Packet Modulation and Mode Hopping in Nonlinear Periodic Structures

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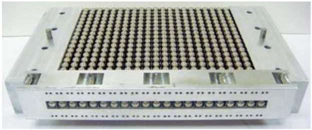
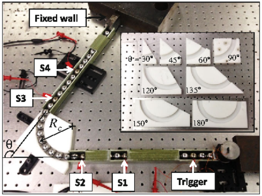
Granular Phononic Crystals - Recent Applications

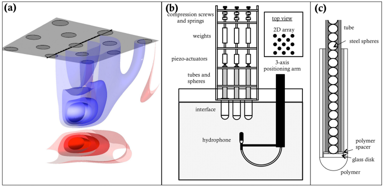
- Granular Phononic Crystals (GPC's) as tunable vibration filters, shock protectors, acoustic lenses, acoustic rectifiers etc. - [Tournat et al., 2004], [Boechler et al.,2011], [Spadoni and Daraio, 2010],[Cai et al. 2013], [Donahue et al., 2014]



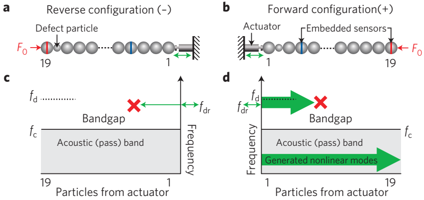
Acoustic Lens [Spadoni and Daraio, 2010]



Curved Granular Chain [Cai et al. 2013]



Acoustic Lens with Tunable Focus [Donahue et al. 2014]



Acoustic Rectification [Boechler et al. 2011]

Waves in Granular Crystals Across Regimes of Nonlinearity

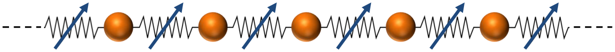
- Linear regime: granular crystals behave as conventional phononic crystals (periodic structures)
  - Bandgaps and directivity are available
- Weakly nonlinear regime: granular chains as FPU systems
  - Continuum limit: solitons (compact pulses traveling without distortion) [Zabusky and Kruskal, 1965]
  - Anticontinuum limit: discrete/dark breathers (localized standing or propagating modes) [Flach and Gorbach, 2005], [Chong et. al.,2013]
- Strongly nonlinear regime:
  - Continuum limit: waves with soliton-like characteristics [Nesterenko, 1983 & 2001]
  - Anticontinuum limit: discrete breathers

Motivation and Objectives

- Long-term objectives: Wave manipulation and control
  - Use nonlinearity to introduce and tune anisotropy and asymmetry in the response
  - Use nonlinearity to realize multifunctional acoustic logic structures and switch between functionalities
- Current objectives: Understand the fundamental mechanisms of wave propagation
  - Decode the interplay between nonlinear and dispersive mechanisms
  - Find descriptors and classifiers of nonlinear effects (packet distortion/modulation)
  - Construct inverse problems to infer parameters of nonlinearity from the wave response
  - Explore interplay between nonlinearity, topology and mode structure in crystals with topological and modal complexity

A Necessary Prequel: Chains With Quadratic Nonlinearity

- Spring-mass chain with quadratic nonlinearity - Fermi-Pasta-Ulam (FPU-α)

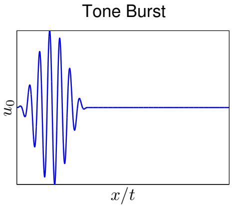


- Weak nonlinearity: $\Rightarrow F = K\,\delta + G\,\delta^2 \;,\; G << K \;,\; \overline{G} = G/K << 1$
  - $m\,\ddot{u}_n + K\,(2\,u_n - u_{n-1} - u_{n+1}) + G\,[(u_n - u_{n-1})^2 + (u_n - u_{n+1})^2] = 0$

- STRATEGY: Parametric analysis (simulations + signal processing)
  - Magnitude of nonlinearity ($\overline{G}$)
  - Dispersion (Wavenumber $\xi \in [0, \pi]$)

