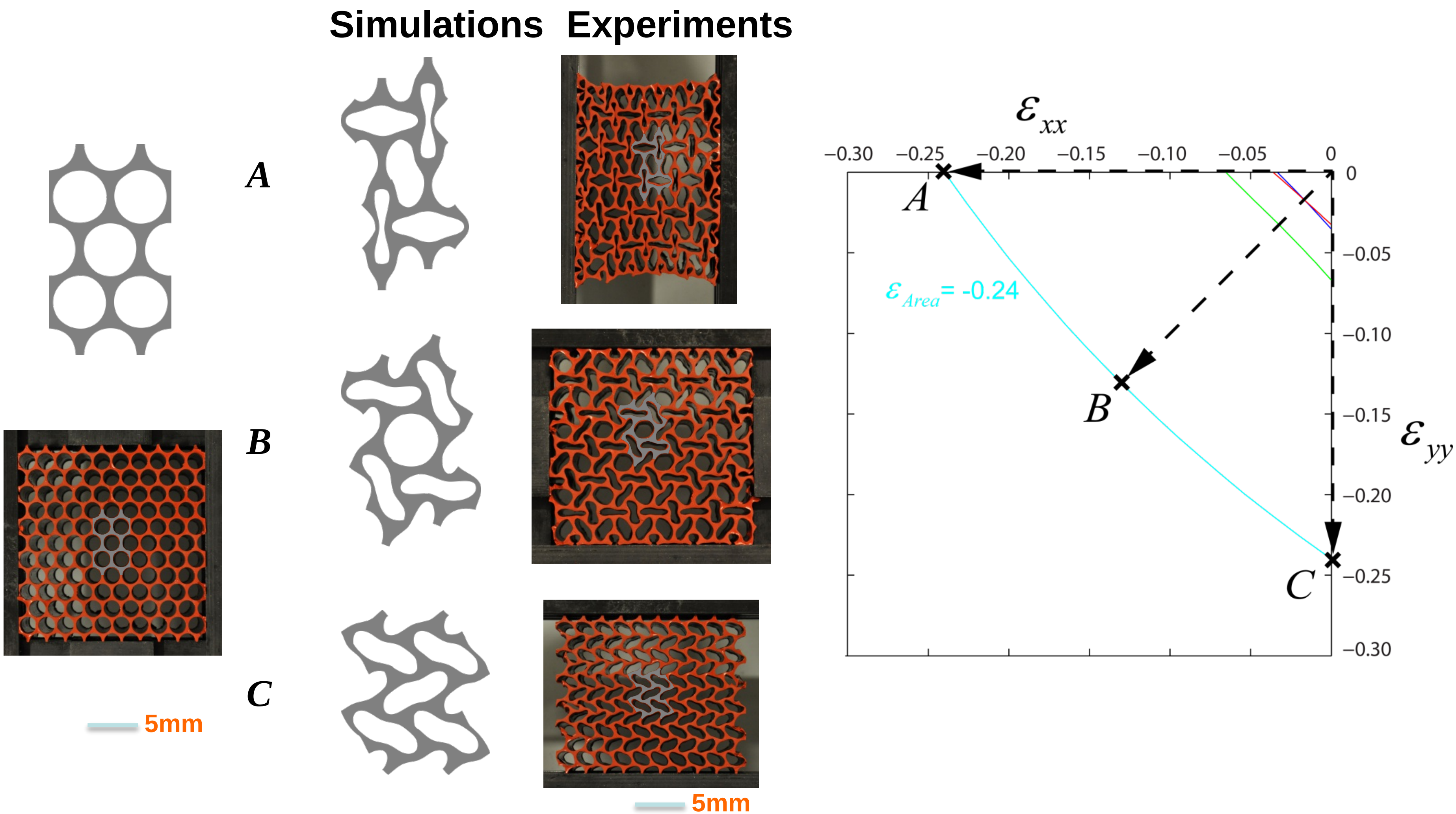


Loading direction

S. Shan, P. Wang, S.H. Kang, P. Wang and K. Bertoldi. Advanced Functional Materials, 2014



?? Questions ??

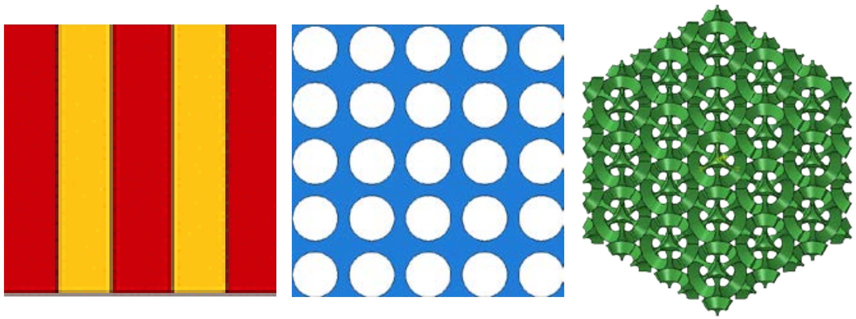
Applications:

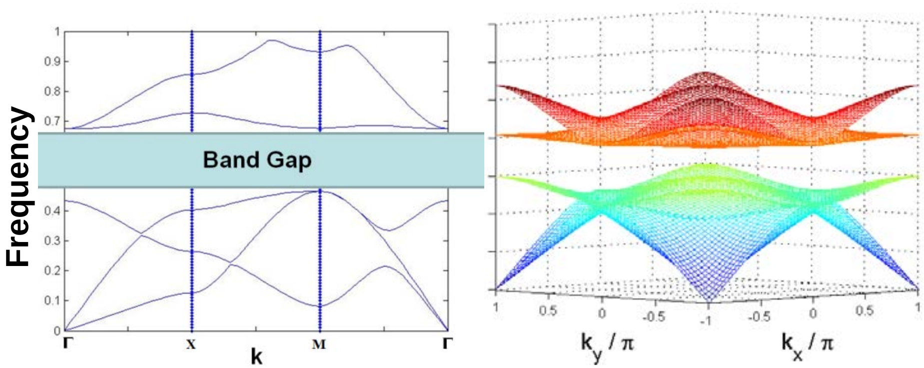
Can we exploit the pattern transformation induced by buckling to design a new class of adaptive materials and devices?

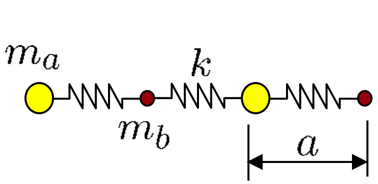
• materials with unusual properties
• color displays
• **phononic switches/ tunable phononic crystals**
• formation of complex pattern

Manipulating elastic waves

Phonic Crystals

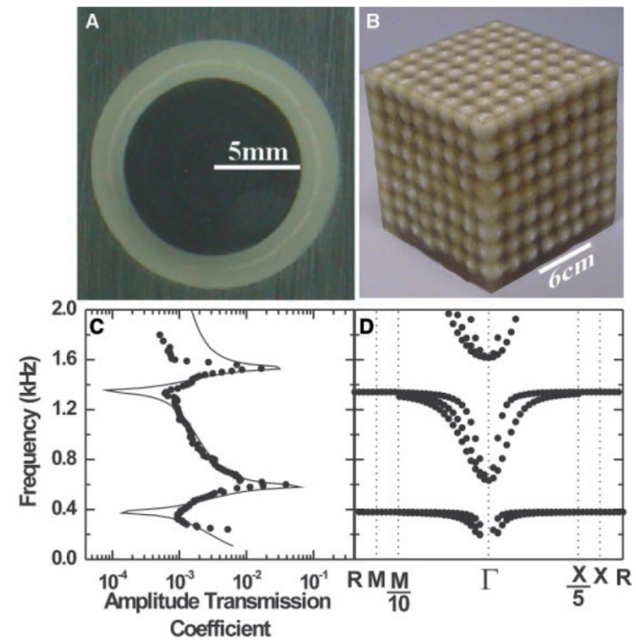




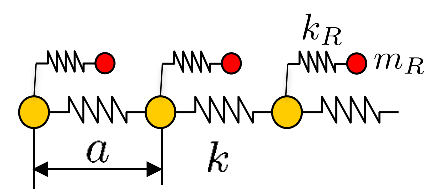


Bragg Scattering

Locally Resonant Metamaterials



(Liu et al., Science, 2000)



Local Resonance

Question: **Can we exploit deformation and instabilities to manipulate the propagation of elastic waves?**

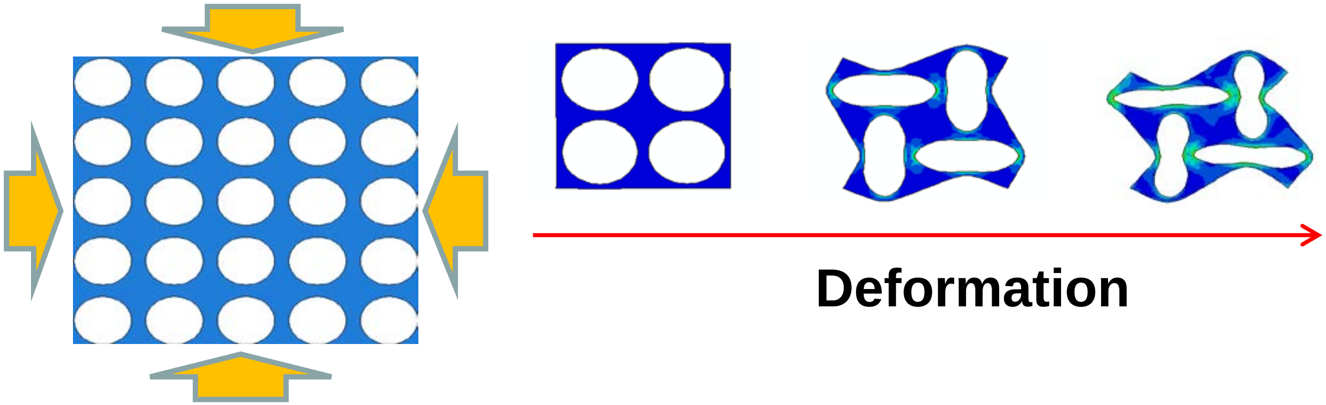
Tunable band-gaps in phononic crystals

Bertoldi K, Boyce MC. Physical Review B, 2008

We exploit large deformation and buckling in phononic crystals to tune the band gaps of the structures.

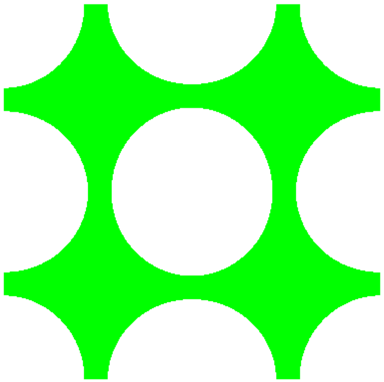
Simulation steps (Finite Element Method):

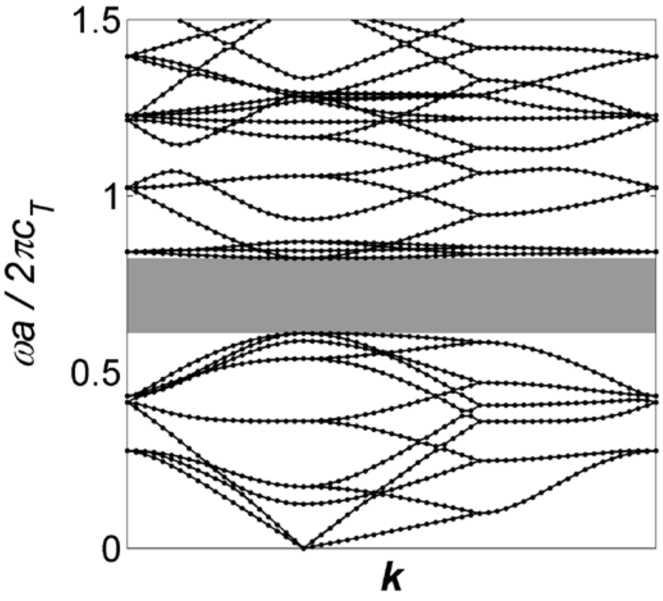
- Buckling analysis
- Postbuckling analysis (finite deformations, periodic boundary conditions)
- Wave propagations (small amplitude waves. Perturbation step)



Tunable Phononic Band-gaps

Effect of pattern transformation on phononic band gaps.





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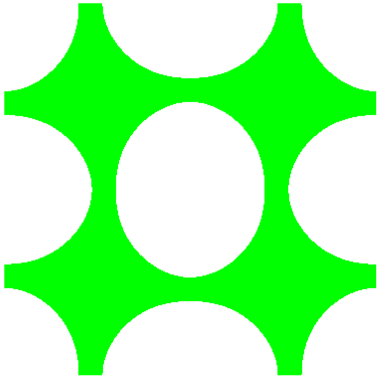

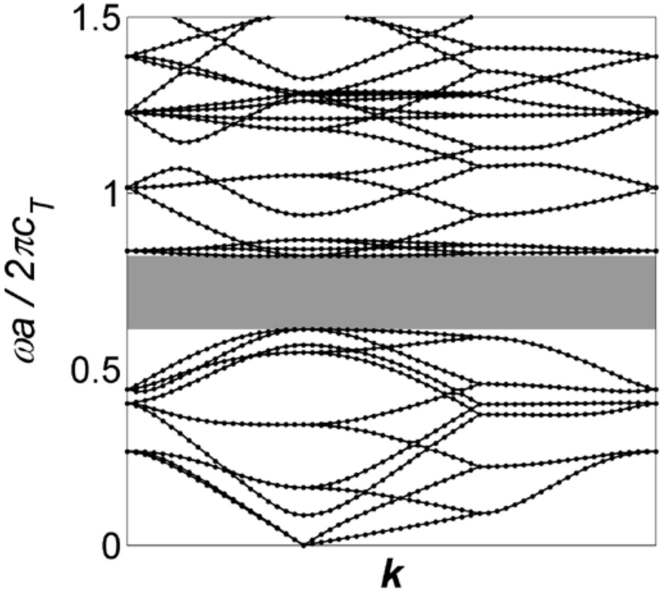

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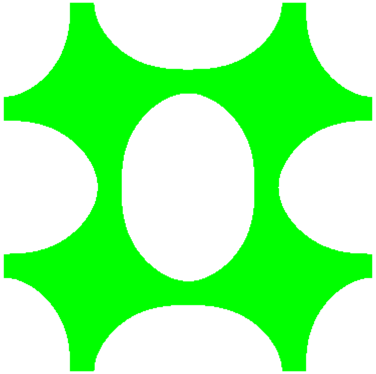

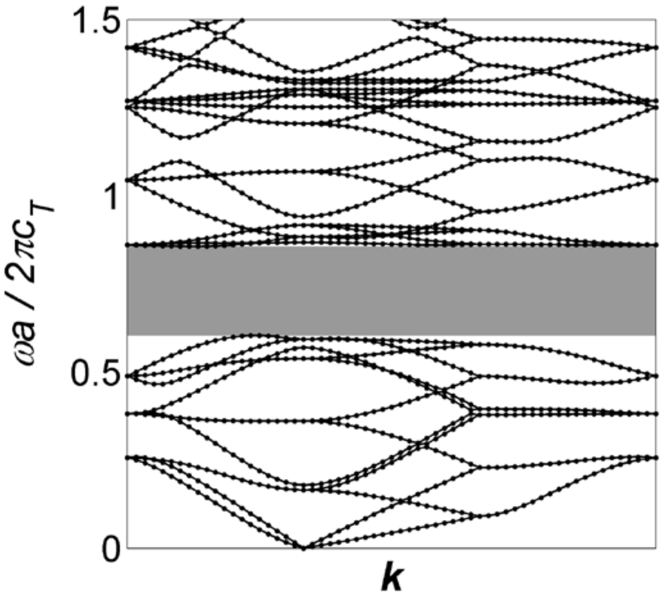

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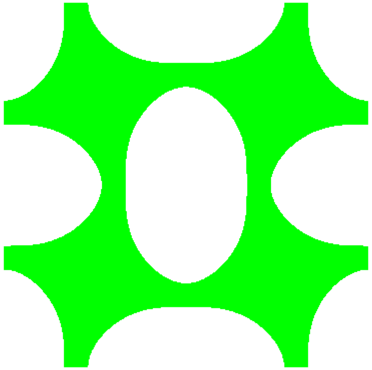

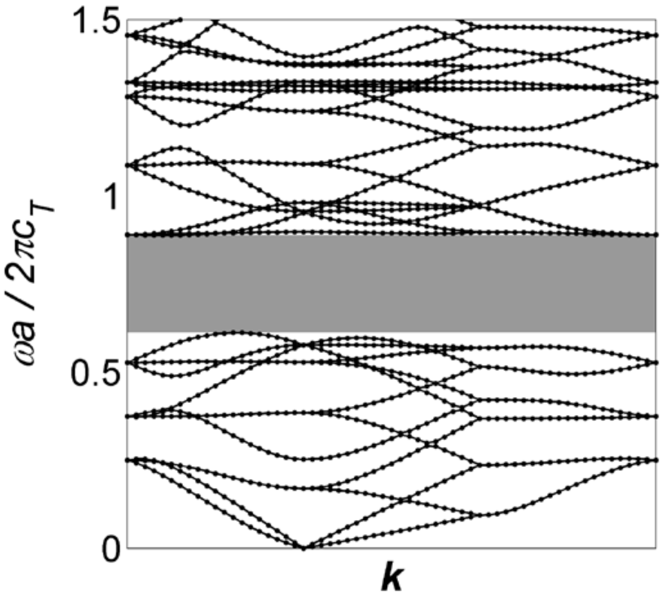

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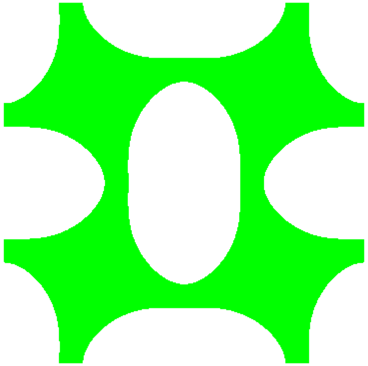

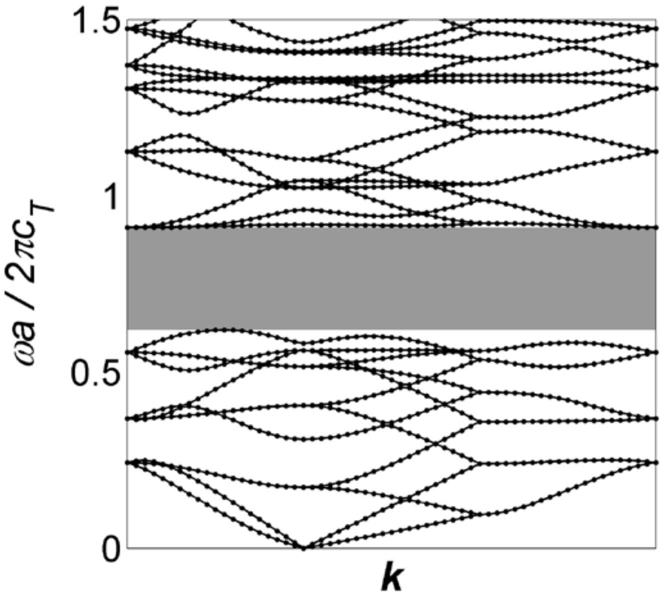

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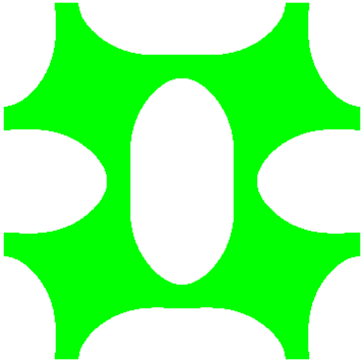

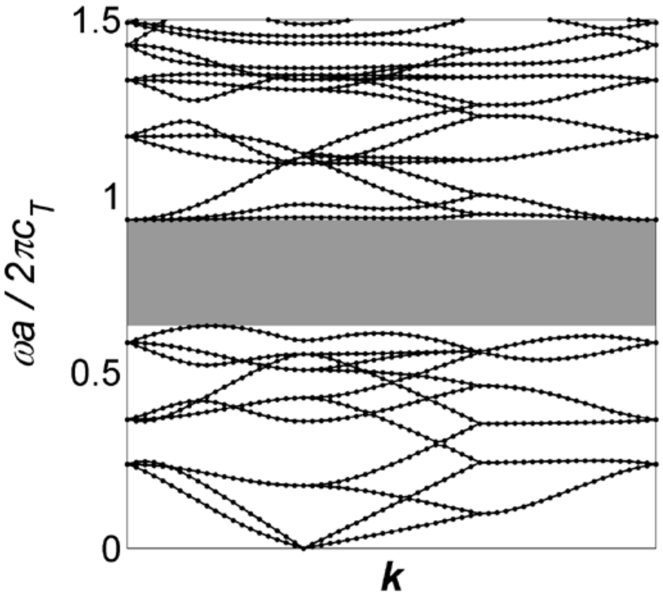

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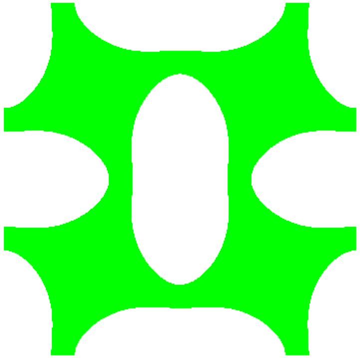

