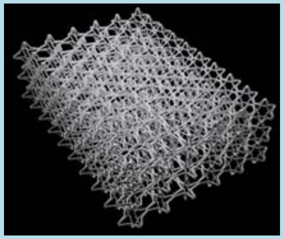
Designing Metal Water

RUTGERS
School of Engineering
Mechanical and Aerospace Engineering
Adam J. Nagy

Metallic Foams and Cellular Materials

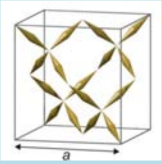
Transformation acoustics relies on the ability of creating materials with unique properties which can be found in designed cellular microstructures

examples

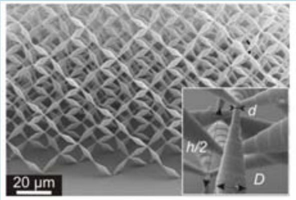


Cellular structure with negative Poisson's ratio

(Friis, E. A., Lakes, R. S., and Park, J. B., 1988)



Cellular structure with pentamode behavior, created by optical lithography



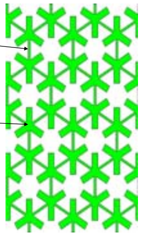
(M. Kadic, T Bückmann, N. Stenger, M.Thiel, M. Wegener, 2012)

Question: How to control effective properties such as elasticity and density of a designed cellular material?

Metal Water

Goal: Controlling the macroscopic properties of designed cellular metallic foams for use in transformation acoustics applications. As a stepping stone Metal Water reproduces the acoustic properties of water and serves as a generic material that can be deformed into anisotropic pentamode structures required for acoustic cloaking.

Main Design



thin struts provide the necessary stiffness (bulk modulus)

islands give the right effective density

Original design for Metal Water made from aluminum. The geometry is dictated by static and dynamic homogenization theory and checked by the FEM package COMSOL.

Lattice Structures to Mimic the Acoustic Behavior of Water

Two Dimensional

Using Bloch-Floquet theory dispersion curves of the original Metal Water and simplified versions are created and the group velocity of the longitudinal branch is compared against the wave speed of water.

Bloch-Floquet Theory

Take nodal displacements and apply Floquet conditions

Nodal displacements: $q_j = \begin{bmatrix} u_j \\ v_j \\ \phi_j \end{bmatrix}$, with condition $q_j = q_i e^{i(\boldsymbol{k}\cdot\boldsymbol{r}_{ij} - \omega t)}$

Example for hexagonal lattice

$$\begin{matrix} q_2 = q_5 e^{i\boldsymbol{k}\cdot\boldsymbol{r}_{25}}, & \boldsymbol{r}_{25} = \boldsymbol{a}_2 - \boldsymbol{a}_5 \\ q_6 = q_1 e^{i\boldsymbol{k}\cdot\boldsymbol{r}_{61}}, & \boldsymbol{r}_{61} = \boldsymbol{a}_6 - \boldsymbol{a}_1 \end{matrix} \rightarrow q = T\tilde{q} \rightarrow \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & Ie^{i\boldsymbol{k}\cdot\boldsymbol{r}_{25}} & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & I & 0 & 0 \\ Ie^{i\boldsymbol{k}\cdot\boldsymbol{r}_{61}} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_5 \\ q_3 \\ q_4 \end{bmatrix}$$

unit cell

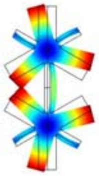
Equilibrium Equation

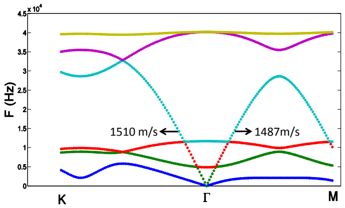
$$[-\omega^2 M + K]q = 0 \rightarrow T^H[-\omega^2 M + K]Tq = 0$$

(Phani, Woodhouse, Fleck 2006)

FEM Dynamic Homogenization

Bloch Floquet conditions applied with COMSOL

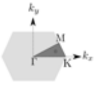




Nearly isotropic, note: $c_{H_2O} \approx 1500\, m/s$

Group velocity dispersion relation

$$c_j = \frac{\mathrm{d}\,\omega(\boldsymbol{k})}{\mathrm{d}\,|\boldsymbol{k}|}$$

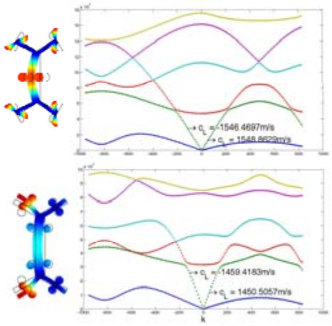


$\mathrm{K} = \frac{\pi}{l}(\frac{4}{3}, 0)$
$\Gamma = (0, 0)$
$\mathrm{M} = \frac{\pi}{l}(1, \frac{1}{\sqrt{3}})$

Brillouin Zone for hexagonal lattice

Metal Water II

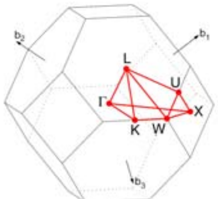
New design constructed with aluminum struts and lead weights.