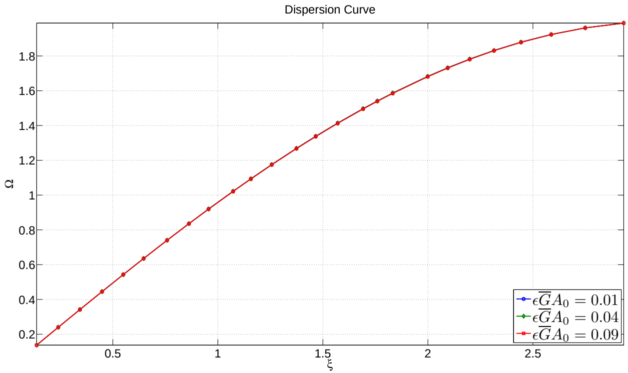
Excitation: Tone bursts (sine modulated by Hann window)

Boundary condition: $u(0,t) = u_0(t)$ (imposed carrier frequency)



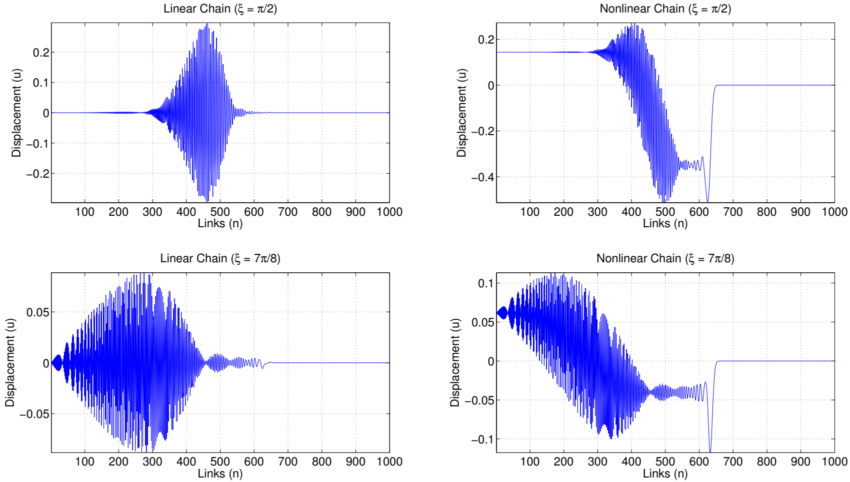
The Elusive Effect of Quadratic Nonlinearity

- The Lindstedt-Poincare method gives second order correction (after Narisetti et al., 2010)
  - $\Omega = \sqrt{2 - 2\cos(\xi)}\left(1 - \frac{1}{6}\overline{G}^2 A_0^2 \sin^2(\xi)\right)$
  - Marginal effect on dispersion curve and filtering properties



- Analytical curves show no change in cutoff frequency
- What are the effects on the spatial features of wave propagation?

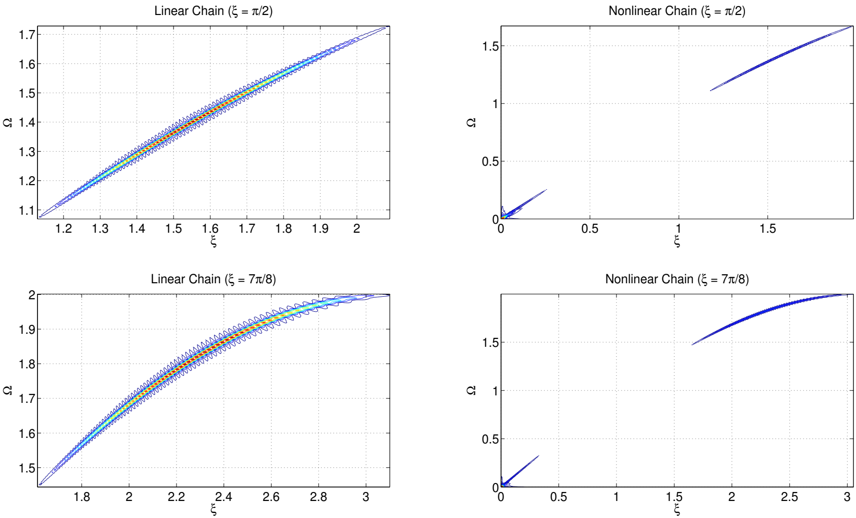
- Monitor distortion in different dispersion and nonlinearity regimes