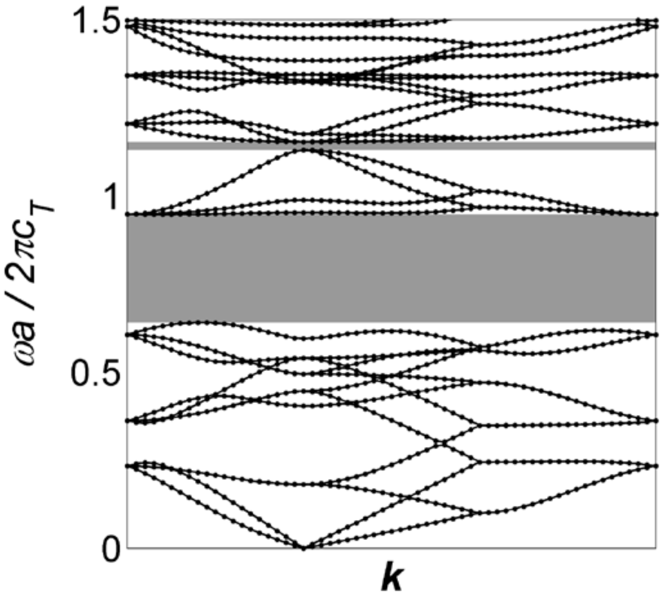

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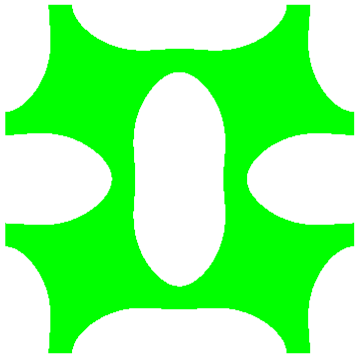

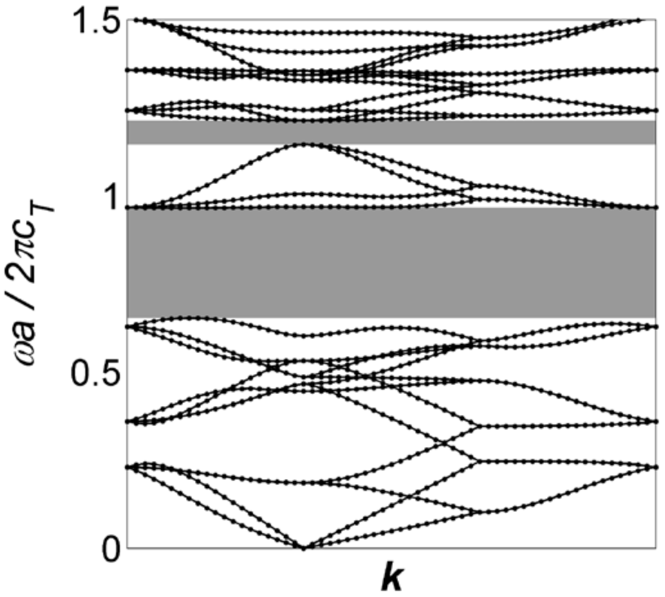

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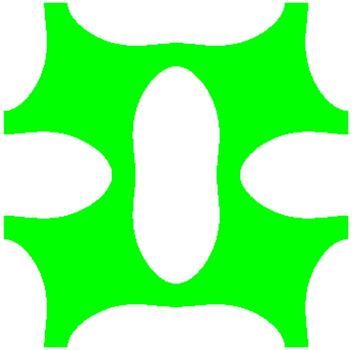

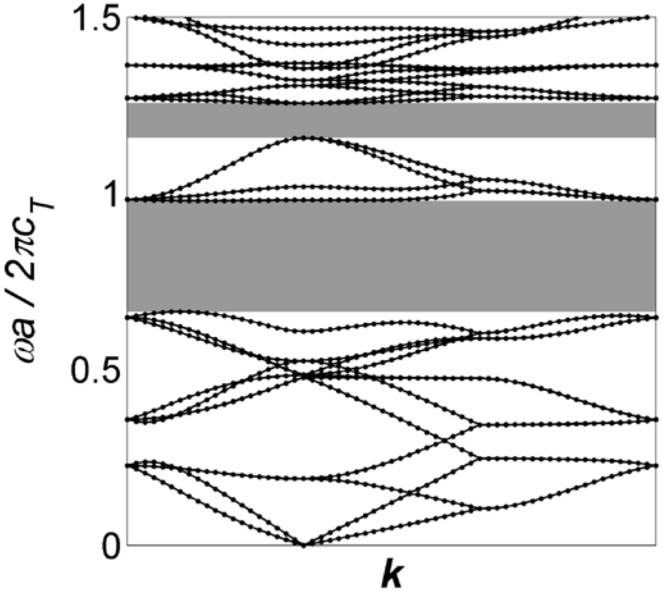

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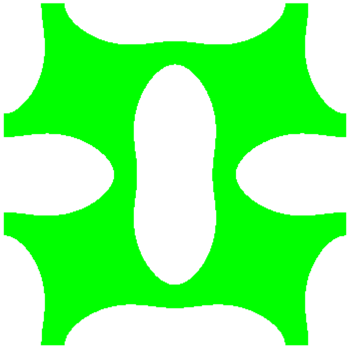

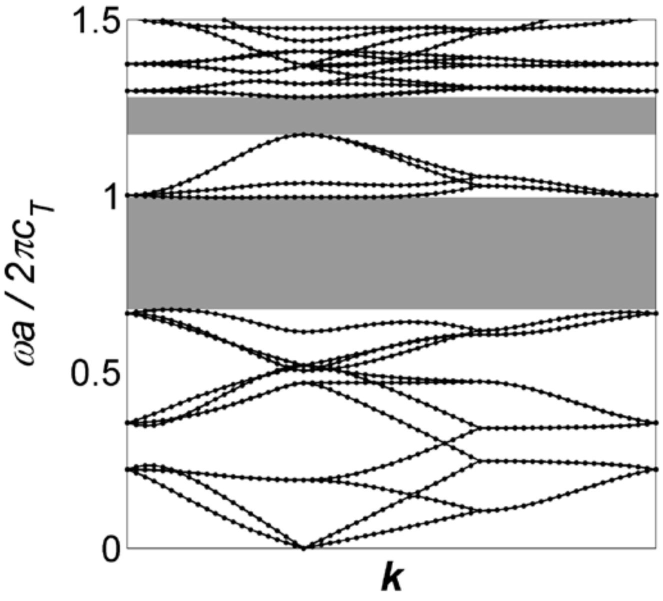

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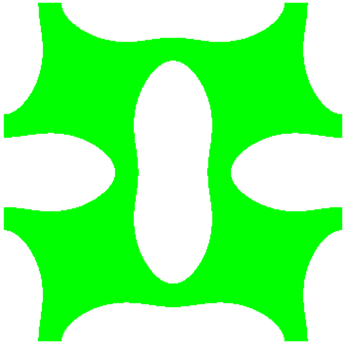

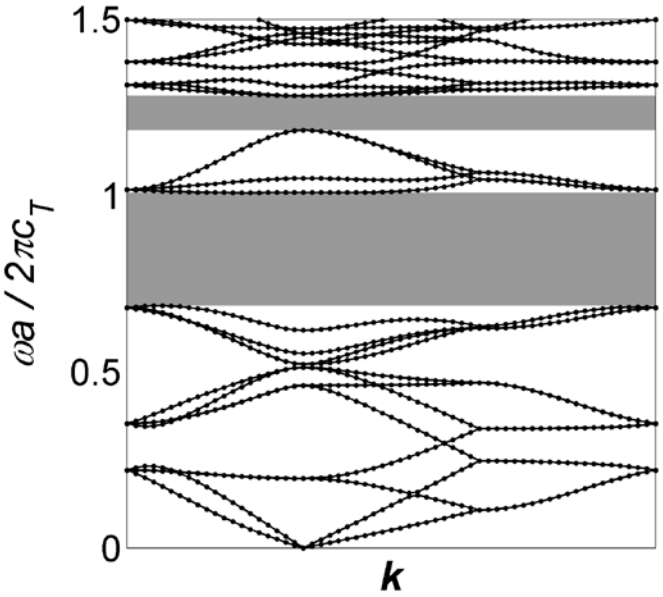

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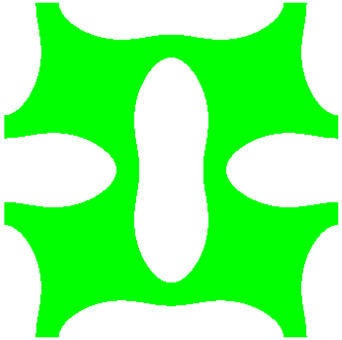

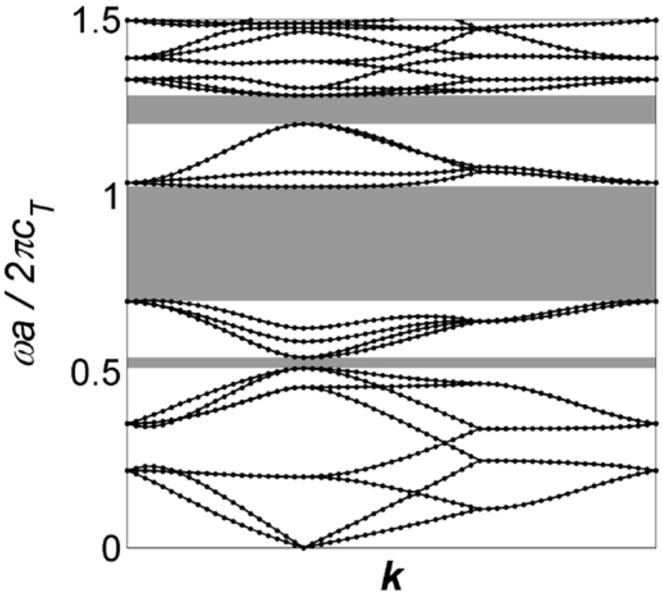

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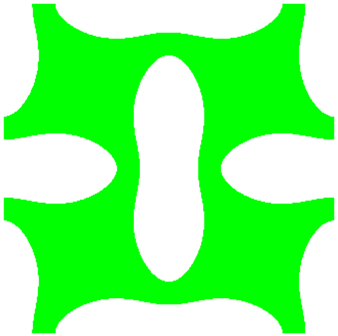

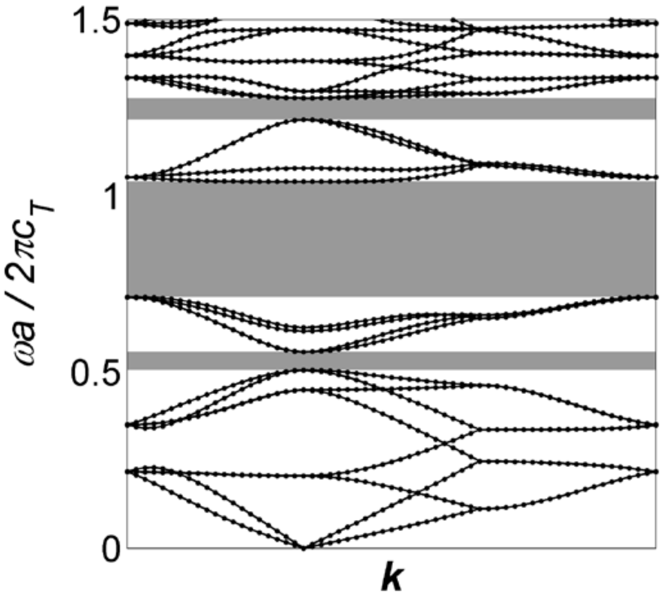

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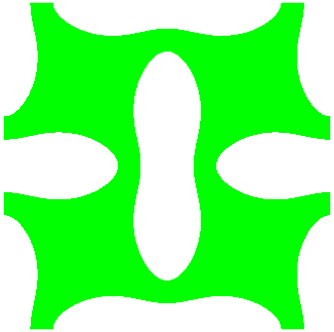

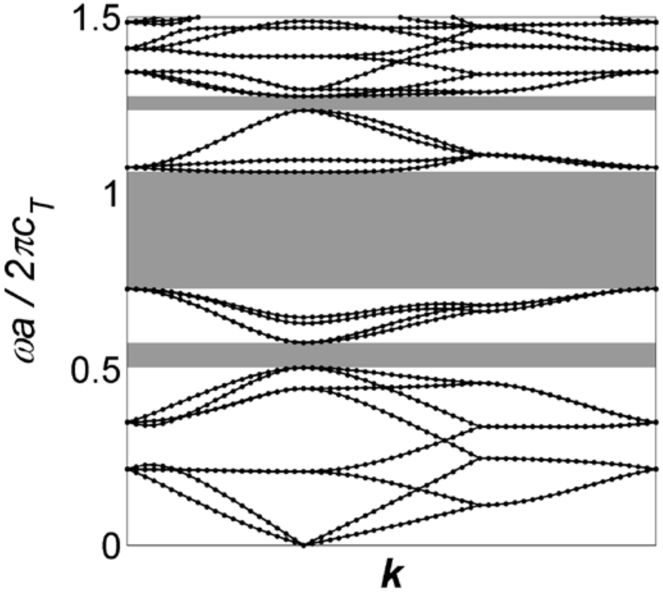

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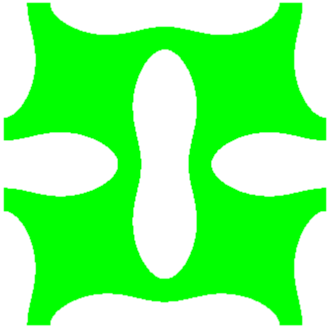

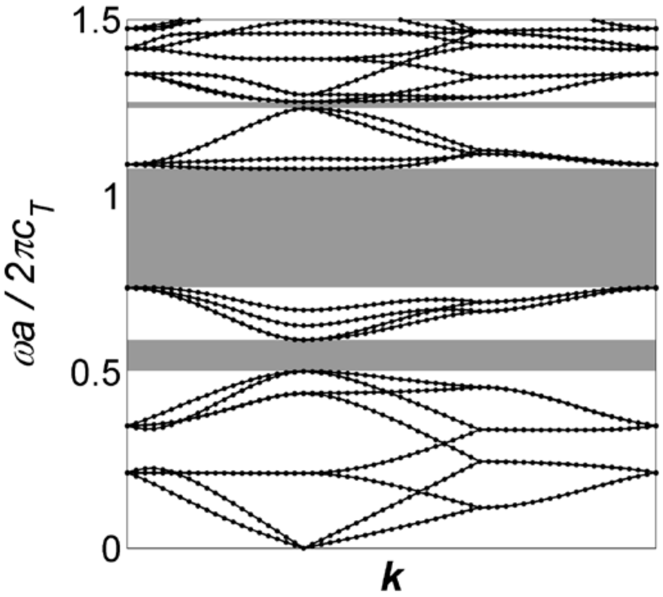

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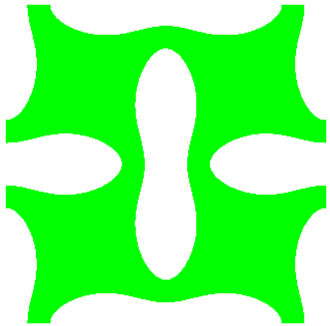

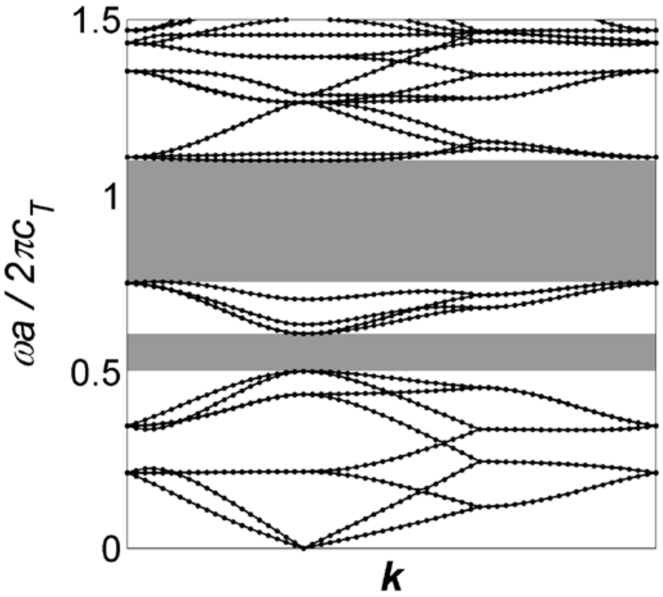

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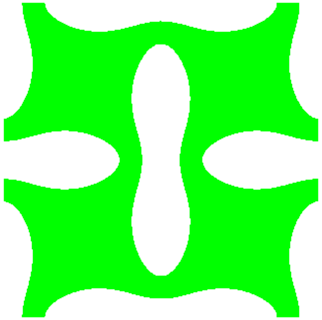

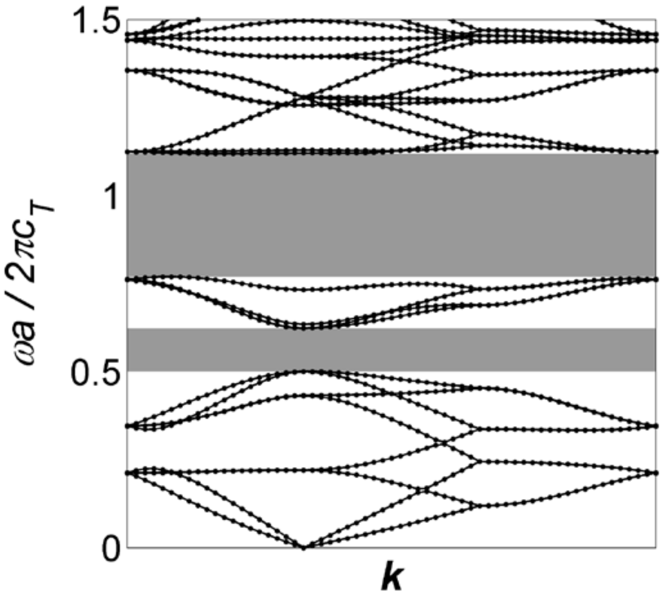

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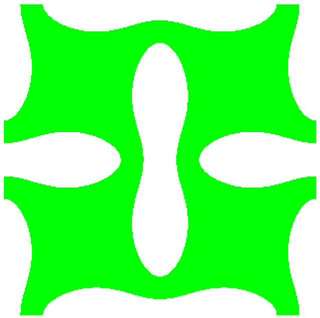
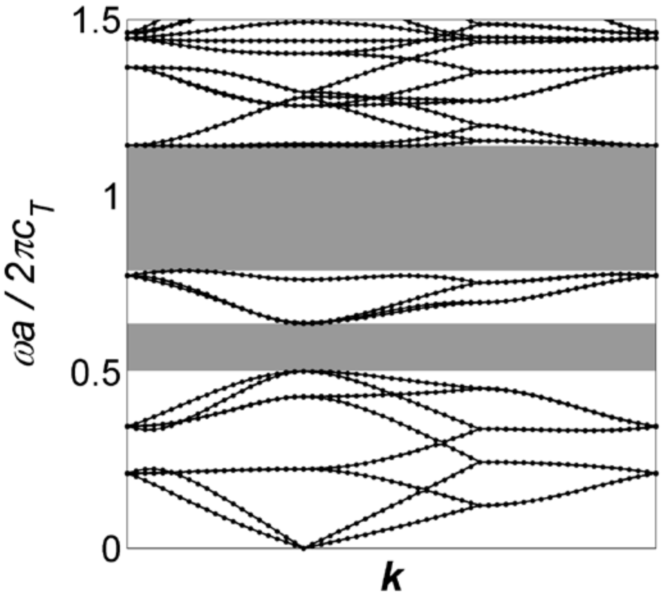
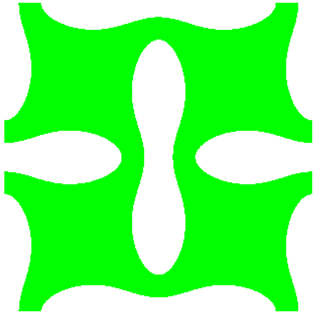
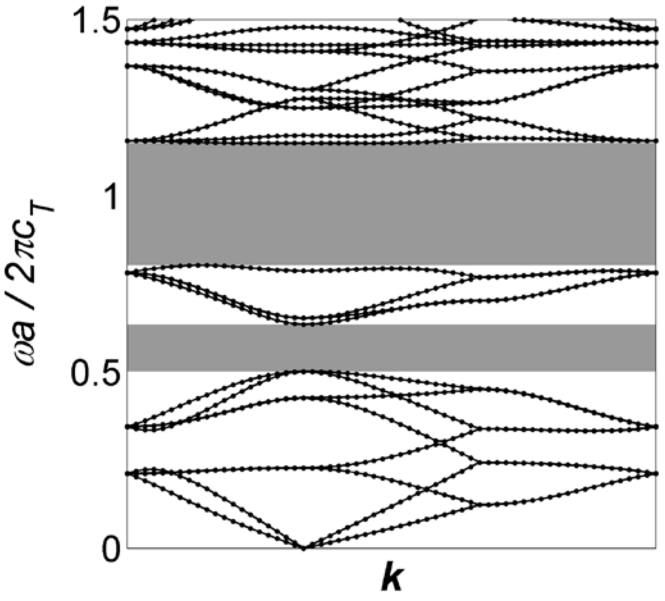




Tunable Phononic Band-gaps

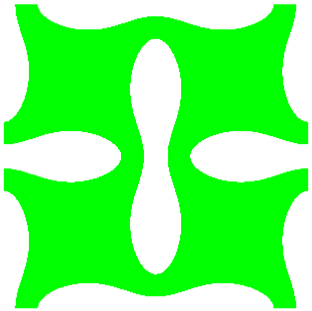
Effect of pattern transformation on phononic band gaps.