Three Dimensional

In three dimensions we use diamond lattice structures offering low shear modulus and isotropic elastic behavior. As a starting guess for geometry the results of Warren and Kraynik (1988) are used and then geometric iteration is done in COMSOL using Bloch-Floquet analysis to match the bulk modulus and density of the lattice structure to water.

Brillouin Zone

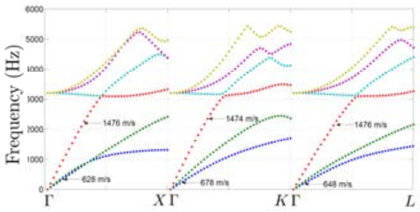


W. Setyawan, S. Curtarolo (2010) ISSN 0927-0256

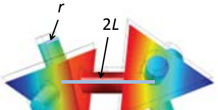
Take wavevector step along origin paths to calculate longitudinal and shear wave speeds.

$$\Gamma \rightarrow X \qquad c_p = \sqrt{\frac{K}{\rho}}$$
$$\Gamma \rightarrow K$$
$$\Gamma \rightarrow L \qquad c_s = \sqrt{\frac{G}{\rho}}$$

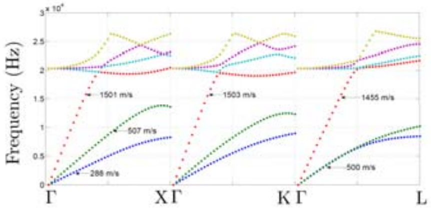
Prototyping with FEM Dynamic Homogenization



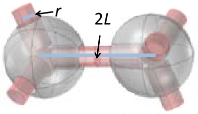
Prototype I uses all steel construction with solid tetrahedral masses to produce desired density. Dispersion behavior along origin paths (left), geometry (right). Lower shear modulus desired.



$\rho = 996\ kg/m^3$
$L/r = 4.4$



Prototype II, uses silicon carbide rods and steel masses. Mass junctions have clearance holes for the rods such that there is minimal contact between junction and rod which helps lower shear. More work to be done to lower shear.



Center junction = steel, Red=SiC

$\rho = 998\ kg/m^3$
$L/r = 5.2$
$G \approx .25\ GPa \quad \nu \approx .45$

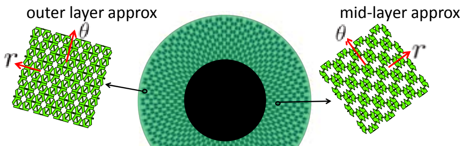
Current and Future Work

Fabricated Metal Water I (right), testing on sample conducted at NRL. Three dimensional manufacturability challenges, sample structure created via lead tetrahedral junctions and rods (left).

Two dimensional sample cut from a one inch thick aluminum block via water jet machining by Whitcraft Group.

Creation of Acoustic Cloaking Device

Anisotropic designs for use in metamaterial devices such as acoustic cloaks



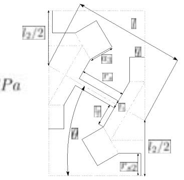
$$C_{FM} = \begin{bmatrix} 1.66 & 2.19 & 0 \\ 2.19 & 3.20 & 0 \\ 0 & 0 & 0.10 \end{bmatrix} GPa \qquad C_{FM} = \begin{bmatrix} 0.99 & 2.00 & 0 \\ 2.00 & 5.14 & 0 \\ 0 & 0 & 0.20 \end{bmatrix} GPa$$

$$\rho = 1533 kg/m^3 \qquad \rho = 1180 kg/m^3$$

Approximate cloak into layers of specifically designed metallic foams based on acoustic transformation elasticity and density needs

Anisotropic Designing

$$C_{FEM} = \begin{bmatrix} 1.173 & 0.255 & 0 \\ 0.255 & .0598 & 0 \\ 0 & 0 & 4.321 * 10^{-5} \end{bmatrix} GPa$$



Adding additional angular parameter to Metal Water gives anisotropic elasticity range to the design while maintaining low shear modulus.