- The packet features an oscillatory part modulated by a DC-like component

Exposing the Dominant Long-Wavelength Feature

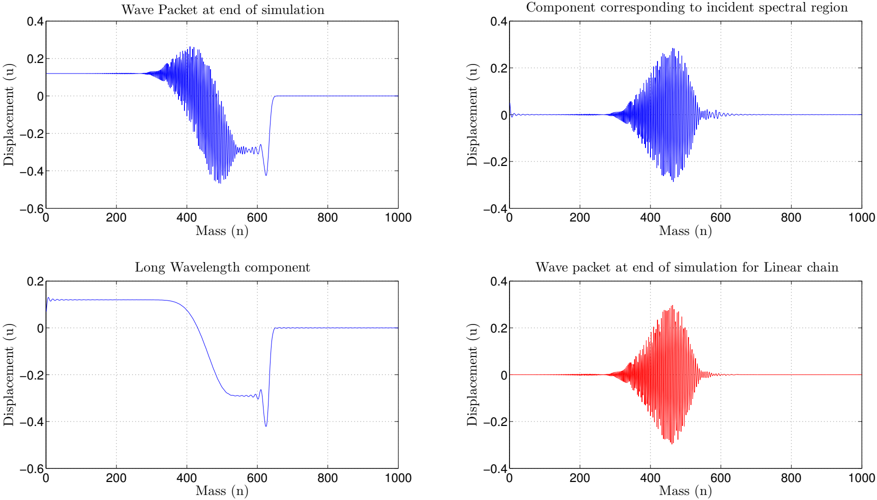
- Fourier spectra via 2D-DFT



- The spectrum is decomposed into two separable features

Isolating the Asymmetric Envelope Soliton

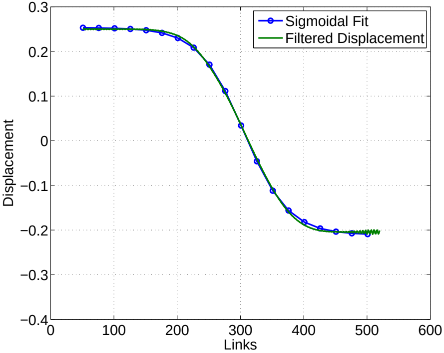
- Use spectral filtering to study/visualize the different spectral contributions



- Modulation of carrier frequency by long-wavelength sigmoidal envelope (Asymmetric Envelope Soliton) [Huang et. al., 1993]

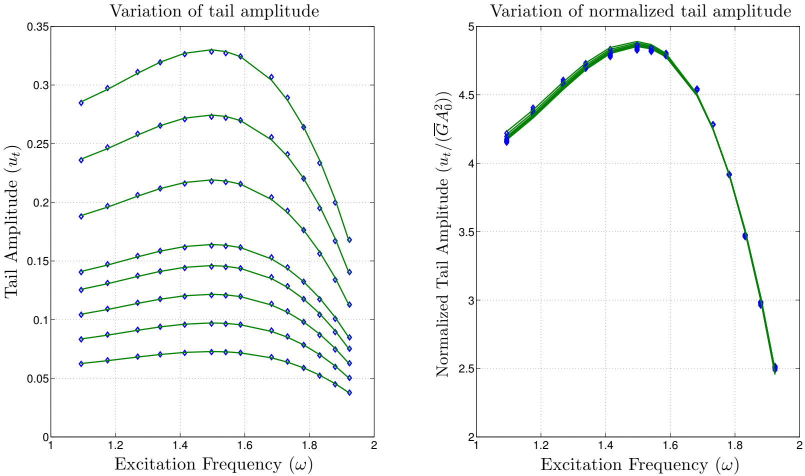
Modeling the Asymmetric Soliton as a Sigmoidal Curve

- The long-wavelength component can be modeled as a sigmoid function
- Equation for Sigmoidal Curve : $y = A + \frac{B-A}{1+e^{-(x-C)/D}}$
  - A : Tail Asymptote
  - B : Front Asymptote
  - C : Inflection point
  - D : Width of the transition zone