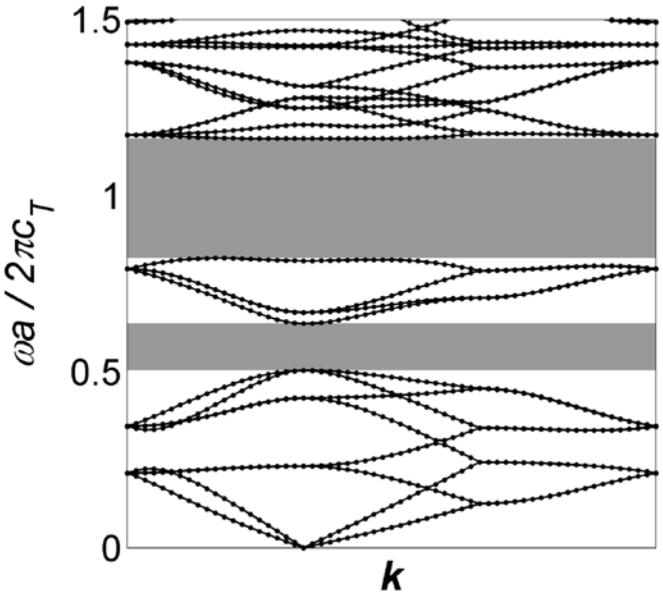




Tunable Phononic Band-gaps

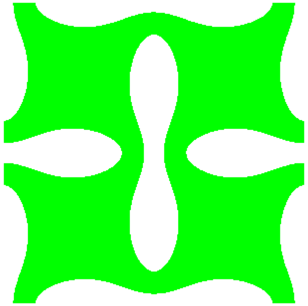
Effect of pattern transformation on phononic band gaps.

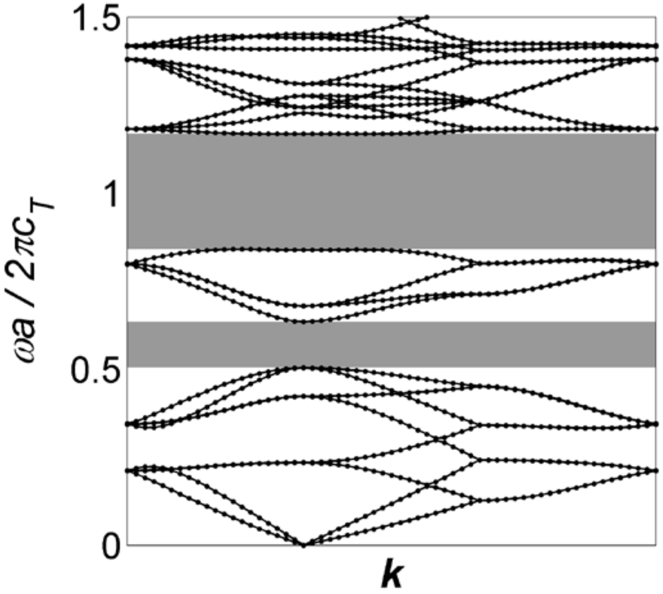




Tunable Phononic Band-gaps

Effect of pattern transformation on phononic band gaps.

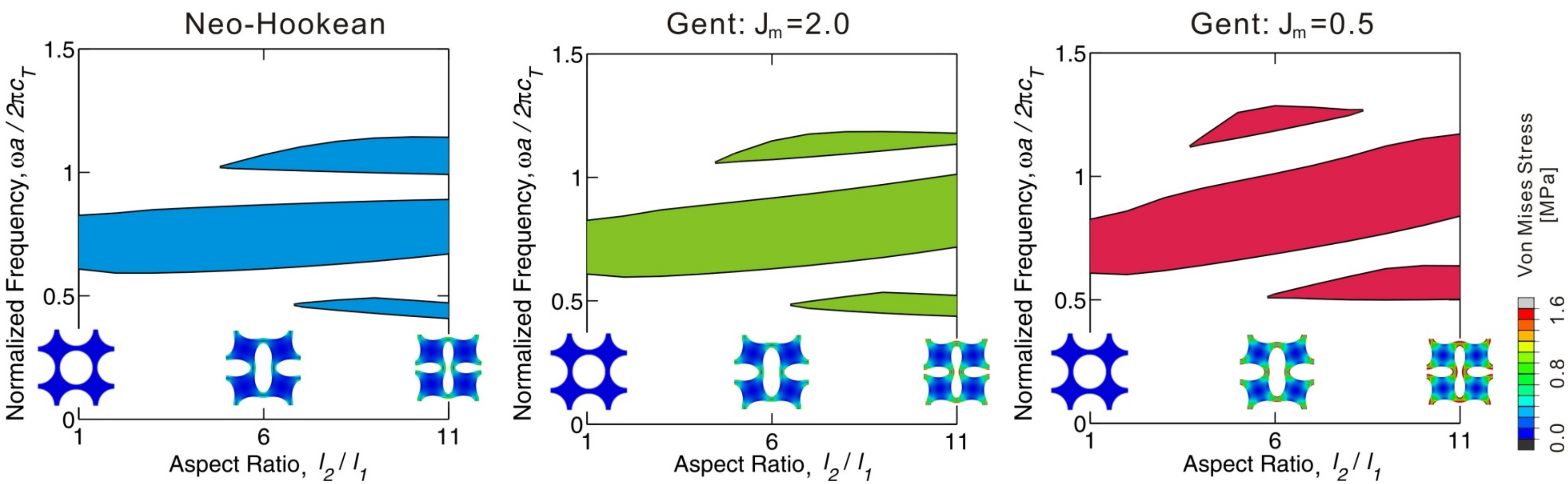




Tunable Phononic Band-gaps

P. Wang, J. Shim and K. Bertoldi. PRB, 2013

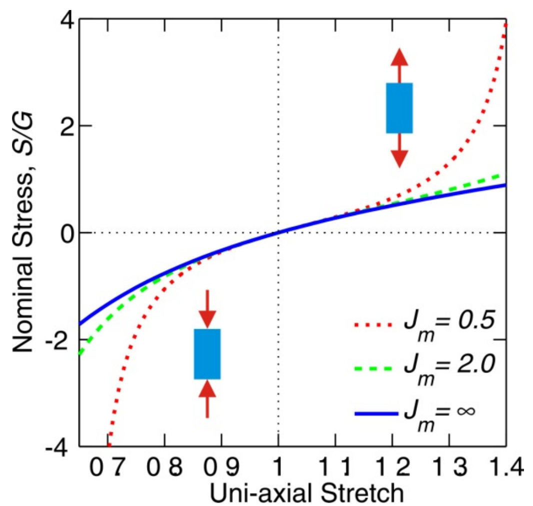
Evolution of the phononic band gaps as a function of the applied strain.



Initial linear response: Band gaps affected marginally by deformation, evolving in an affine and monotonic way

Pattern transformation: The in-plane modes undergo a transformation and 2 new band gaps are opened

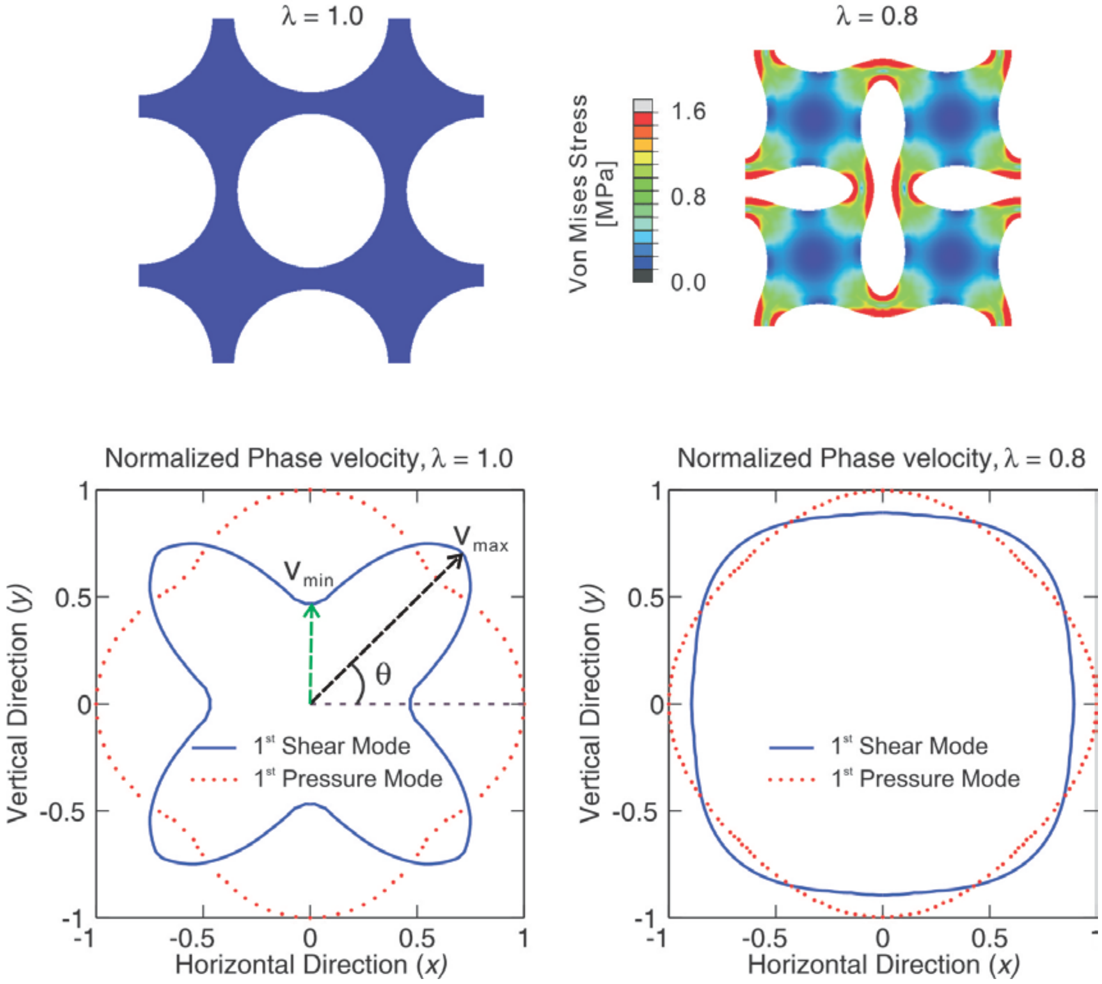
Geometric non-linearities play a major role



Tunable Phononic Band-gaps

P. Wang, J. Shim and K. Bertoldi. PRB, 2013

Phononic crystals are characterized by a low frequency directional behavior that can be exploited to steer or redirect waves in specific directions**.**

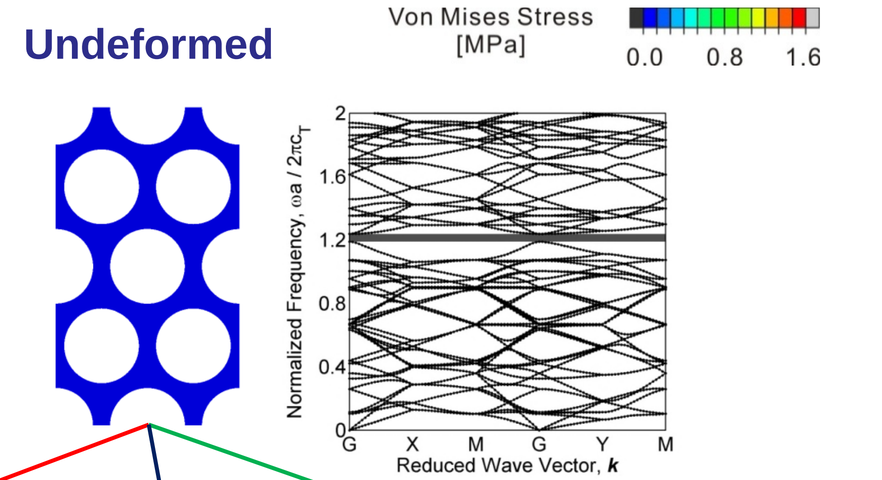


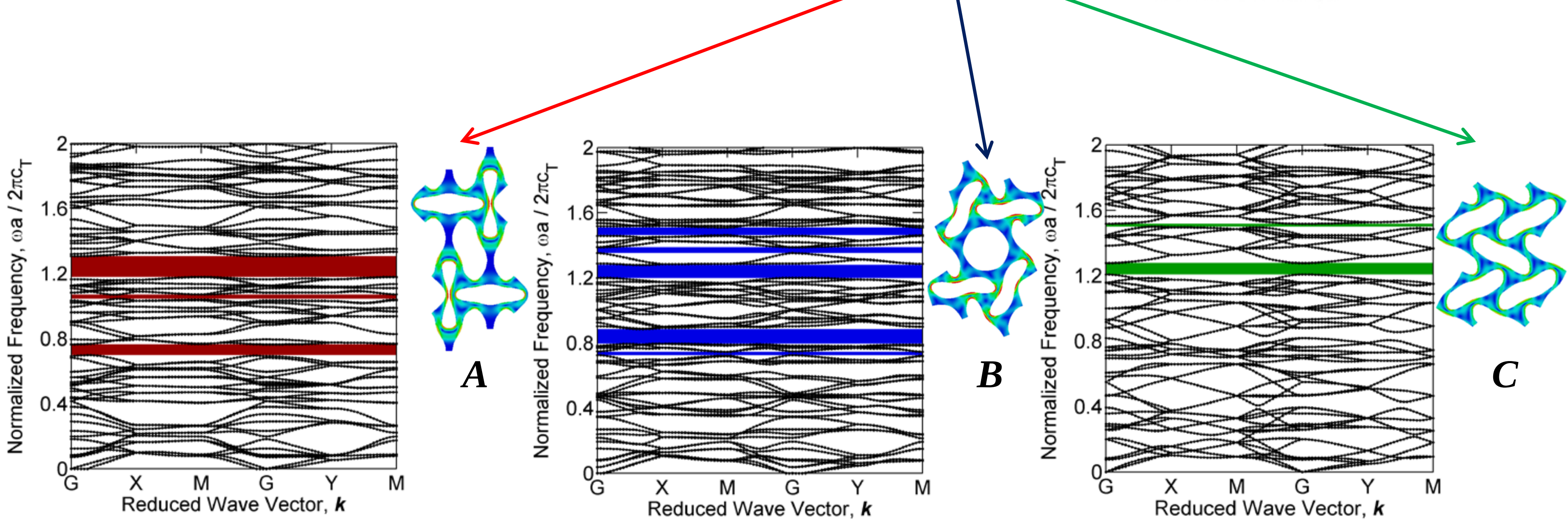
Triangular array of holes: Enhanced tunability

S. Shan, P. Wang, S.H. Kang, P. Wang and K. Bertoldi. Advanced Functional Materials, 2014

Simulations (Finite Element Method):

- Buckling analysis
- Postbuckling analysis (finite deformations, periodic boundary conditions)
- Wave propagations (small amplitude waves. Perturbation step)