Invariants of Quadratic Nonlinearity

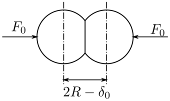
- Normalization of the curves by $\bar{G}A_0^2$ reveals interesting functional parameters



- Functions converge to a single cubic curve $u_t = \bar{G}A_0^2(\bar{a}\omega^3 + \bar{b}\omega^2 + \bar{c}\omega + \bar{d})$
- Coefficients of the curve are invariants of weak quadratic nonlinearity

Monoatomic Granular Chains

Granular chain of spheres

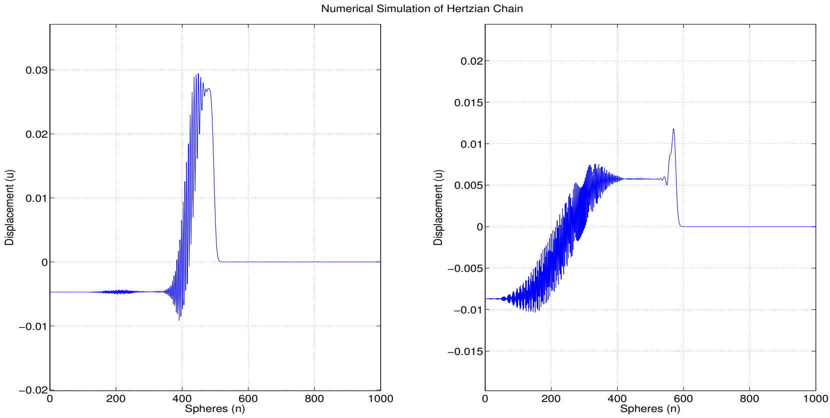


Precompression

- $m\,\ddot{u}_n \;-\; A\,[\delta_0 + u_{n-1} - u_n]_+^p \;+\; A\,[\delta_0 + u_n - u_{n+1}]_+^p = 0$
- Hertzian power law : $p = 3/2$, $A = \frac{E\sqrt{2R}}{3(1-\nu^2)}$
- Strong precompression : $\|u_{n\pm1} - u_n\| \ll \delta_0$ recovers the Fermi-Pasta-Ulam problem
  - $m\ddot{u}_n + K(2u_n - u_{n-1} - u_{n+1}) + K_2((u_n - u_{n-1})^2 - (u_{n+1} - u_n)^2) + K_3((u_n - u_{n-1})^3 - (u_{n+1} - u_n)^3) = 0$
  - $K = \frac{3}{2}A\delta_0^{1/2}$ $K_2 = -\frac{3}{8}A\delta_0^{-1/2}$ $K_3 = -\frac{1}{16}A\delta_0^{-3/2}$

Asymmetric Envelope Solitons in Granular Chains

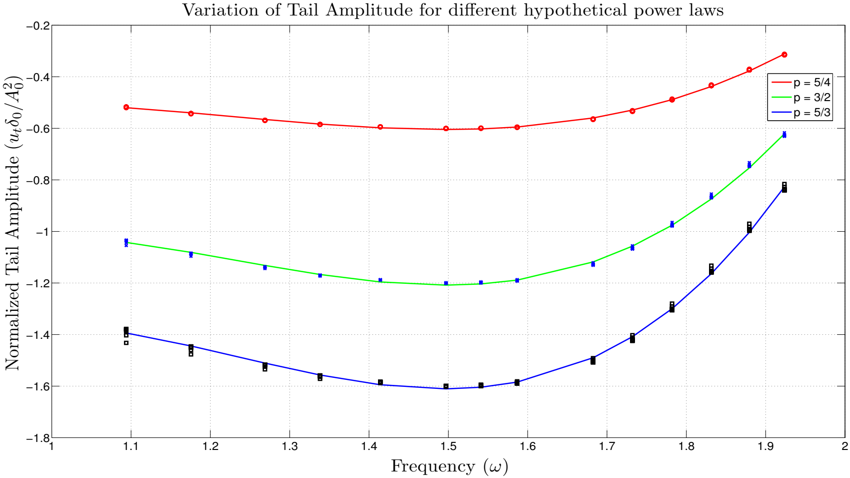
- Long chain: 1350 steel spheres: $D = 9.525$mm; $E = 210$GPa; $\nu = 0.3$; $\rho = 8000$kg$/$m^3
- Burst specified as boundary condition
- Amplitude of burst : $1/10^{th}$ of initial precompression



Wave motion characteristics similar to FPU-α (quadratic nonlinearity) chains

Confirming the Cubic Frequency Dependence

- Simulation of monoatomic granular chains with variable p



$$u_t = a\omega^3 + b\omega^2 + c\omega + d$$

An Inverse Problem for Granular Chains

- Quadratic term (K_2) dominates the response!
- Equivalent quadratic nonlinear chain

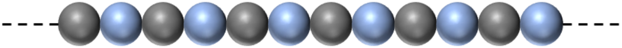
  $$u_t = \overline{G} A_0^2 (\overline{a}\omega^3 + \overline{b}\omega^2 + \overline{c}\omega + \overline{d})$$

  - $\overline{G} = -\frac{K_2}{K} = -\frac{p-1}{2\delta_0}$
  - $\overline{a}, \overline{b}, \overline{c}, \overline{d}$ are predetermined invariants for quadratic nonlinearity

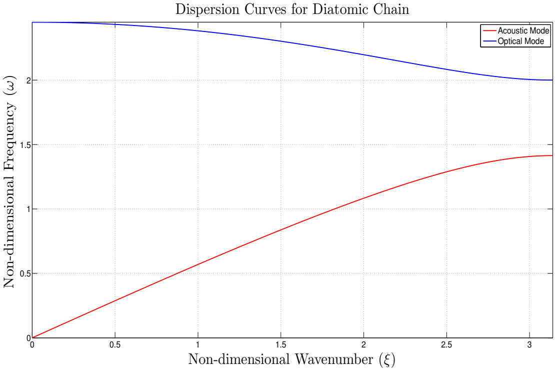
- $u_t = a\omega^3 + b\omega^2 + c\omega + d$
- $\frac{a}{\overline{a}}$ gives the scaling due to the nonlinearity associated with the established power law
- Closed form expression to estimate power law : $p = 1 - \frac{2\delta_0}{A_0^2}\frac{a}{\overline{a}}$

Imposed power law	Reconstructed from curve	Error %
$p = 5/4 = 1.25$	1.2473	0.2159
$p = 3/2 = 1.5$	1.4948	0.3401
$p = 5/3 = 1.667$	1.6613	0.3198

Diatomic Granular Chains: The Effects of Mass Contrast

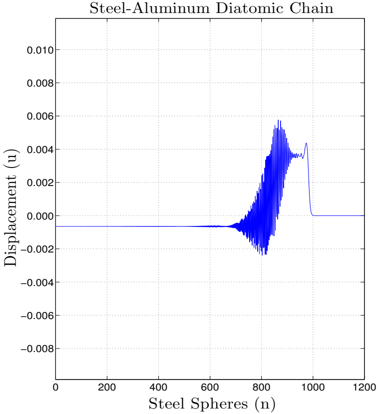


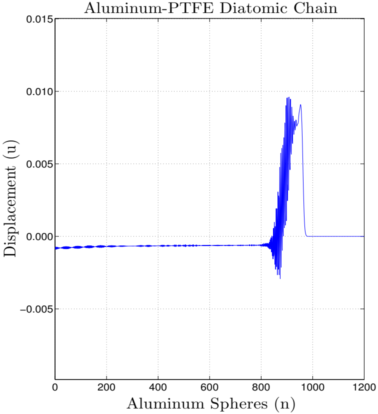
- $M\,\ddot{u}_n \;-\; A\,[\delta_0 + v_{n-1} - u_n]_+^p \;+\; A\,[\delta_0 + u_n - v_{n+1}]_+^p = 0$
- $m\,\ddot{v}_n \;-\; A\,[\delta_0 + u_{n-1} - v_n]_+^p \;+\; A\,[\delta_0 + v_n - u_{n+1}]_+^p = 0$
- Two particle unit cell gives rise to two modes of wave propagation : Acoustic and Optical