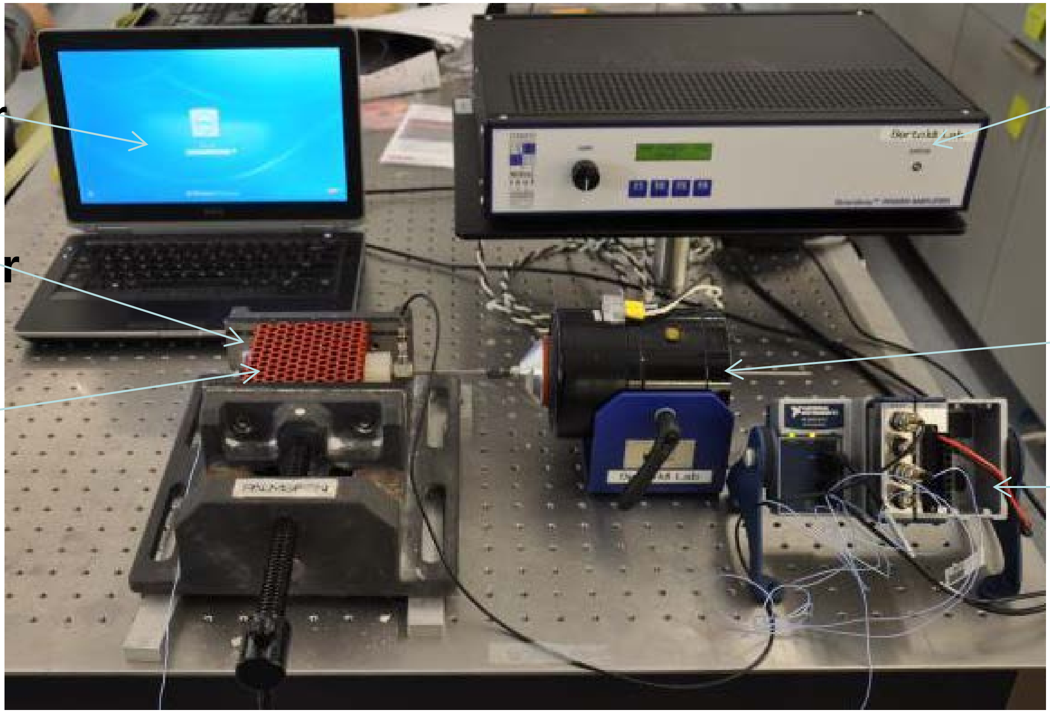


ε_{xx} = - 0.24

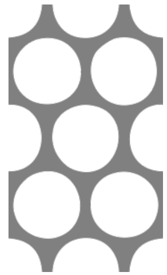
ε_{xx} = ε_{yy} = - 0.14

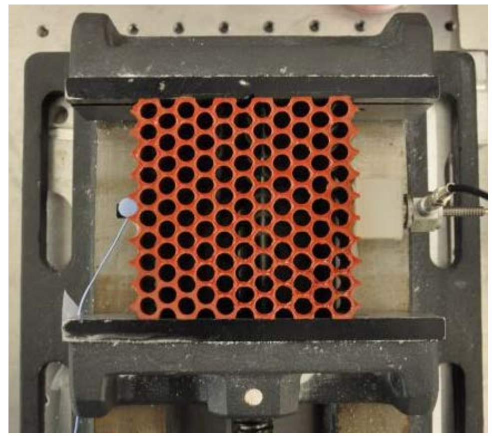
ε_{yy} = - 0.24

Experiments

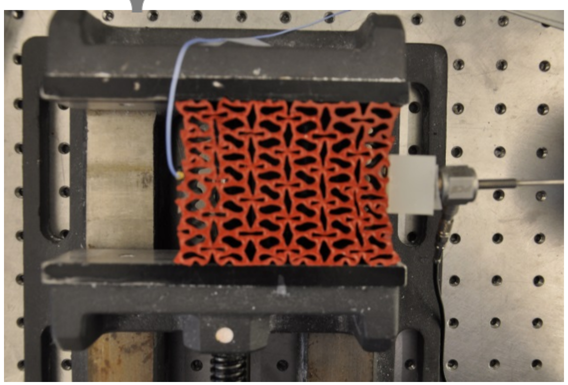


Experiments

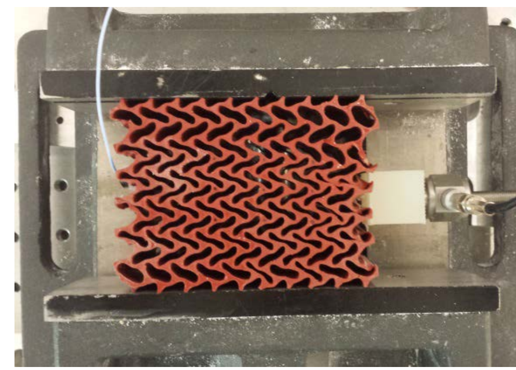




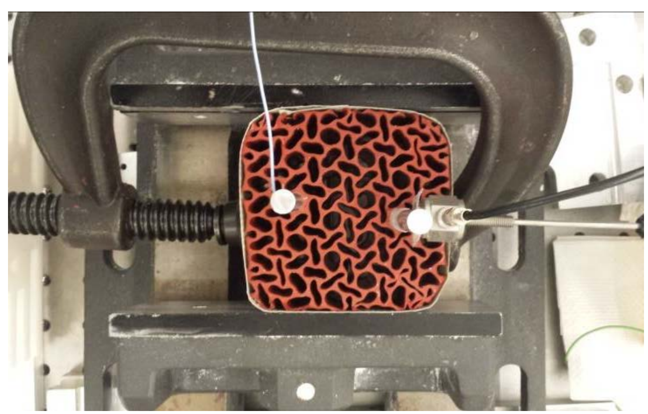
A



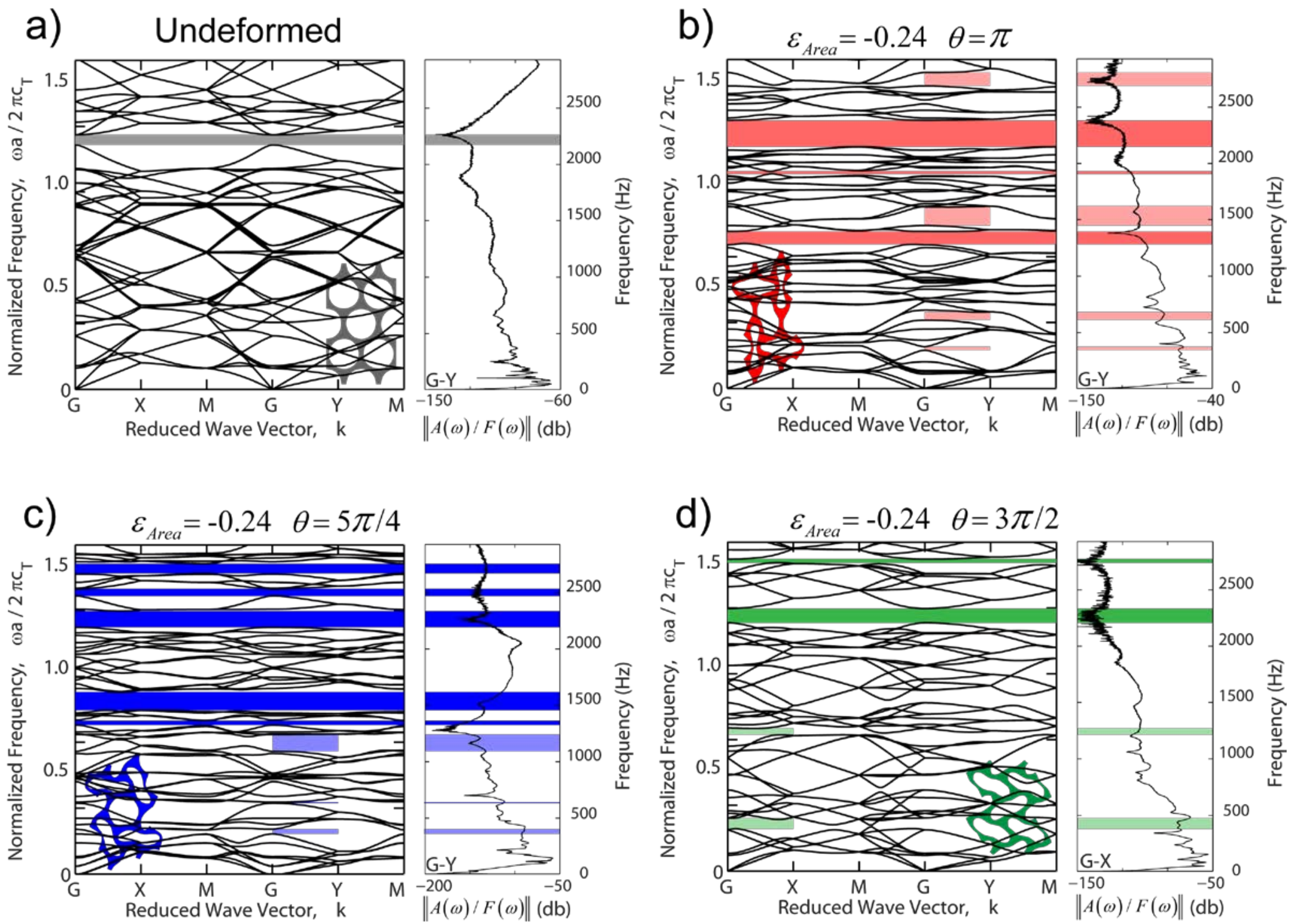
C



B



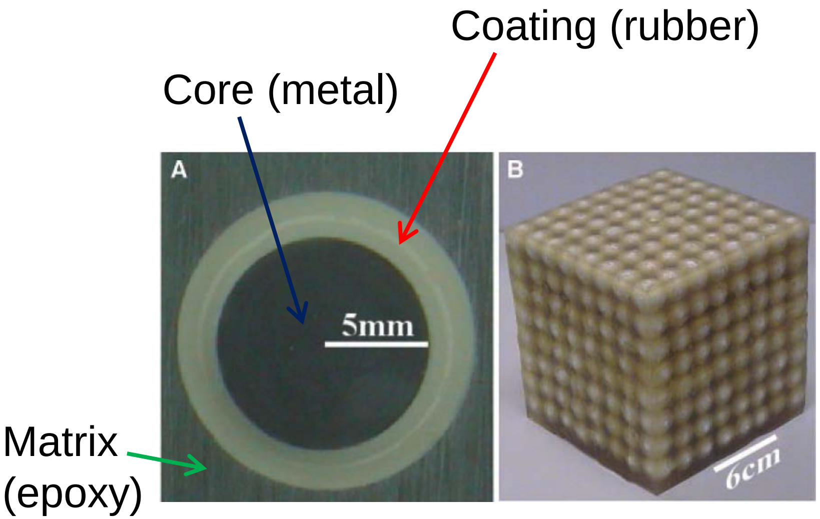
Experiments



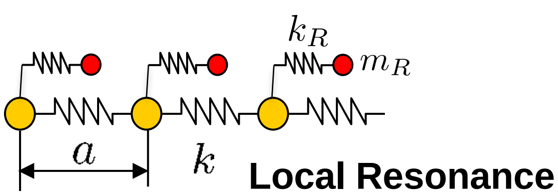
Instabilities and large deformation can be effectively used to manipulate the propagation of elastic waves in periodic structures

Tunable band-gaps in locally resonant metamaterials

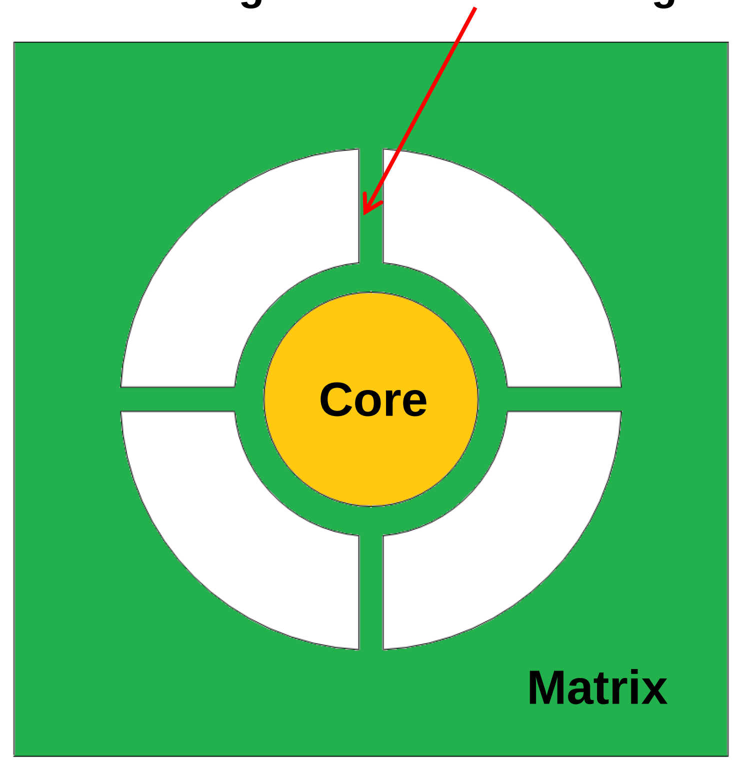
P. Wang, F. Casadei and K. Bertoldi. Submitted



(Liu et al., *Science*, 2000)



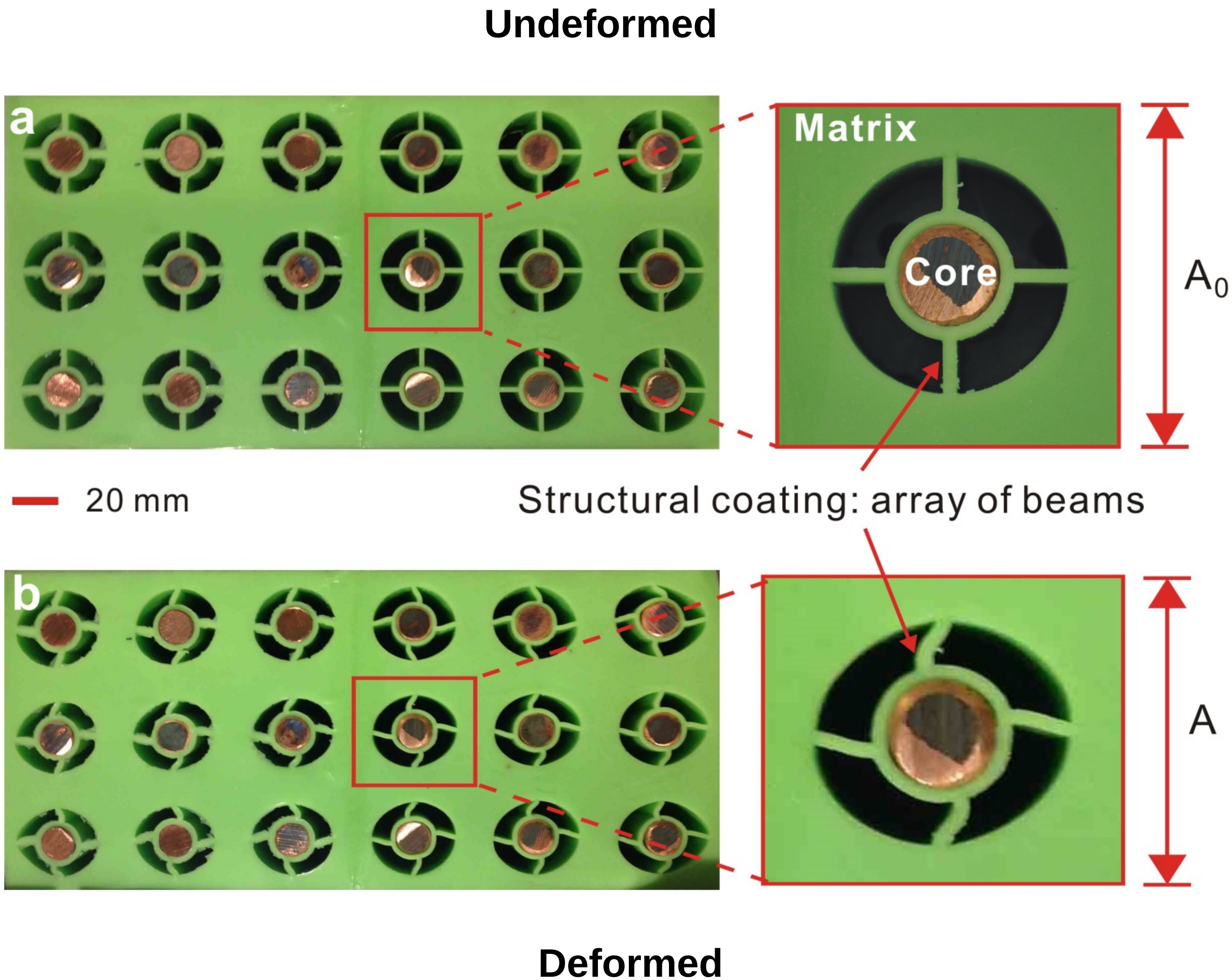
Our design: structural coating



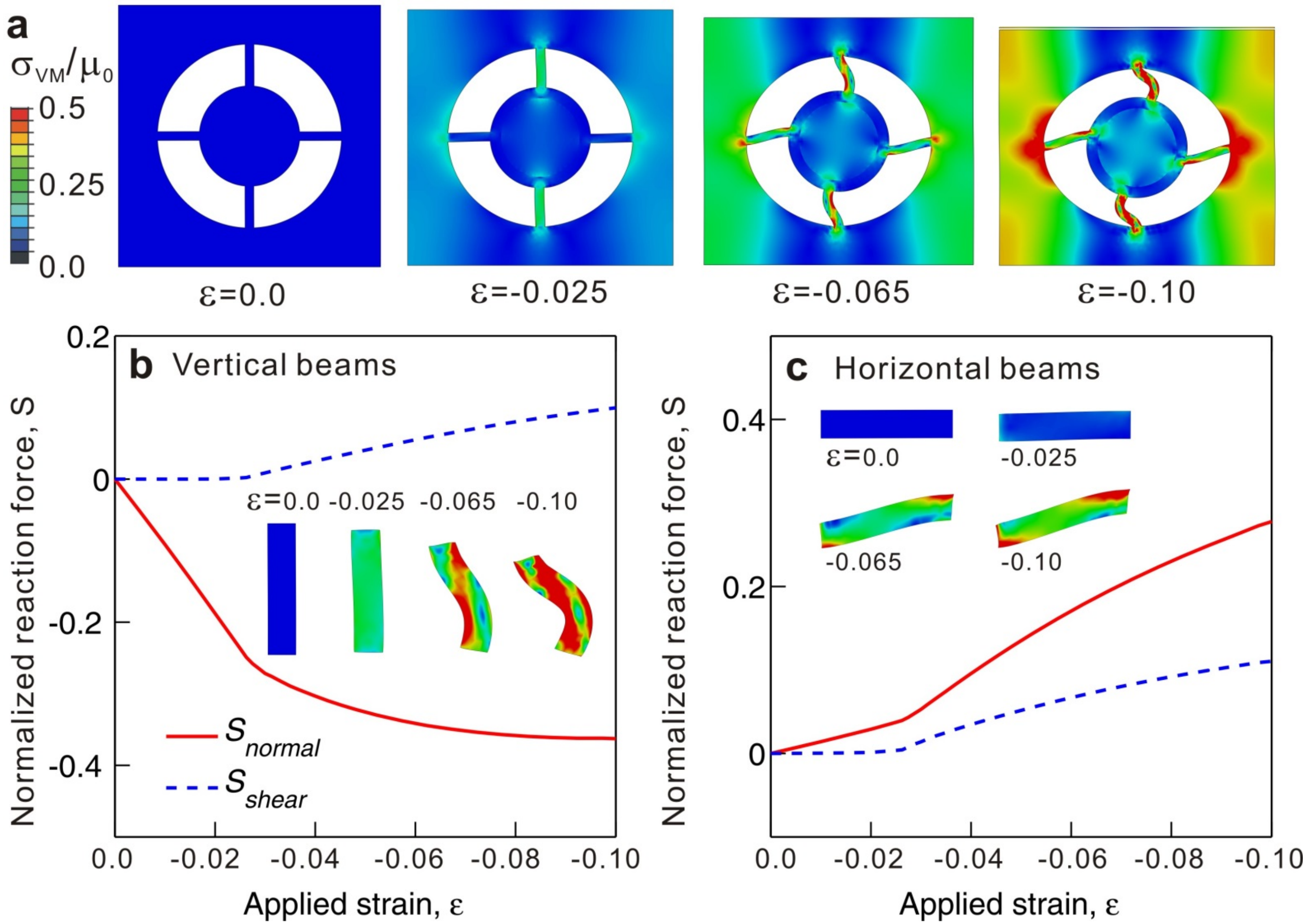
Structural coating:

- Use elastic and highly deformable beams
- Use buckling to significantly alter the beam stiffness and – in turns – the frequency of the bandgap

Our design


Undeformed
a
Matrix
Core
A_0
20 mm
Structural coating: array of beams
b
A
Deformed

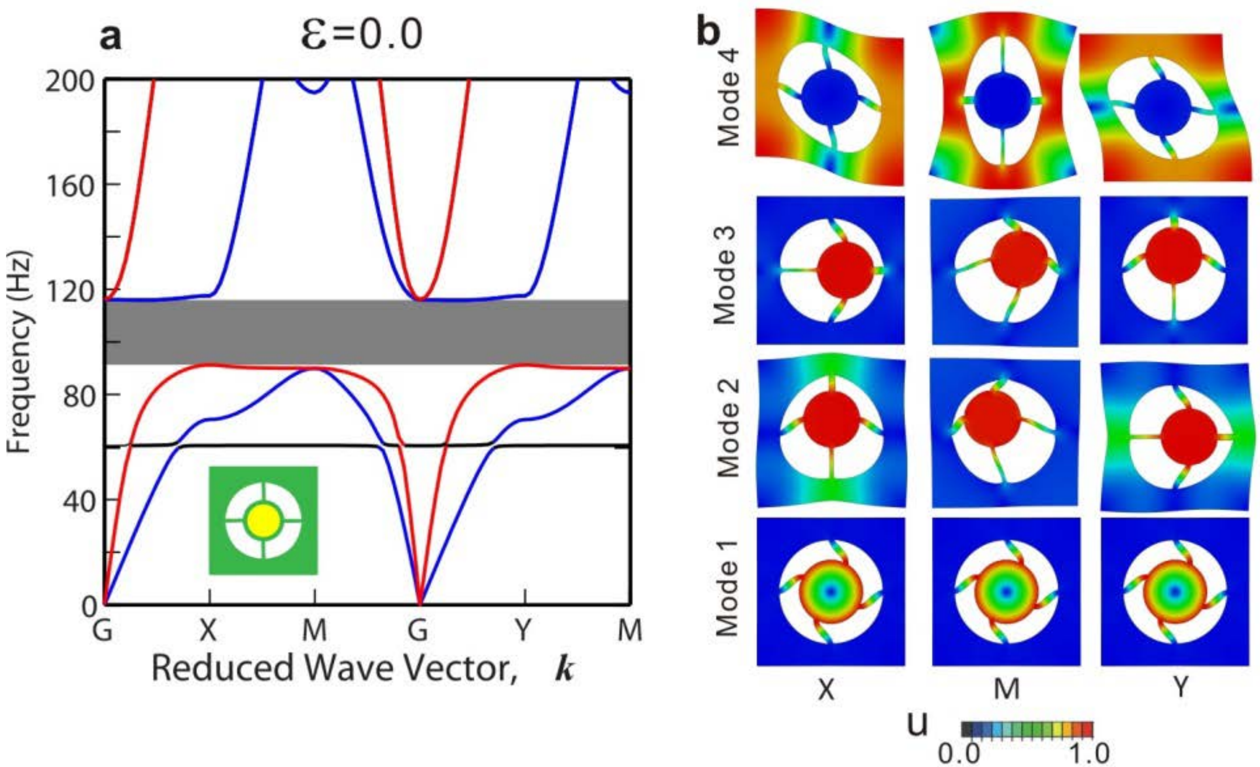
Static response



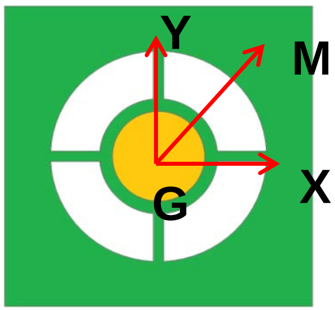
Buckling and large deformation significantly alter the stiffness of the beams