Packet Modulation in Diatomic Granular Chains

- Three different diatomic Chains : Steel-Aluminum(Al), Steel-PTFE, Al-PTFE

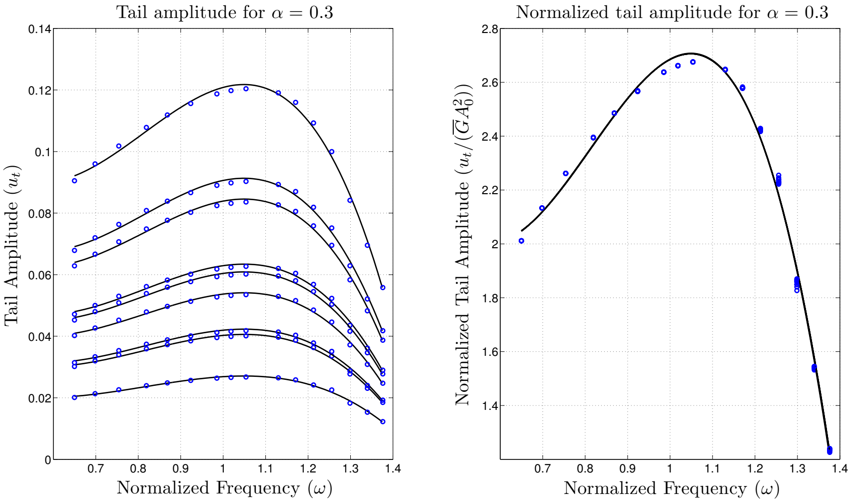




- Long-wavelength component similar to monoatomic chain
- Effect of mass ratio ($m/M = \alpha$)?

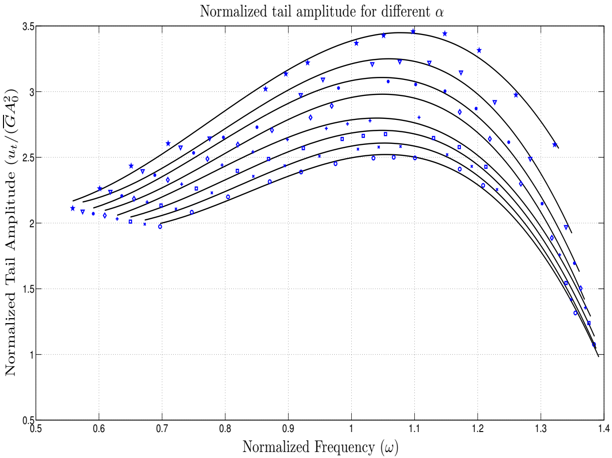
Another Short Digression into Quadratic Nonlinearity

- Diatomic Spring-Mass Chain with quadratic nonlinearity for a given α



- Tail Amplitude has same dependence on nonlinearity and amplitude of excitation as the monoatomic chain

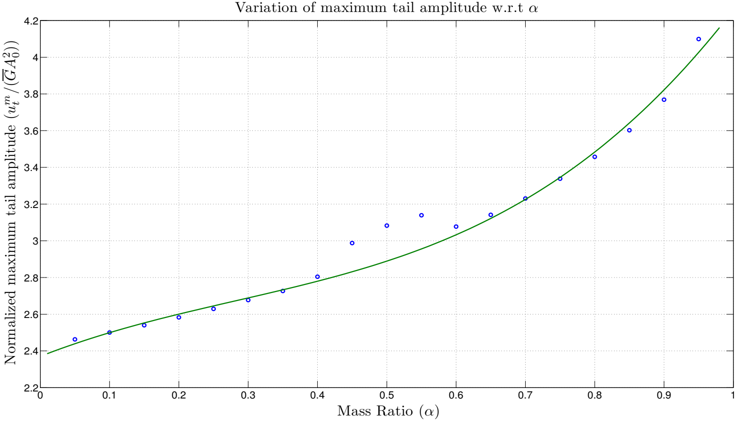
Parametrization of Tail Amplitude for Different α



α	$\omega\|_{u_t^m}$
0.1	1.0537
0.15	1.0511
0.2	1.0479
0.25	1.0469
0.3	1.0486
0.35	1.0479
0.4	1.0566
0.45	1.0620
0.5	1.0532
0.55	1.0505
0.6	1.0576
0.65	1.0636
0.7	1.0679
0.75	1.0730
0.8	1.0826