Dynamic response: undeformed configuration

FEM: Undeformed configuration

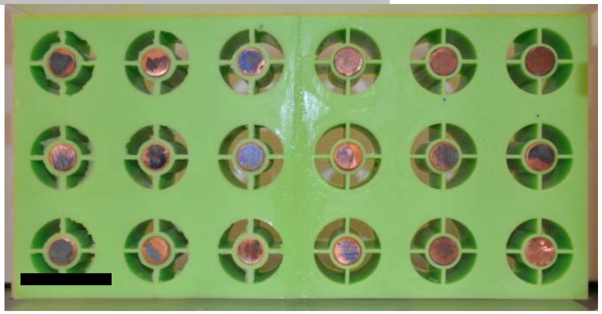


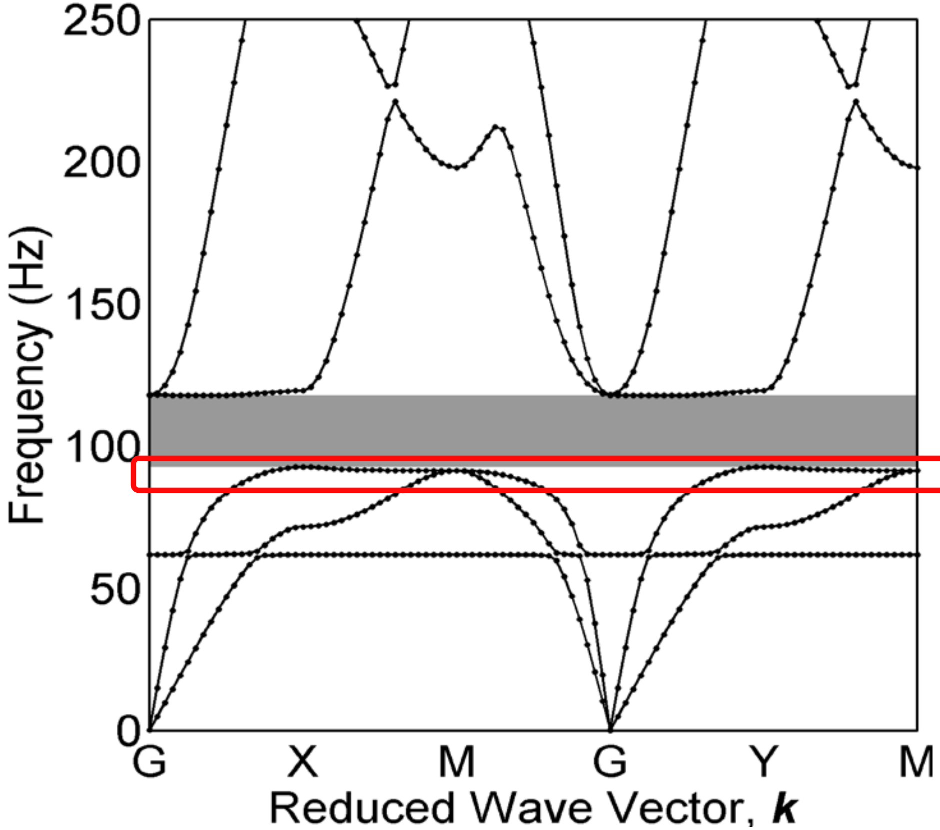
Flat band → Local mode

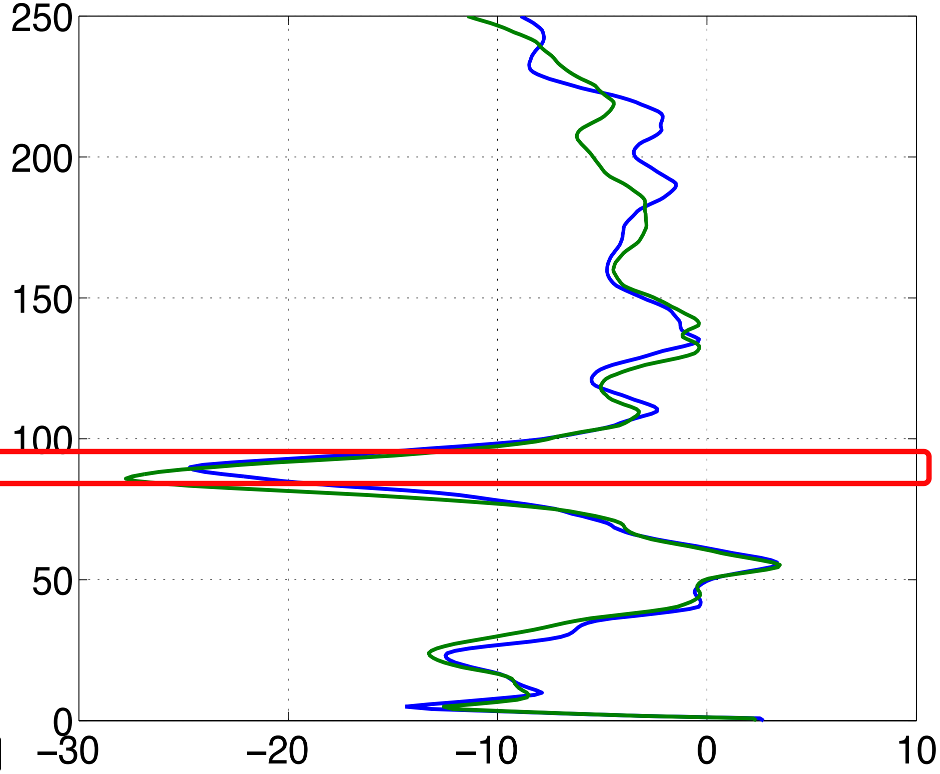


Experimental results: Undeformed configuration

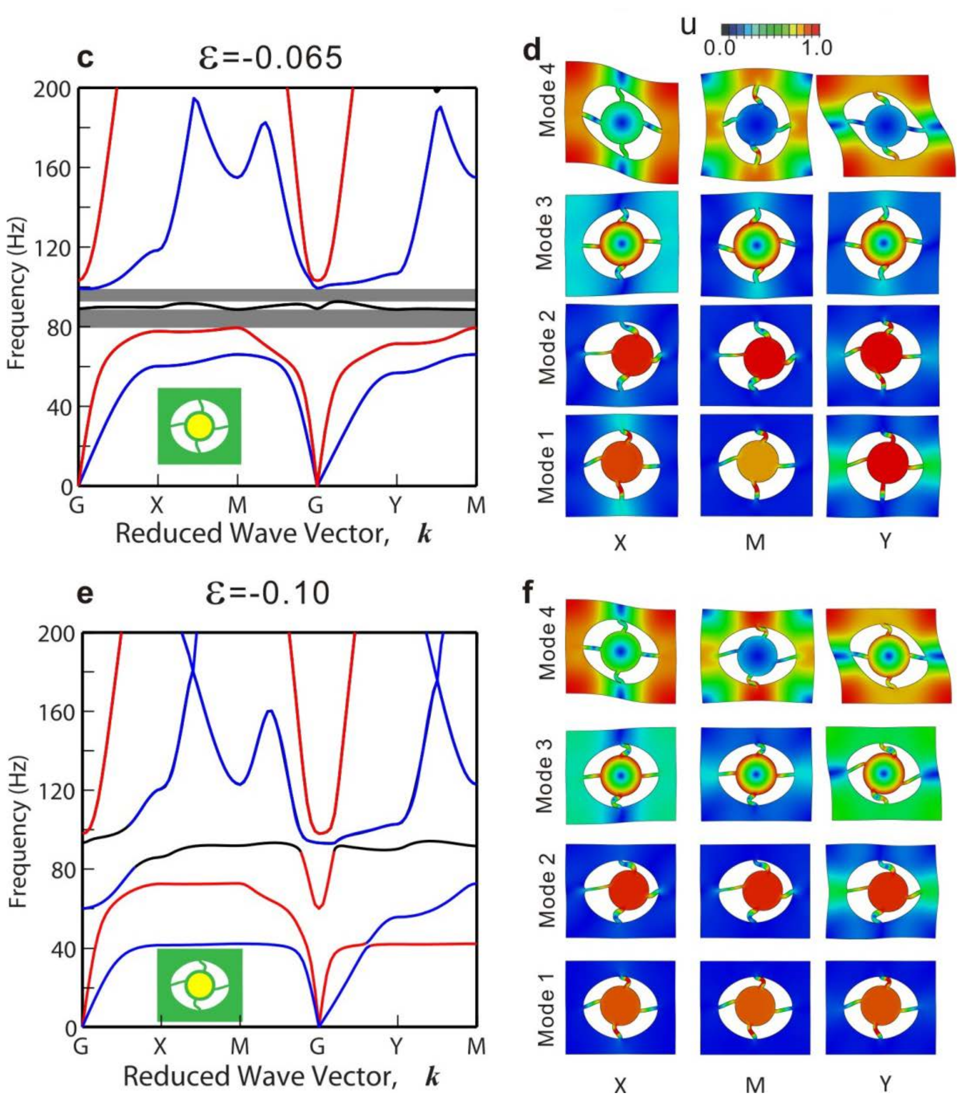
Undeformed configuration







Dynamic response: deformed configuration



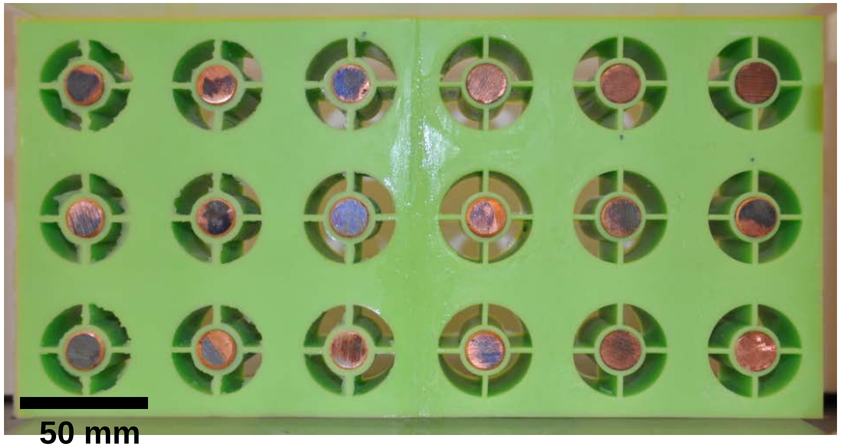
Effect of deformation:

Because of the softening of the two vertical beams induced by buckling the bangap frequency decreases

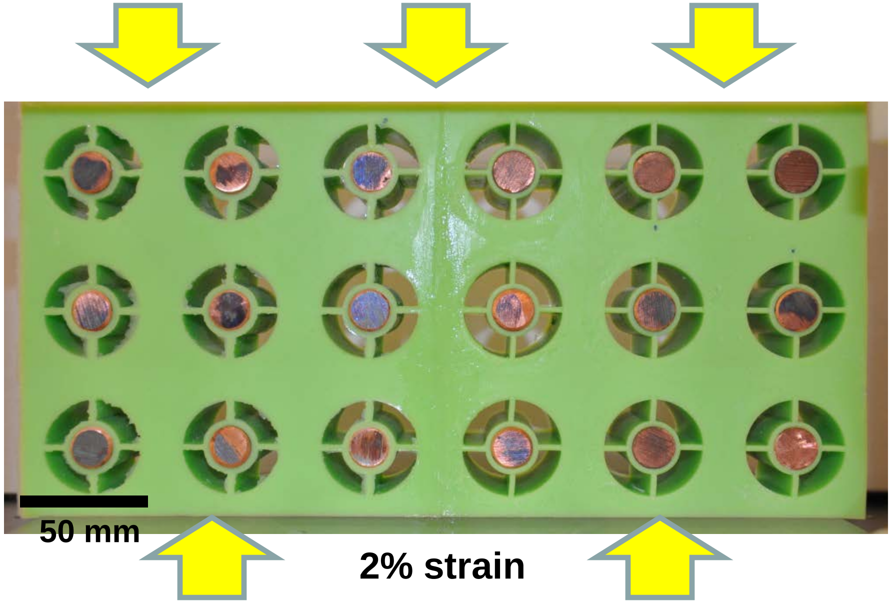
Because of the increase in tangential stiffness induced by buckling the rotational mode rises