- $u_t = \overline{G}A_0^2(A(\alpha)\omega^3 + B(\alpha)\omega^2 + C(\alpha)\omega + D(\alpha))$
- $\omega|_{u_t^m} = \frac{-B(\alpha)+\sqrt{B^2(\alpha)-3A(\alpha)C(\alpha)}}{3A(\alpha)}$
- ω for u_t^m is independent of α

Maximum Tail Amplitude as Invariant of Nonlinearity

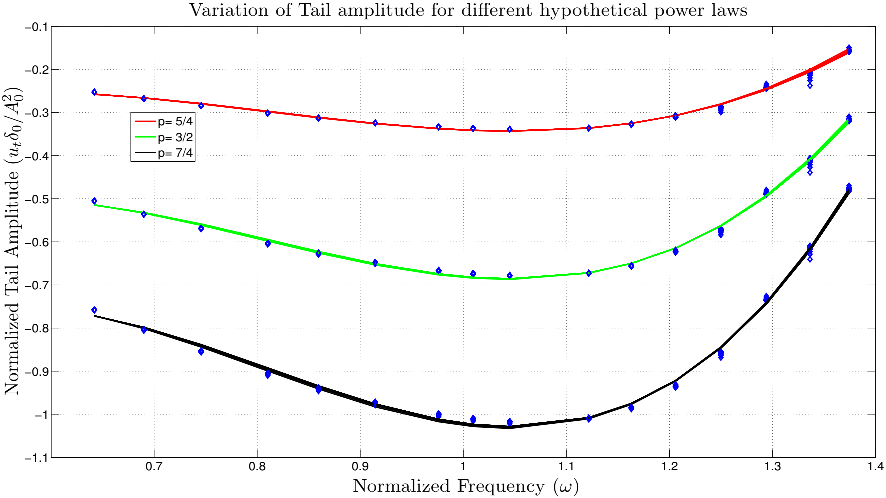


$$u_t^m = \overline{G}A_0^2(P\alpha^3 + Q\alpha^2 + R\alpha + S)$$

- P, Q, R, S qualify as invariants of quadratic nonlinearity in diatomic quadratic chains.

Estimating Nonlinearity in Diatomic Granular Chains

- Simulation of diatomic granular chains with variable p



A Second Inverse Problem for Diatomic Granular Chains

- For Diatomic Granular Chains with strong precompression

$$u_t^m = -\frac{(p-1)A_0^2}{2\delta_0} F(\alpha_0)$$

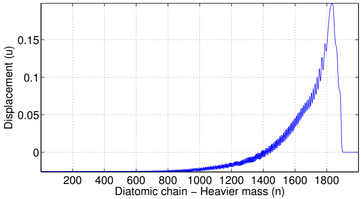
  - $F(\alpha_0) = (P\alpha_0^3 + Q\alpha_0^2 + R\alpha_0 + S)$
  - $\alpha_0 = m/M$ - known parameter
  - $P,\ Q,\ R$ and S obtained from Quadratic Nonlinear Chain

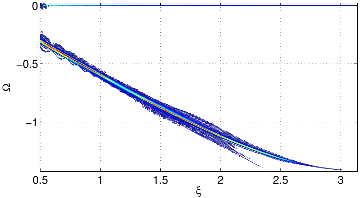
- Closed form expression to estimate power law : $p = 1 - \frac{u_t^m 2\delta_0}{A_0^2 F(\alpha_0)}$

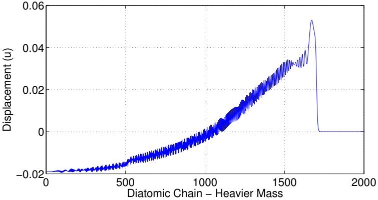
Imposed power law	Reconstructed from Steel-Al chain	Reconstructed from Steel-PTFE chain	Reconstructed from Al-PTFE chain
$p = 5/4 = 1.25$	1.2493	1.2482	1.2473
$p = 3/2 = 1.5$	1.4984	1.4967	1.4948
$p = 7/4 = 1.75$	1.7478	1.7454	1.7426

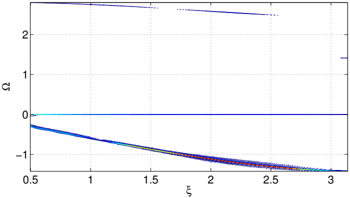
Strong Nonlinearity: Multi-Mode Activation

- Diatomic granular chain with $u/\delta_0 = 0.95$
  - Where do higher harmonics live in the band diagram of the crystal?
  - What are the distortion characteristics that the higher harmonics exhibit?