The bandgap is completely suppressed → phononic switch

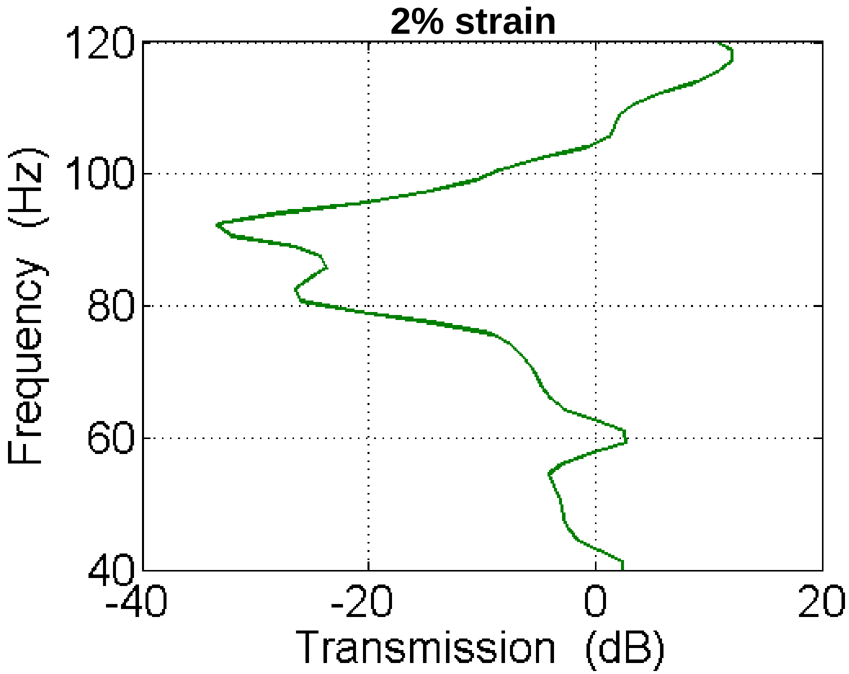
Experimental results



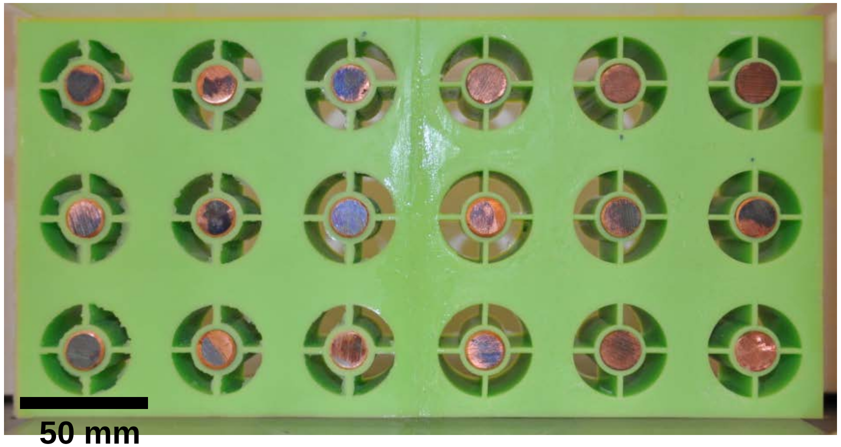
No strain



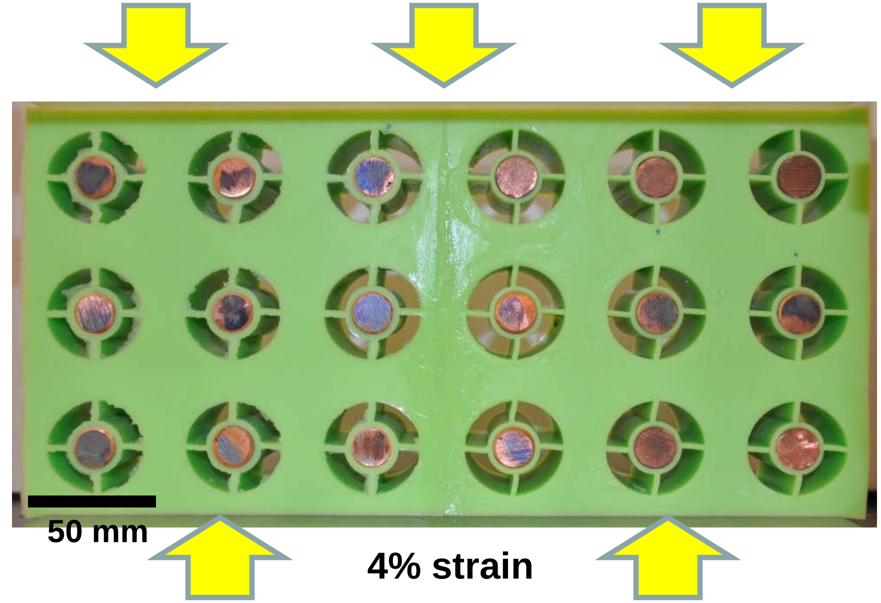
2% strain

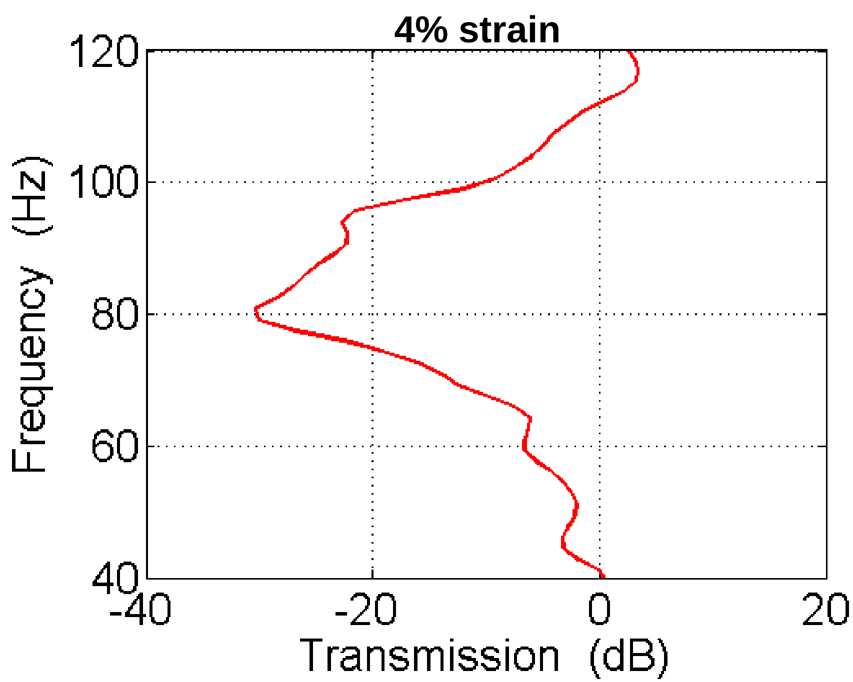


Experimental results

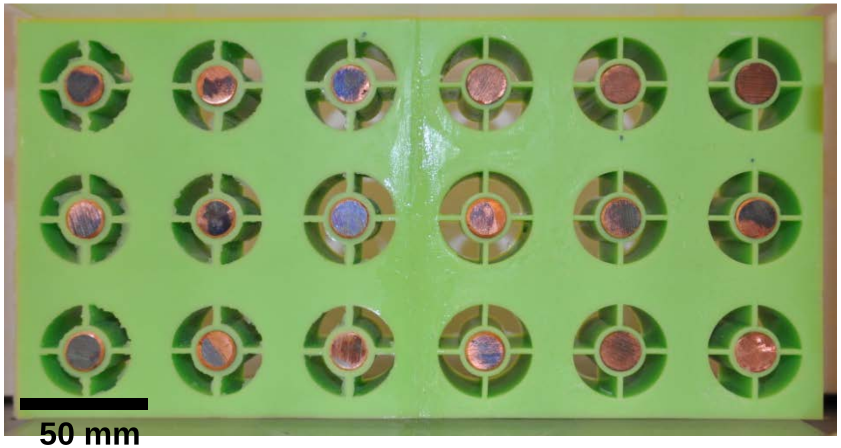


No strain

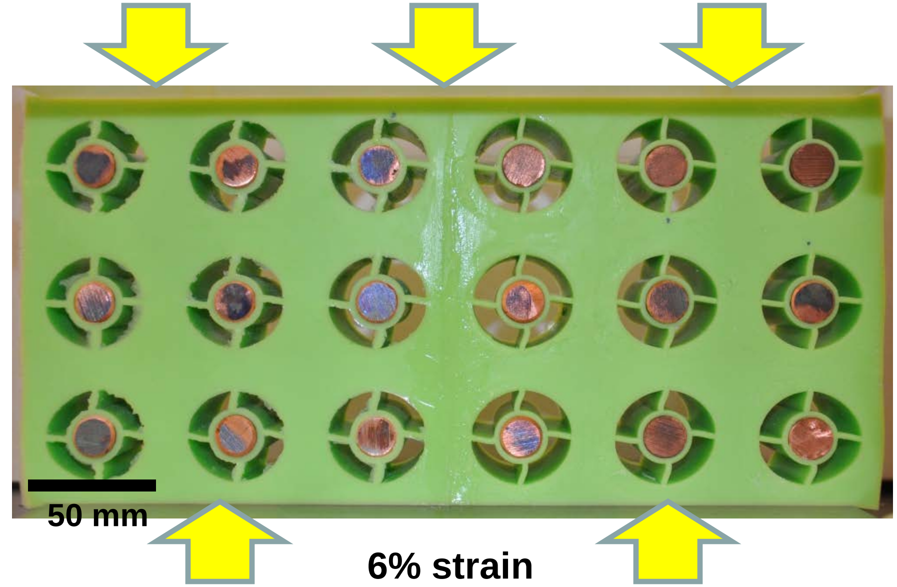




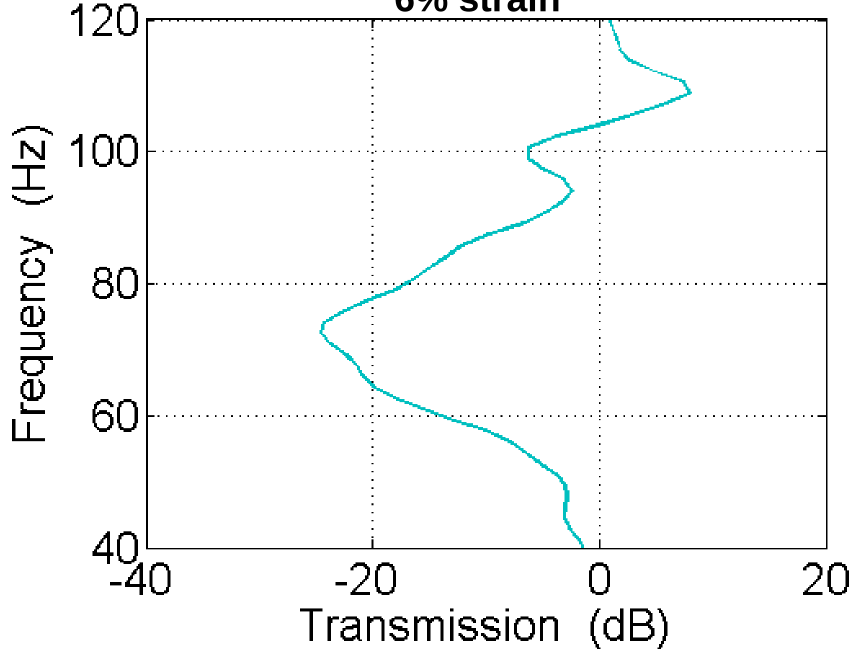
Experimental results



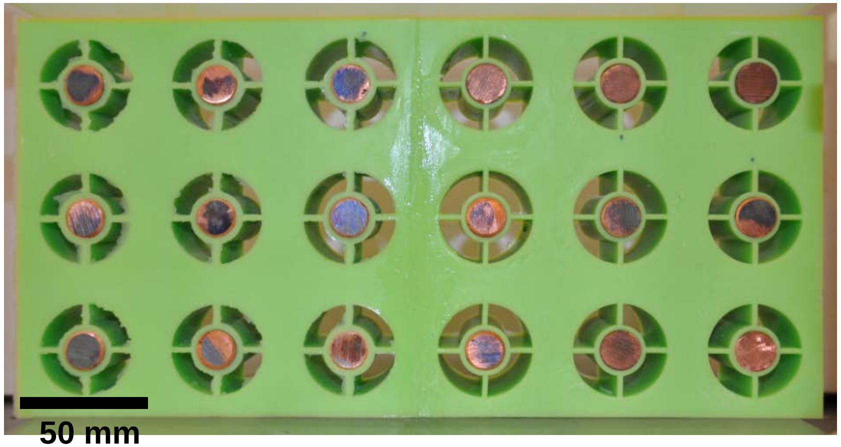
No strain



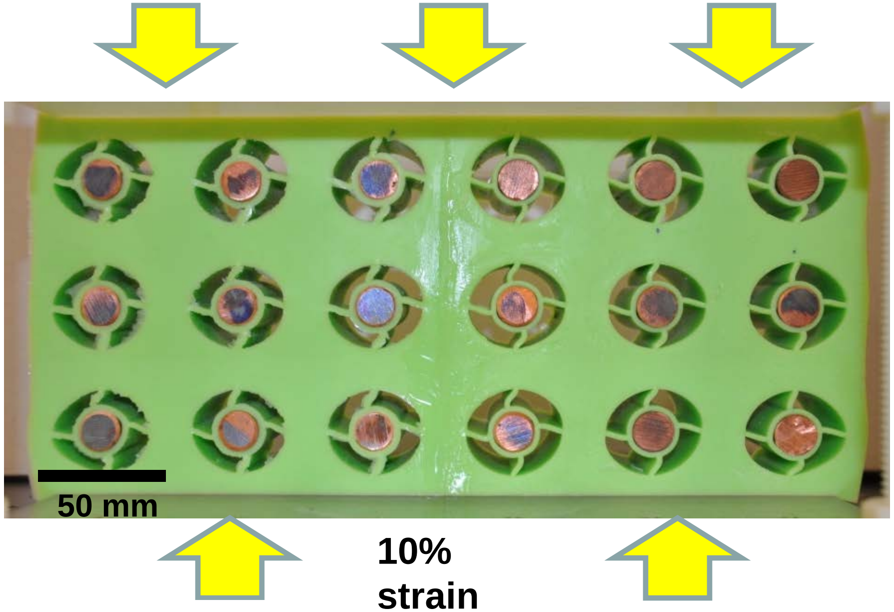
6% strain



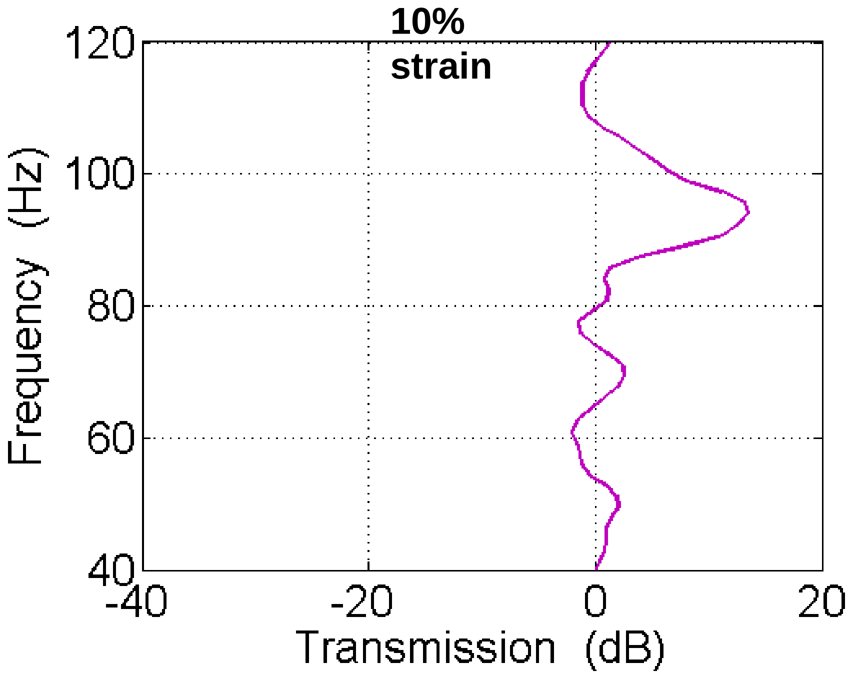
Experimental results



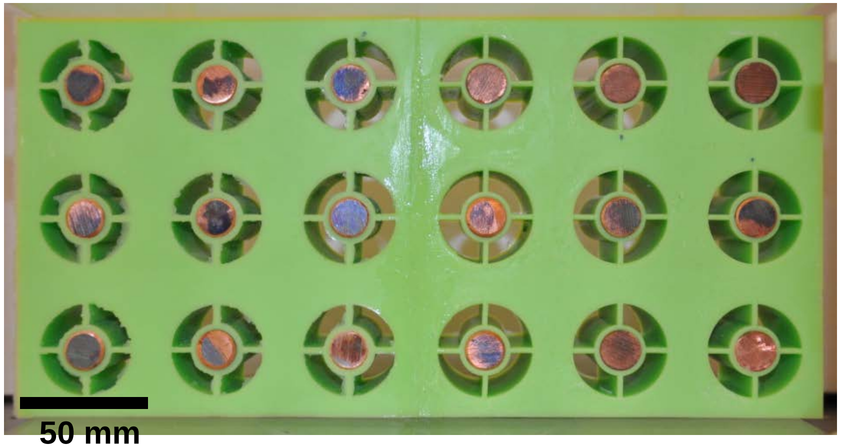
No strain

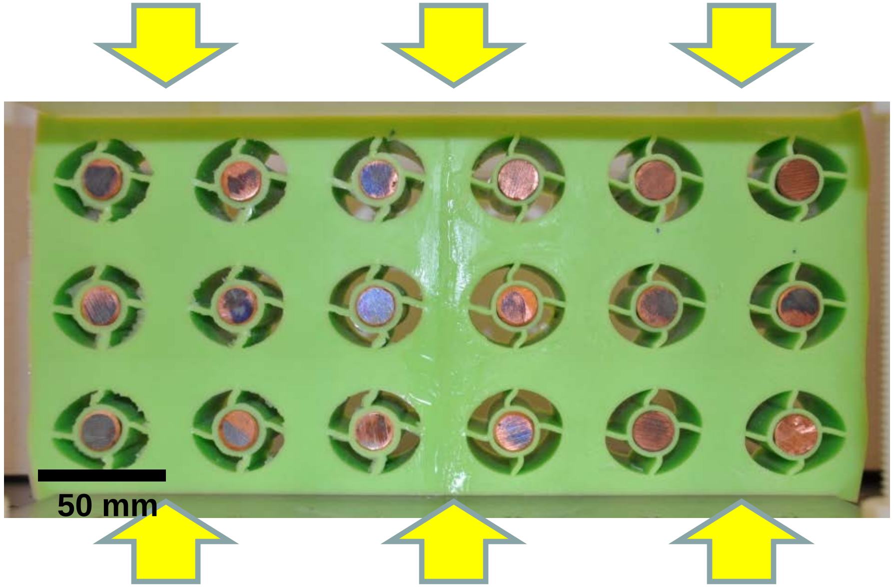


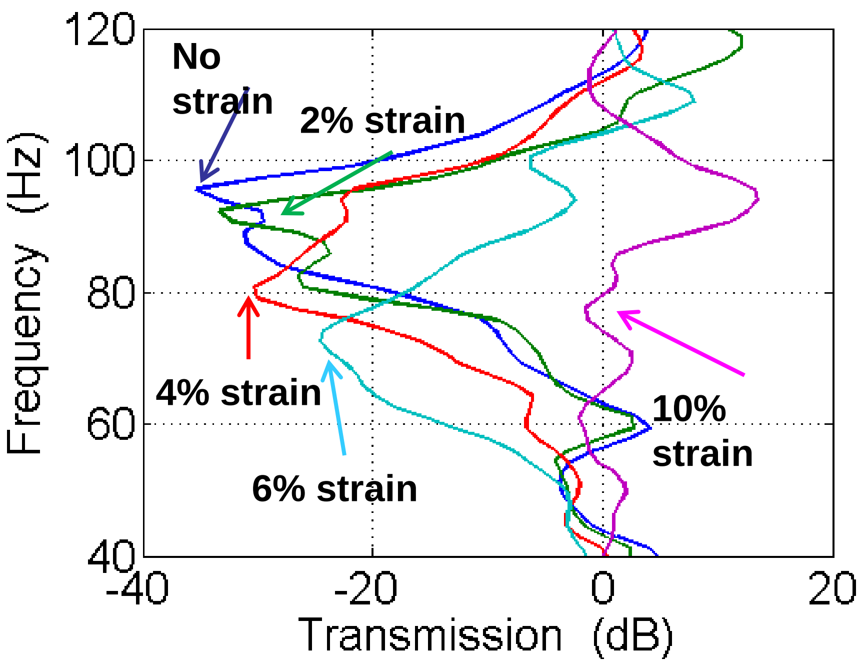
10% strain

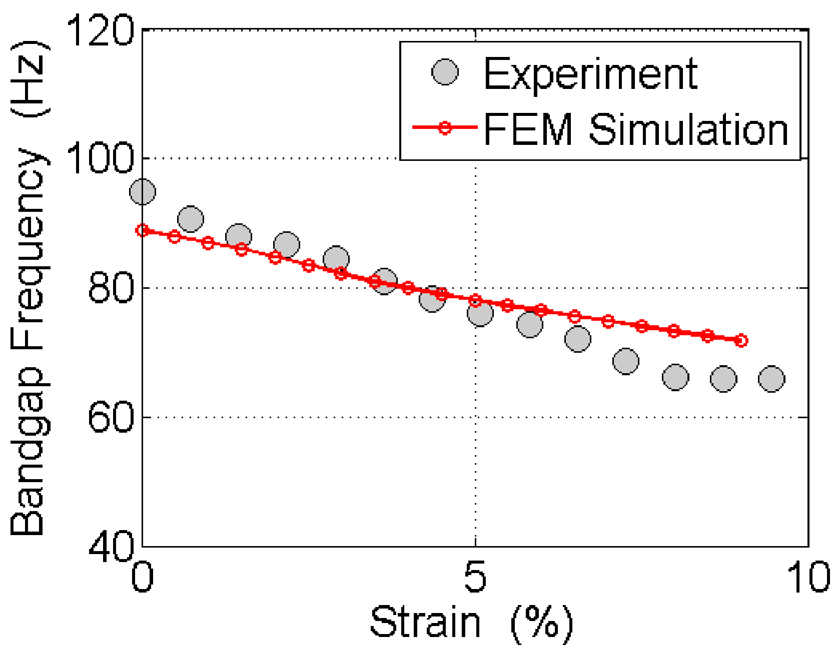


Experimental results

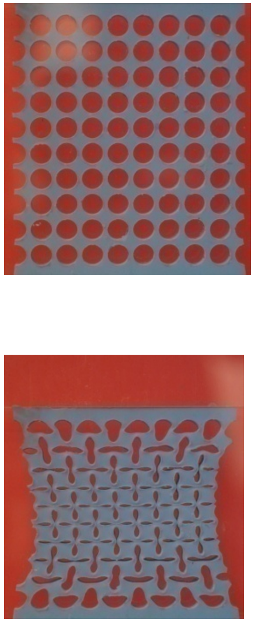








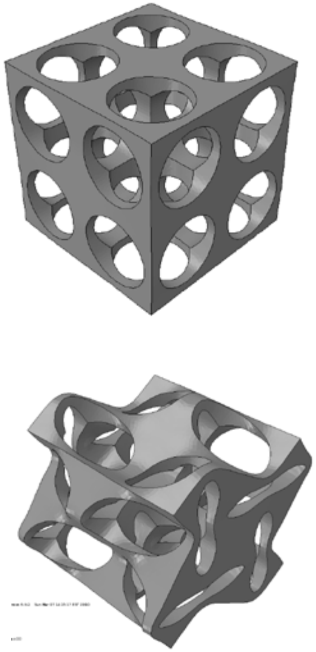
Structured Spherical Shells



2D Continuum Structure

Continuum Version?

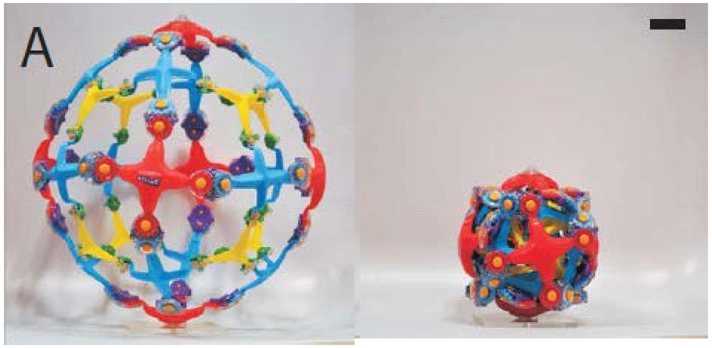
Hoberman Twist-O



3D Continuum Structure

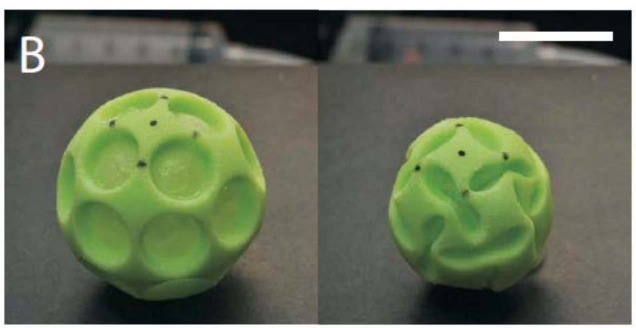
Continuum structure

……from "rigid" Hoberman Twist-O



- Opportunities for reversible encapsulation
- Large number of hinges required
- Challenging fabrication at the micro and nano scale

……to Buckliball: a continuum shell that uses **buckling** to expand/contract



- No hinges
- Actuation mechanism works over a wide range of scales

Design parameters:

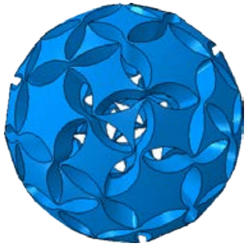
Arrangement of holes

$$\frac{\textit{Volume of Void}}{\textit{Volume of Sphere}}$$

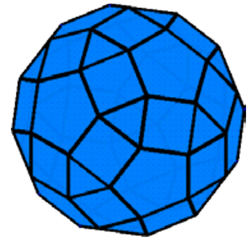
$$\frac{\textit{Shell Thickness}}{\textit{Sphere Inner Radius}}$$

Arrangement of holes

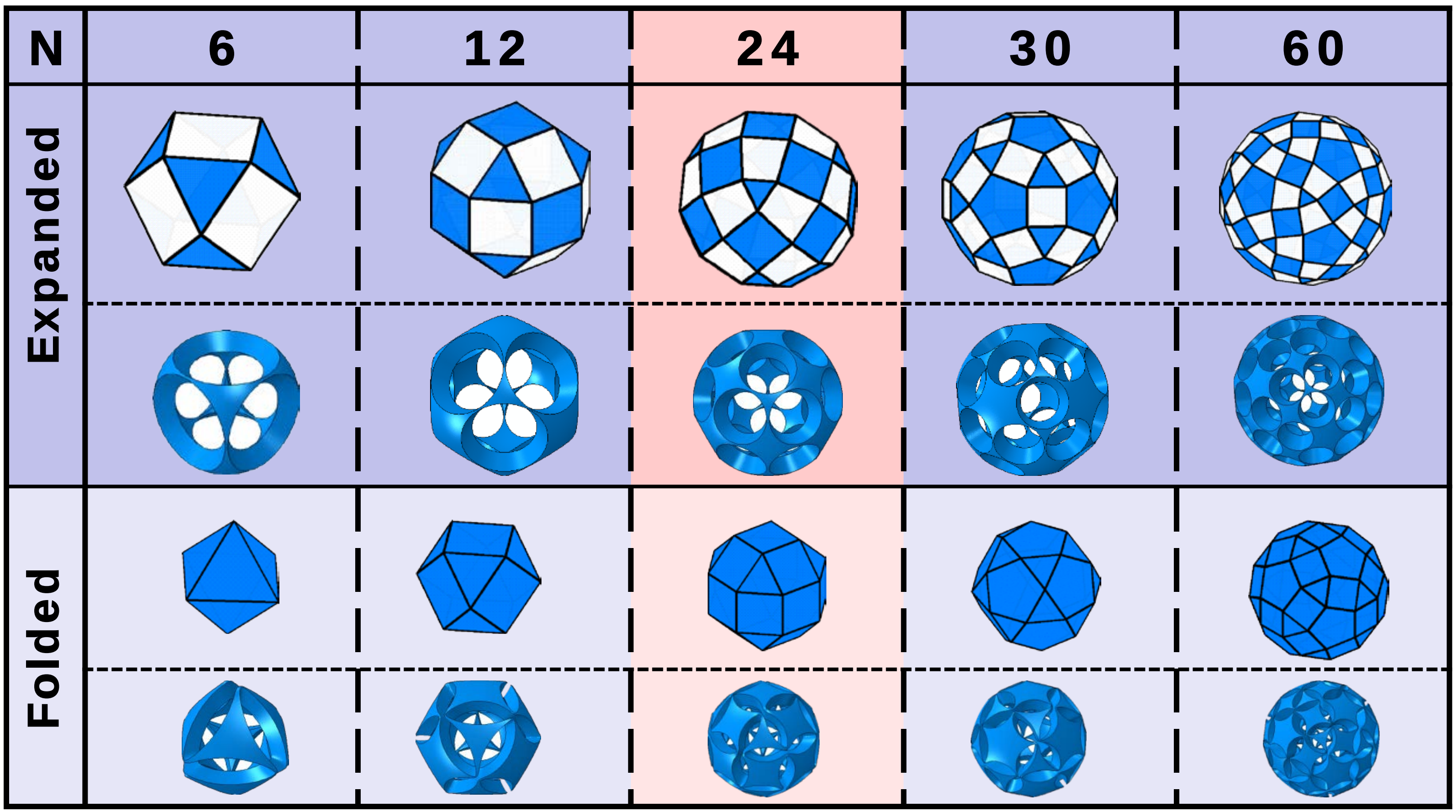
Shim J, Perdigou C, Chen ER, Bertoldi K, Reis P, PNAS, 2012



Folded sphere:
1) spherical shape
2) all holes with the same shape
3) equally distributed holes
4) closed holes

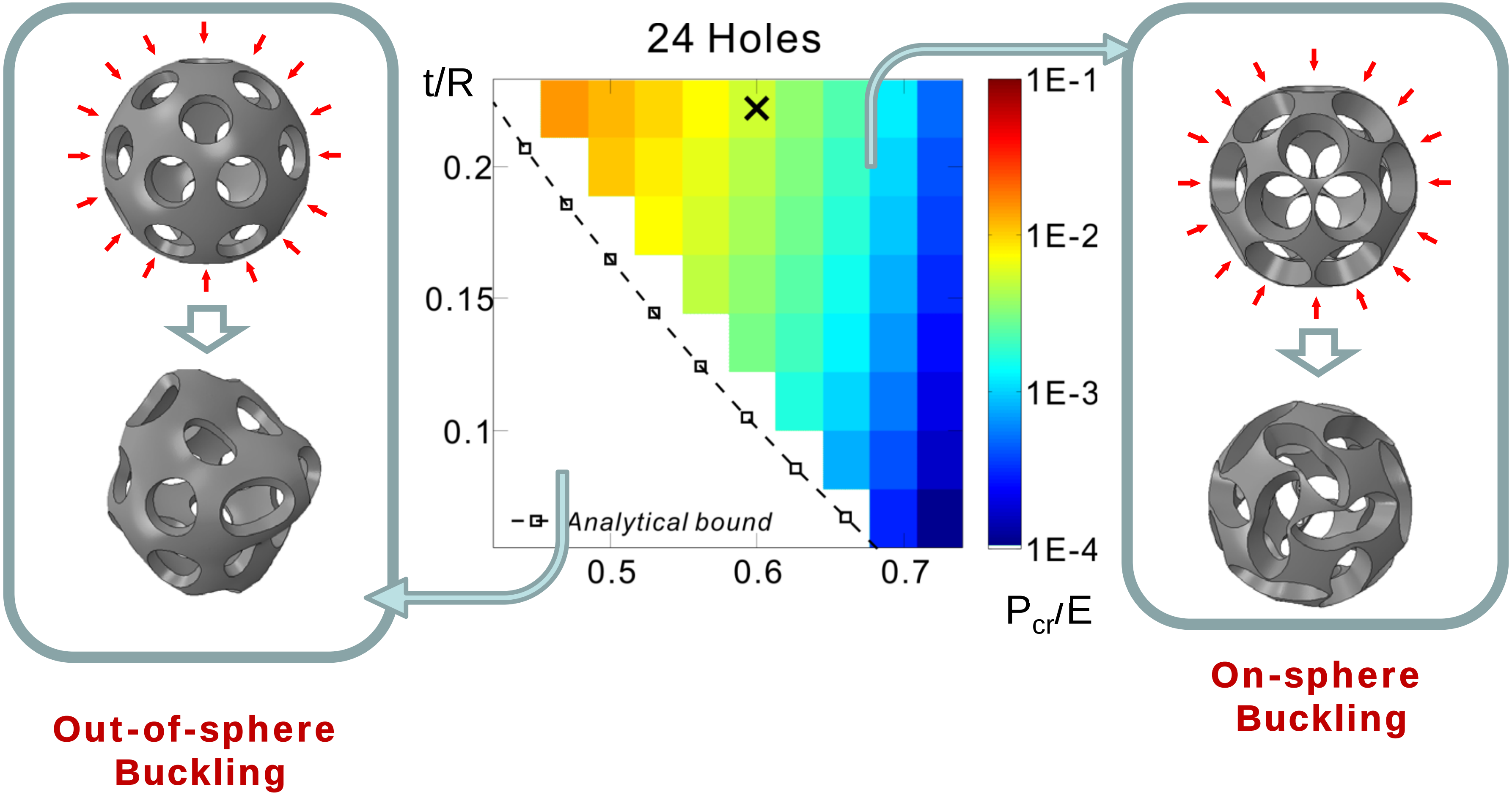


Polyhedra
1) convex polyhedra
2) vertex-transitive
3) regular face
4) quadrilateral vertex figure



24 holes: Shell thickness and Void volume fraction

Finite Element Simulations: Buckling analysis



Experiment: Buckliball

$$R_{in} = 22.5\,mm$$

$$t_{shell} = 5.0\,mm$$

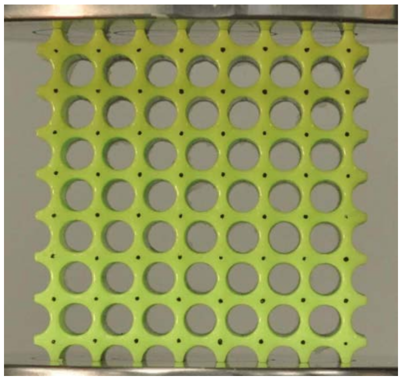
$$\bar{h}_{membrane} = 0.4 \pm 0.1 mm$$

$$Porosity = 60\%$$

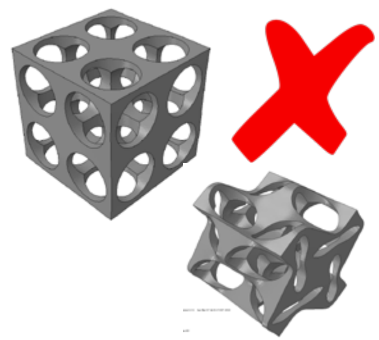
3D soft and reconfigurable materials

Goal: design a new class of 3-D materials whose architecture can be dramatically changed in response to an external stimulus

2D

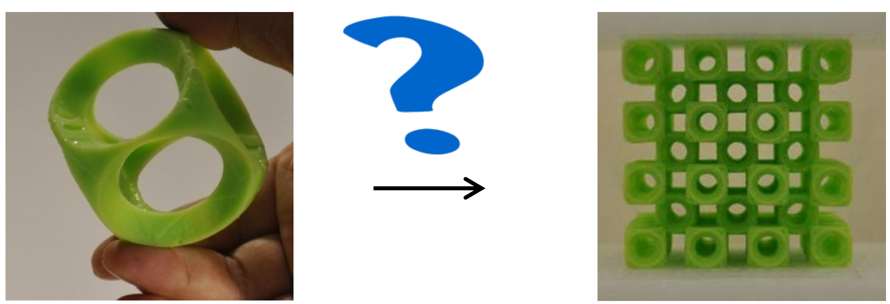


3D



Shells/Buckliball

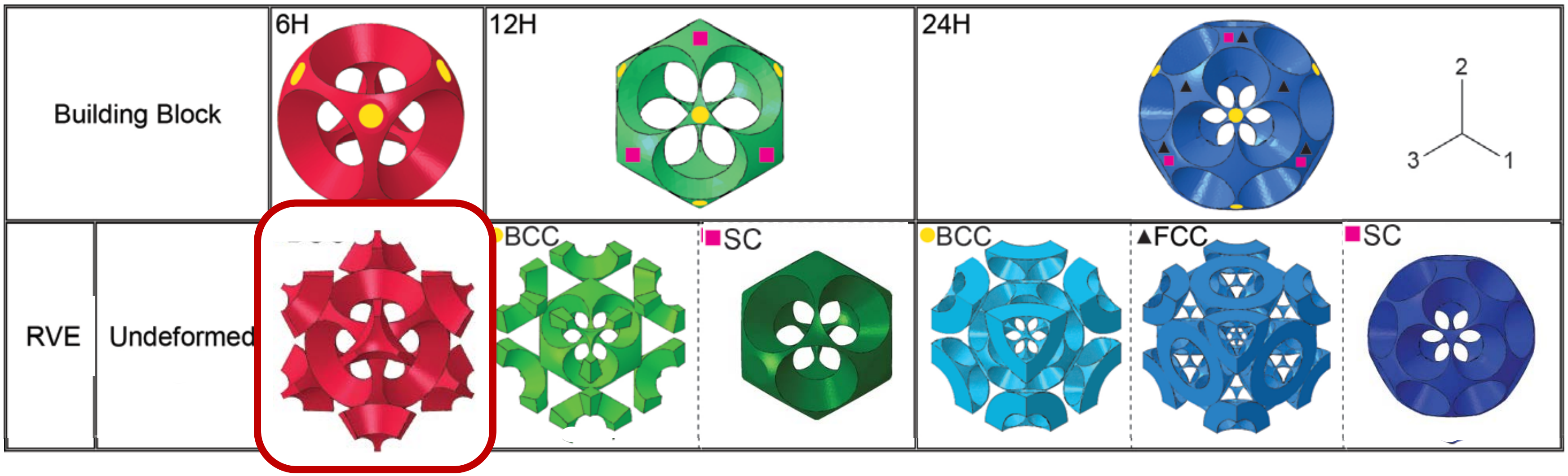
Idea: We use elastomeric Buckliballs as building blocks for 3-D reconfigurable structures.



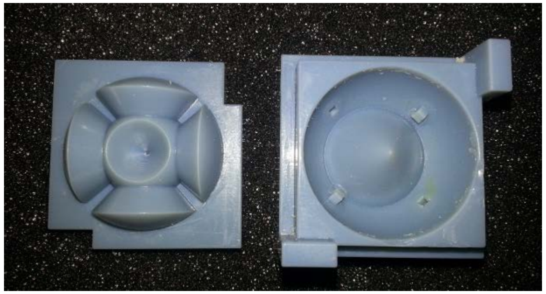
Packing

Babaee S, Shim J, Weaver JC, Patel N, Bertoldi K. Advanced Materials, 2013

Building blocks: Buckliballs
Packing: cubic lattice systems (*sc, bcc, fcc*) ⟶ **6 bucklicrystals**



Experiments



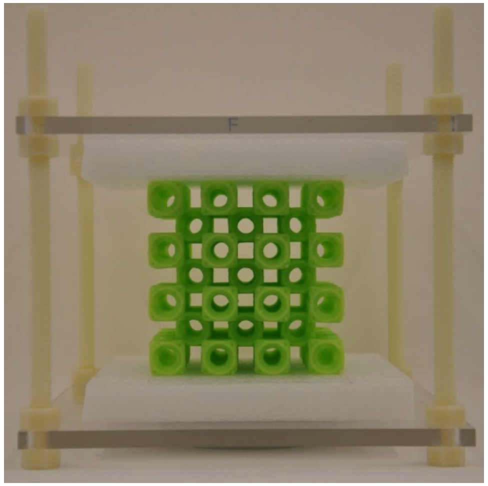
Bucklicrystal

Building block (6H)

$$\tau = \frac{t}{R_i} = \frac{7.1\ mm}{9.9\ mm} = 0.71$$

$$\psi = 0.73$$

BCC →

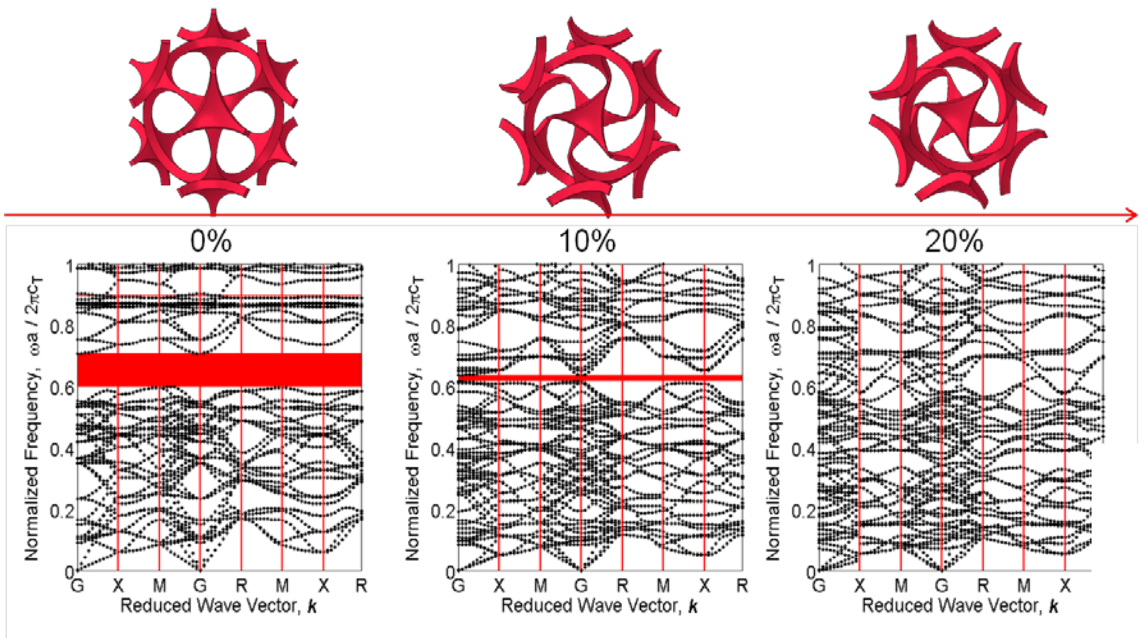


91 building blocks

Cube with edge length = 141.3 mm

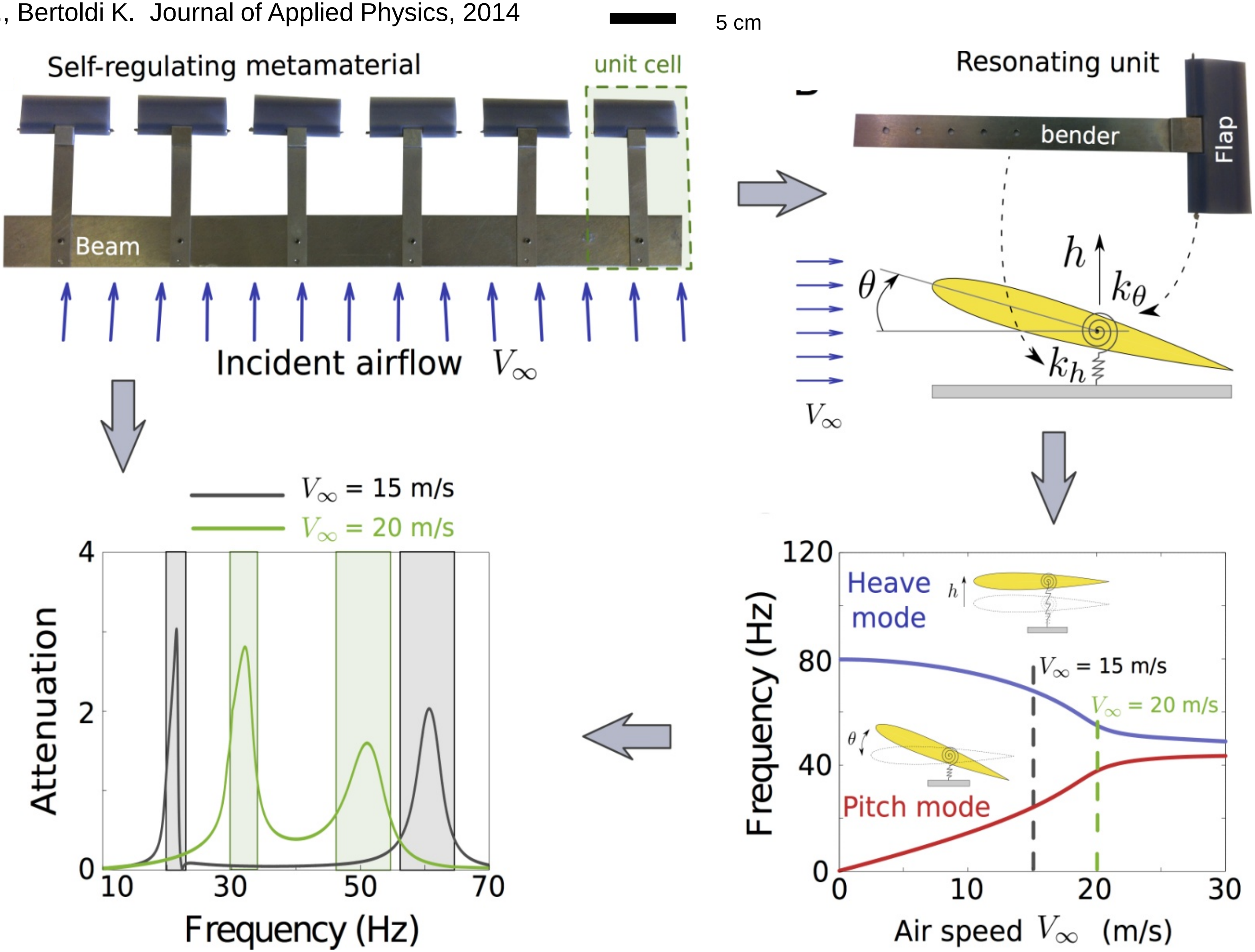
Wave propagation

Can we exploit the large deformation induced by buckling to tune the propagation of waves both in the matrix and in air?

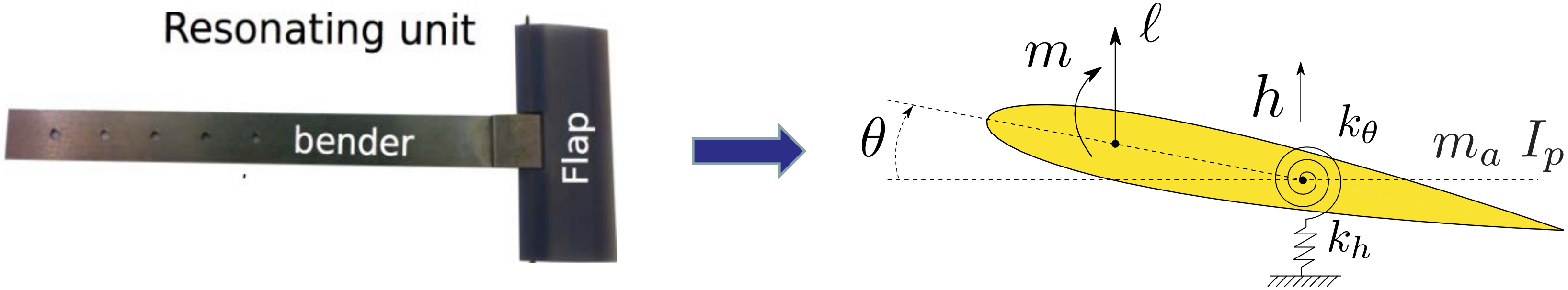


Harnessing fluid-structure interactions

Casadei F., Bertoldi K. Journal of Applied Physics, 2014



Airfoil Resonating Unit



The resonating unit is modeled as a rigid airfoil with two degrees of freedom

$$m_a \ddot{h} + m_a\, b\, x_\theta\, \ddot{\theta}(t) + k_h h = \ell(t),$$

$$m_a\, b\, x_\theta\, \ddot{h}(t) + I_a \ddot{\theta}(t) + k_\theta \theta(t) = m(t) + b\left(\frac{1}{2} + a_f\right)\ell(t),$$

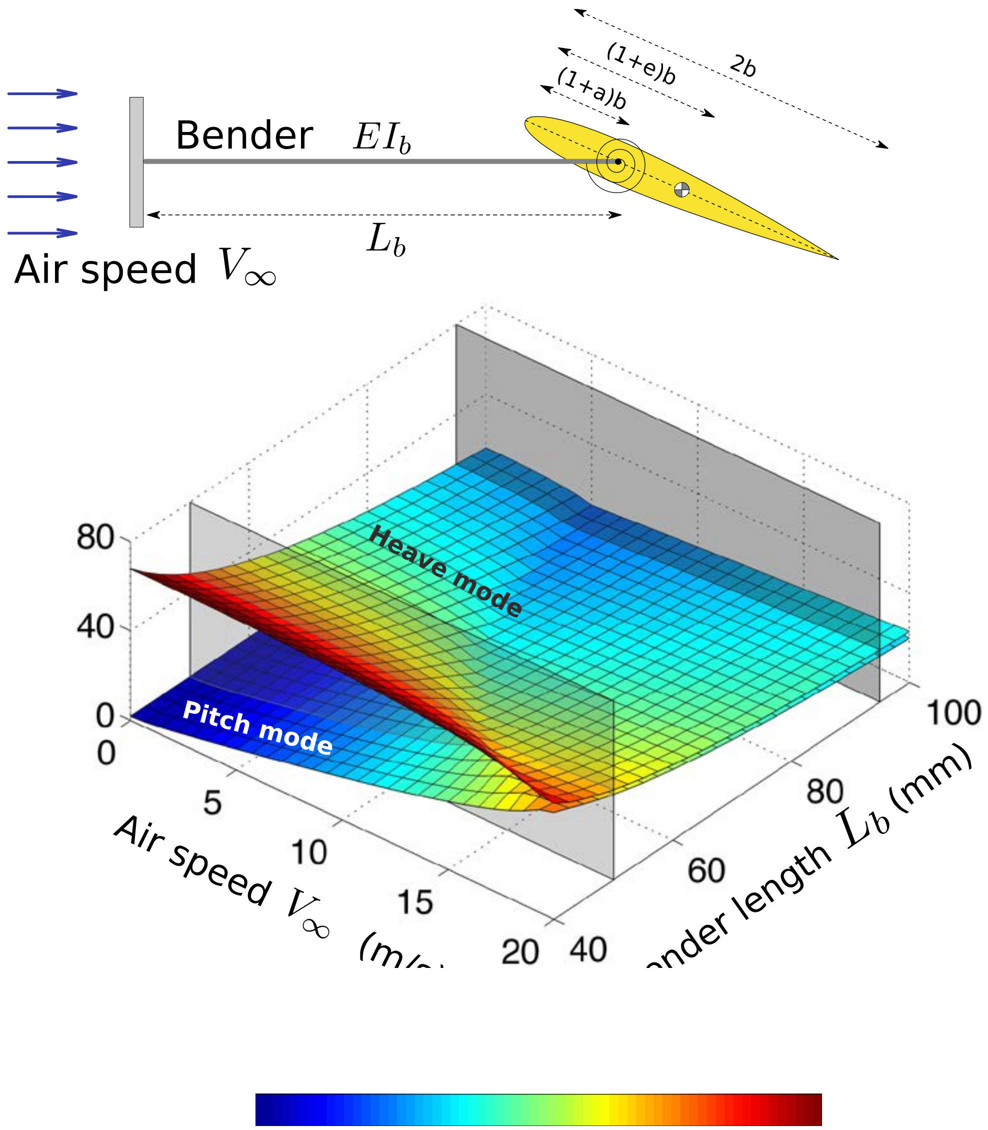
A finite-state induced flow theory is used to model the unsteady aerodynamic loads on the airfoil (Peters et al. 1995)

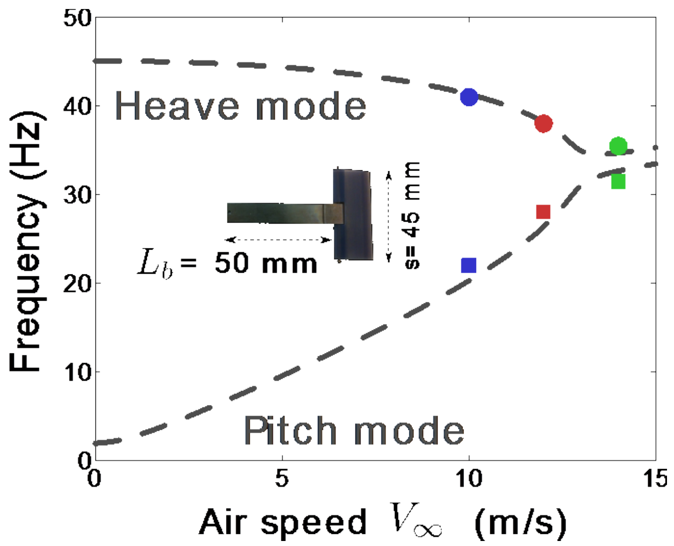
$$\ell(t) = \pi\rho_\infty b^2\left(\ddot{h}(t) + V_\infty\dot{\theta}(t) - ba\ddot{\theta}(t)\right) + 2\pi\rho_\infty s V_\infty b\left[\dot{h}(t) + V_\infty\theta(t) + b\left(\frac{1}{2} - a\right)\dot{\theta}(t) - \frac{1}{2}\boldsymbol{b}^T\boldsymbol{\lambda}(t)\right],$$

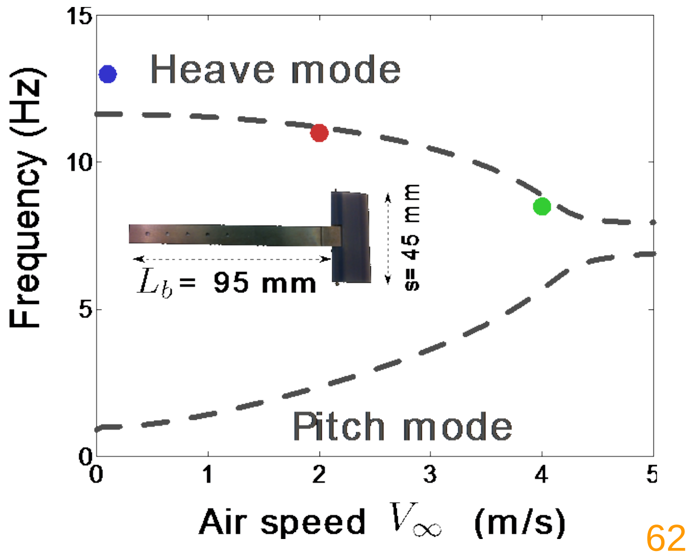
$$m(t) = -\pi\rho_\infty s b^3\left[\frac{1}{2}\ddot{h}(t) + V_\infty\dot{\theta}(t) + b\left(\frac{1}{8} - \frac{a}{2}\right)\ddot{\theta}(t)\right],$$

with $\boldsymbol{A}\,\dot{\boldsymbol{\lambda}}(t) + \dfrac{V_\infty}{b}\boldsymbol{\lambda} = \boldsymbol{c}\left[\ddot{h}(t) + V_\infty\dot{\theta}(t) + b\left(\frac{1}{2} - a\right)\ddot{\theta}(t)\right]$