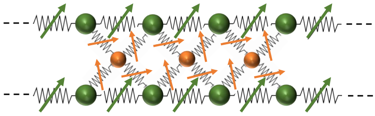


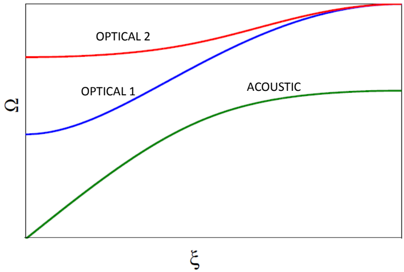


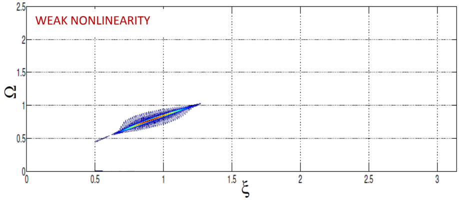


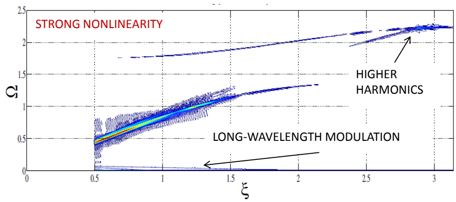
Looking Ahead: Mode Hopping in Complex Crystals

- Can we achieve multi-modal wave propagation from a single-frequency excitation?
- Can we achieve mode hopping? Can we activate multiple functionalities via amplitude tuning?



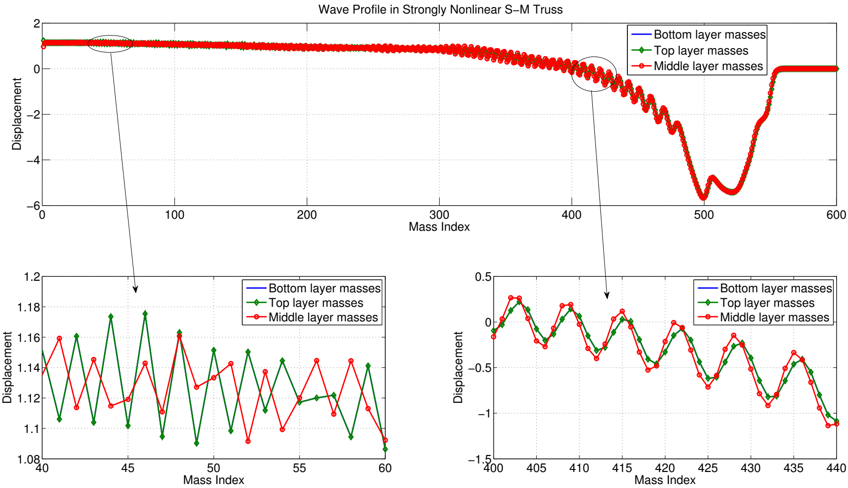






Evidence of Modal Mix Via Higher Harmonics

- Strongly nonlinear conditions trigger a blend of modes



Summary of Findings

- We have identified the signature of quadratic nonlinearity in granular chains as an asymmetric envelope soliton packet modulation
- We have formulated an inverse problem to extract the parameters of nonlinearity for granular chains (with unknown, arbitrary power laws) from their spectro-spatial response
- We have observed mode hopping phenomena in complex strongly nonlinear granular systems with multi-modal dispersion relations and proposed this approach for nonlinear switches

- Related publications:

  - R. Ganesh, S. Gonella, Spectro-spatial wave features as detectors and classifiers of nonlinearity in periodic chains, *Wave Motion*, v. 50, pp. 821 - 835, 2013
  - R. Ganesh, S. Gonella, Invariants of nonlinearity in the phononic characteristics of granular chains, *Physical Review E*, under review