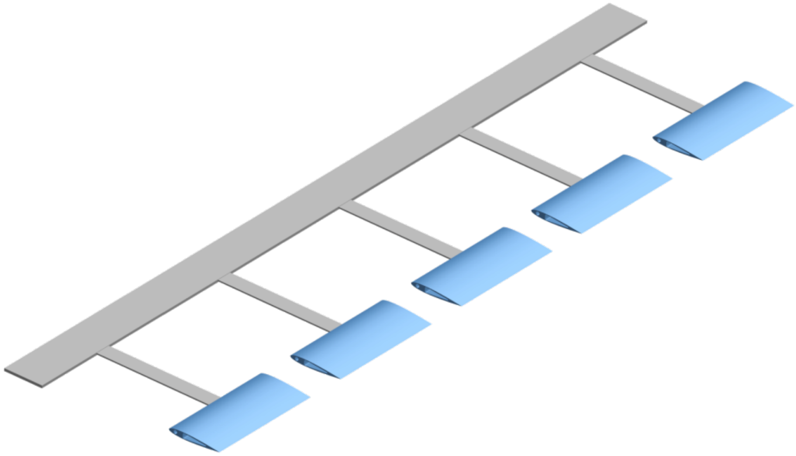
Aeroelastic response of the Flaps





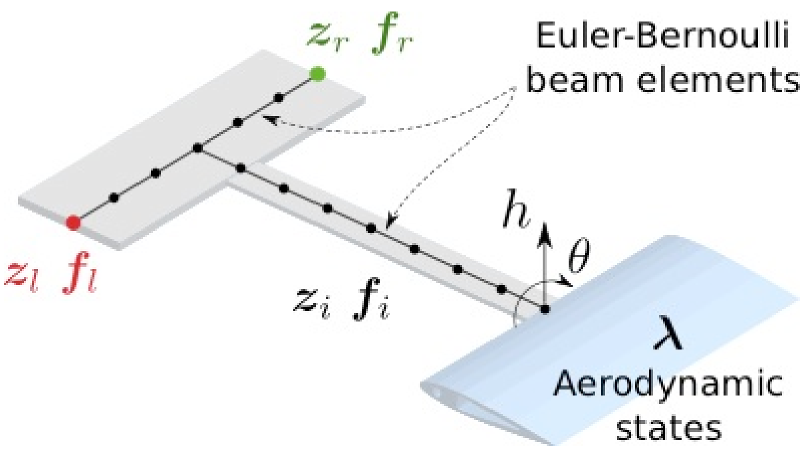


Dispersion Relations of the Beam



A transfer matrix approach is used to compute the dispersion relations of a beam with periodic airfoil-type resonators

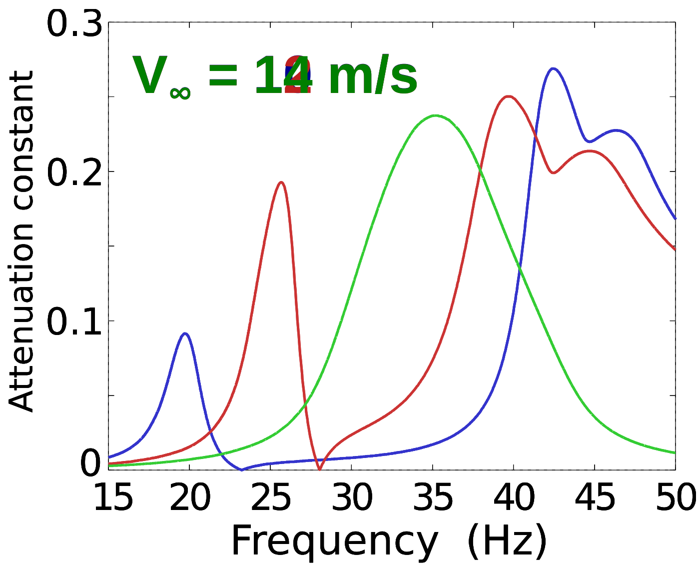
$$\left[\boldsymbol{K}_a + i\omega\,\boldsymbol{C}_a - \omega^2\,\boldsymbol{M}_a\right]\,\boldsymbol{z}(\omega) = \boldsymbol{f}(\omega)$$

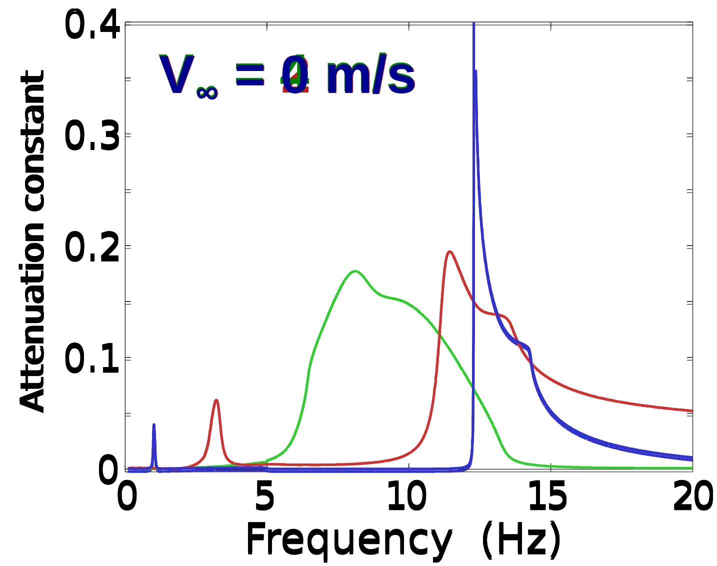


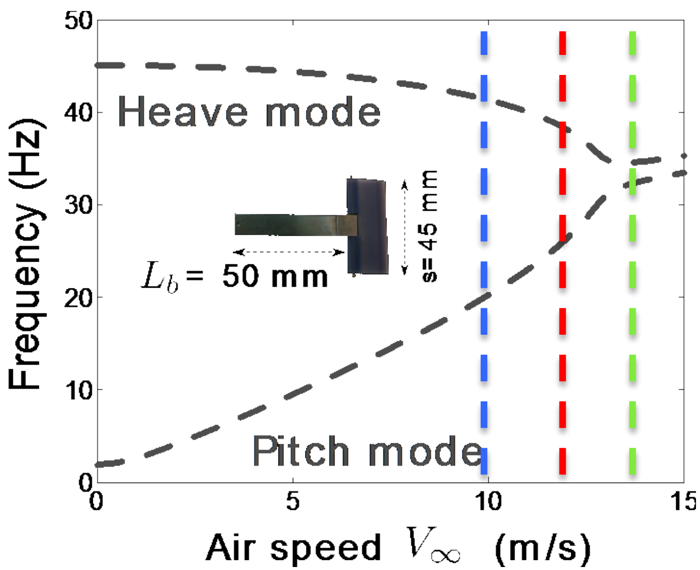
$$\begin{pmatrix} \boldsymbol{z}_r \\ \boldsymbol{f}_r \end{pmatrix} = \boldsymbol{T}(\omega, V_\infty)\begin{pmatrix} \boldsymbol{z}_l \\ \boldsymbol{f}_l \end{pmatrix} = e^{\mu(\omega)}\begin{pmatrix} \boldsymbol{z}_l \\ \boldsymbol{f}_l \end{pmatrix}$$

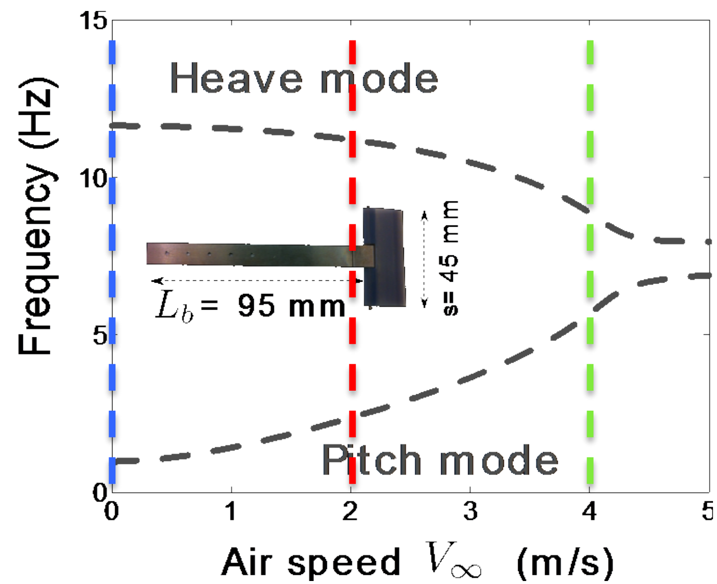
Complex propagation constant

Numerical Results

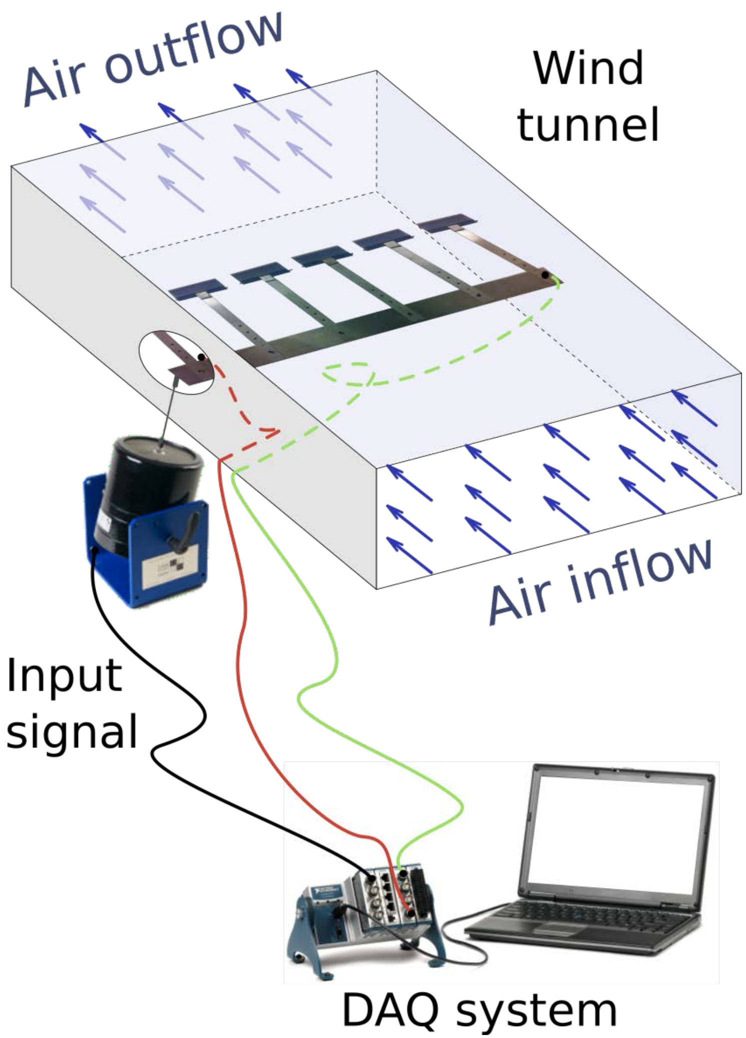


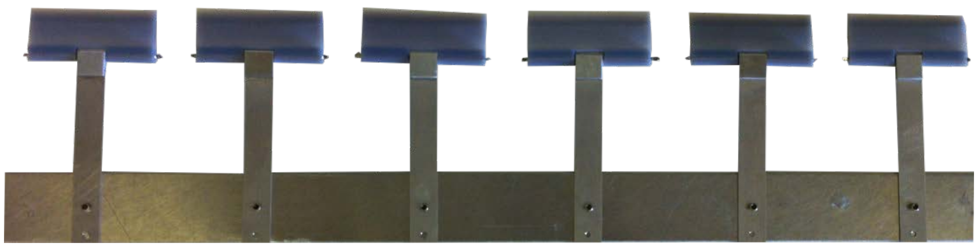






Experimental Setup



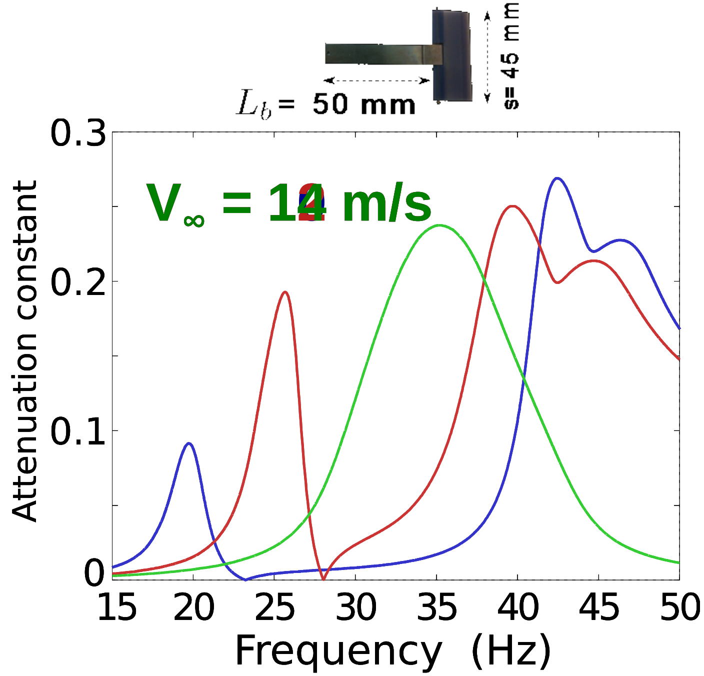


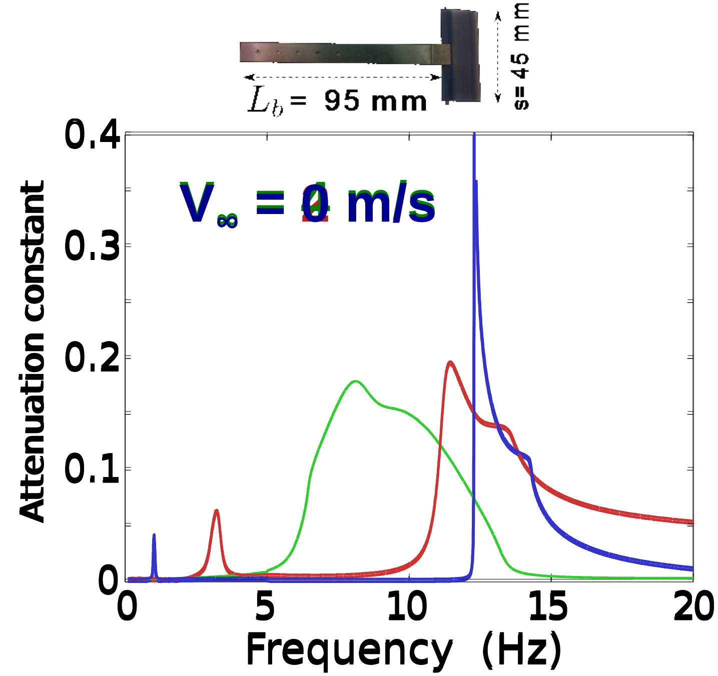
Aluminum beam
(L=360mm, w=25.4mm, t=1.27mm)

- Measure the transmission coefficient of a beam with six airfoil-type resonators
- Repeat measurements at different wind speeds

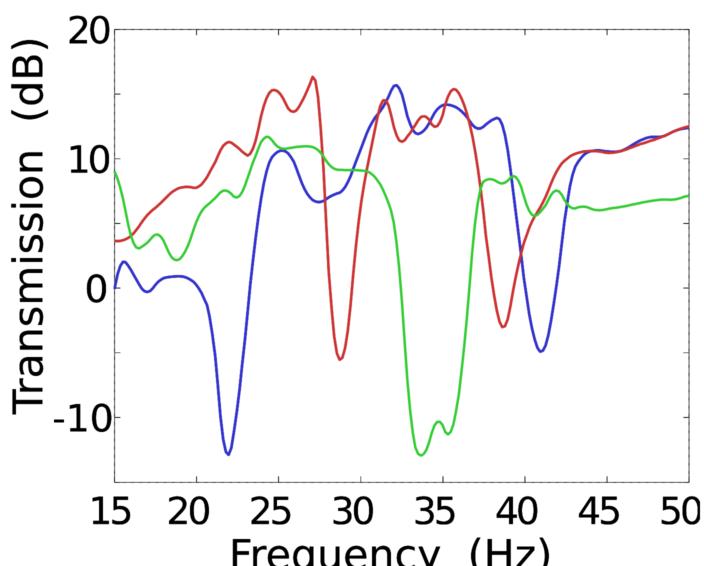
Experimental Results

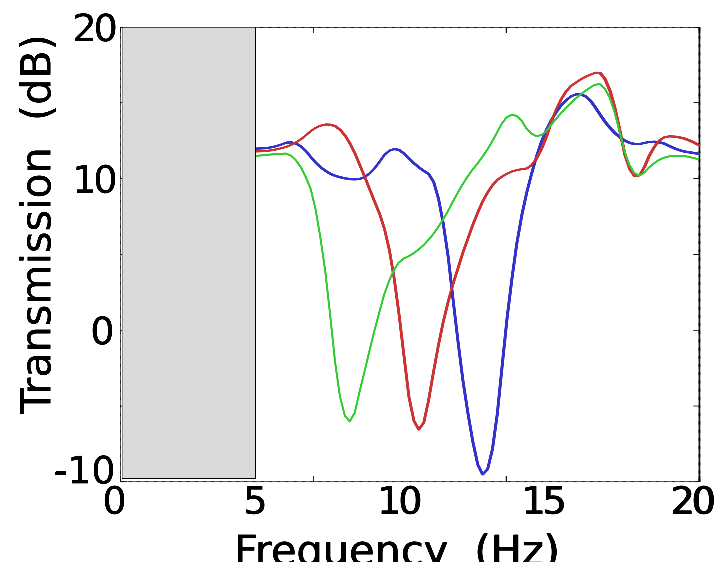
FEM





Experiments

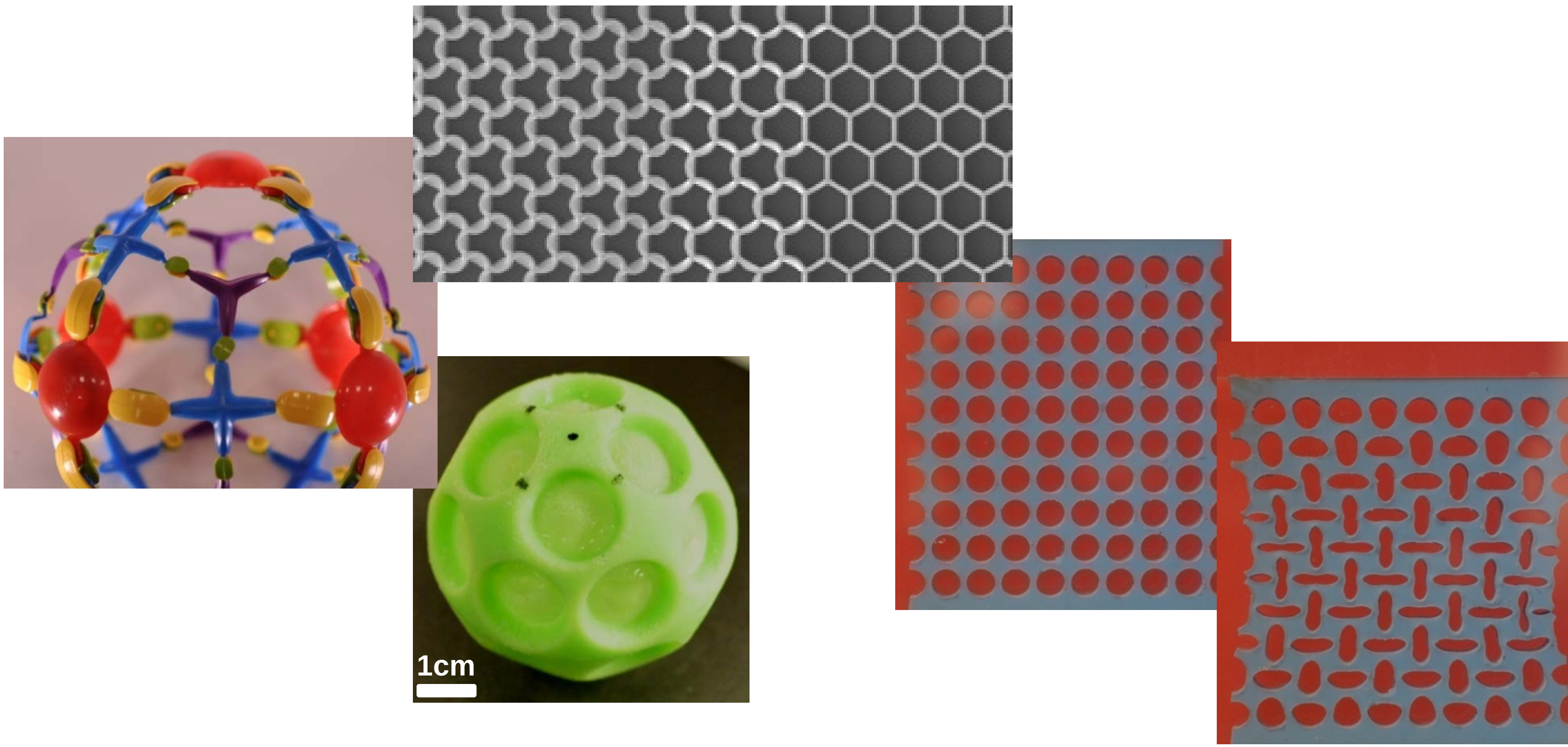




Conclusions

Non linear response of structures: Exciting playground due to the interplay of geometry and large deformation – buckling

Large deformation / instabilities and other stimuli (such as flow speed) can be exploited to design material with novel and tunable properties



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