DEBATES

*Spectral Analysis and the Study of Seasonal Fluctuations in Historical Demography**

Barbara Spencer - Derek Hum - Paul Deprez
Department of Economics
University of Manitoba

> If we look with a historian's interest at the development of a science, . . . we find that tools also have a life of their own. They may even come to dominate an entire period or school of thought. The solution of important problems may be delayed because the requisite tools are not perceived. Or the availability of certain tools may lead to an awareness of problems, important or not, that can be solved with their help. Our servants may thus become our guides . . .
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> (T. C. Koopmans, *The Interaction of Tools and Problems in Economics*, in « Three Essays on the State of Economic Science »)

Introduction.

Our entire approach to historical demography was radically changed from 1956 onwards when M. Fleury and L. Henry published their now-famous *Manuel de Démographie Historique.*[1] As with all advances in techniques, the period immediately following its introduction met with a harvest of results based on the family reconstitution method. Indeed, the number of studies carried out with the aid of their methods is truly impressive and it is reasonable to assume that many more studies, currently under way in various countries, will be adding to our wealth of information. With each increase in the number of studies available, however, one becomes more and more

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1 M. Fleury and L. Henry, *Des registres paroissiaux à l'histoire de la population. Manuel de dépouillement et d'exploitation de l'état civil ancien,* Paris 1956, pp. 84.

aware of the fact that there remains a considerable gap in the methodology. While considerable efforts have been made to refine and perfect the initial method of Fleury and Henry, including investigations into possible ways to computerize the approach, amazingly little effort has been spent towards the design of methods that would enable us to compare and to synthesize the various results. This failing has reduced considerably the usefulness of the results obtained to date, consequently leading to disenchantment on the part of many researchers.

The present paper is a first and partial attempt to narrow that methodological gap. We have turned our attention towards the analysis of seasonal fluctuations in vital statistics series since this is a concern found in almost all contemporary studies dealing with pre-industrial populations. Currently, the method of inquiry concerning seasonal fluctuations makes use of data which are normalized by a rather simple procedure proposed by L. Henry.[2] This normalization amounts to dividing the number of observations each month by the number of days in that month. The daily averages for each month thus obtained are aggregated over the year. The figure 1200 is then subsequently divided by the sum of the daily averages to obtain a multiplier that is used for each of the daily averages. The resulting set of normalized monthly figures then allows for a comparison between months since variations resulting from the different lengths of months have been eliminated. Without further analysis however, this normalization method allows only for a more visual presentation of the fluctuations and of the changes that can be inspected. In other words, the importance of the differences and of the changes can only be measured in a very subjective fashion, with the result that most authors have had to content themselves with drawing the most obvious and hence the most superficial conclusions. It is clear that given the complexity of the phenomenon, any study restricted to subjective observation methods, is inadequate.

In this paper we employ spectral techniques to compare the pattern of seasonal fluctuations in births between urban and rural villages, central and peripheral parishes, areas in which economic activities are primarily agricultural and those where one suspects a significant level of cottage industry, etc. We also indicate some of the difficulties of aggregation when combining small villages to increase sample size. The methods afforded by spectral techniques allow an unambiguous, careful and standard depiction of the seasonal pattern of fluctuations, thereby enabling us to compare in an objective and precise manner the nature of the variations. When understood and properly interpreted, spectral methods are vastly superior to calculations based upon mere visual inspection of lengthily charted data series. Although working with an extremely limited data base we are able to obtain a characterization of a

[2] *Ibid.*, pp. 49-51.

typical pattern of seasonality and find some differences between urban and rural settings. Furthermore there is an indication that towns which are closely associated with the countryside have a birth pattern similar to the rural villages and that the pattern may differ between villages grouped by economic activity. The findings reported here give partial confirmation to the hypothesis that as a region becomes less involved in agricultural activity, its seasonal pattern of births can be seen to smooth out in the autumn months so that there is less of an increase in August and September. Our data indicate that the births peak in February and March still remains for the city as a whole.

It is curious that in the most extreme urban setting, for which we have one example: the central parish of St. Jacobs, Ghent, there is no discernable seasonal pattern so that it might seem that extreme urbanization caused a reduction in seasonality in this period. Naturally no confidence can be placed on this hypothesis without further data.

A cursory summary of spectral techniques is provided in the next section. The following sections of the paper report on the data analyzed and discuss the results. Our conclusions, together with an outline of our tentative hypothesis, are given in the final section.

Spectral Analysis of Time Series Data - Methodology.

Recently developed statistical techniques known as spectral methods have been used with considerable success in a number of economic applications.[3] As a guide to interpreting our results, a brief, non-mathematical exposition of the basic ideas and procedures underlying spectral analysis is given and this is followed by some remarks on practical considerations.[4]

In order to analyze a time series; that is, a chronological sequence of dated observations, one must have both a statistical theory as well as a formal technique for describing time series. In somewhat oversimplified terms, the method (spectral analysis) is based upon the fundamental idea that a time series, for example monthly births, can be usefully decomposed into a number of component cycles, each of which is associated with a specific period or frequency. Without the use of spectral methods, time series have commonly been analyzed in terms of such notions as "trend", "cycles", "seasonal", "error" components as well as visualizing movements through time as

[3] See F. C. Nold, *A Bibliography of Applications of Techniques of Spectral Analysis to Economic Time Series*, « Technical Report » 6, Institute for Math. Studies in the Social Sciences, Stanford, 1972.

[4] An excellent study on the theory and estimation procedure at the general level is C. Granger and M. Hatanaka, *Spectral Analysis of Economic Time Series*, Princeton University Press, Princeton 1964, pp. 299.

"long run", "short run" or "medium run". These methods, based on regressions and moving averages, of simply comparing peaks and troughs can be very crude. Spectral methods, which are based upon rigorous foundations, produce more detailed description and involve careful estimation procedures. Theorems exist stating that if a series is stationary (essentially trend free).[5] then it can be decomposed uniquely into many uncorrelated frequency or period components. "Frequency" indicates the number of cycles completed per unit time period, and "period" describes the length of time required for one complete cycle. For example, a frequency centred at 0.10 cycles per month corresponds to a period of 10 months. The term "frequency" is simply the inverse of the intuitive concept of "period". For technical reasons it is often more convenient to represent or characterize a data series with reference to its component frequencies of cycles than to work with time periods of differing lengths.

Having decomposed a data series into its component frequencies, one may then speak of the amplitude or the strength of various cyclical components. The power spectrum is simply the plot of the amplitude of various components against frequency. The estimated power spectrum diagram is one of the basic tools with which the properties of a single series are analyzed. For example, if the estimated power spectrum reveals clear peaks at various frequencies or groups of frequencies, this suggests that cycles of particular lengths are significant. On the other hand, if the estimated spectrum is flat, this indicates that fluctuations about the mean are completely random; that is, the series is a sequence of uncorrelated readings since all components are equally present.[6] Spectral analysis is, in essence, an analysis of the variance of a time series in terms of frequency. In other words, the measure of importance adopted is the amount contributed to the overall variance by the component frequency we are considering. Thus, for a variable containing a cycle of specific period (or frequency) the importance of the cycle is the reduction in variance effected when this component is removed. Spectral techniques are therefore appropriate for detecting regular periodicities in data composed of a mix of different cycles ranging from short-term seasonal fluctuations to longer-term cycles extending perhaps over periods of several years.

In observed time series data it may be clear, for example, that it takes just twelve months for the price of some good to move smoothly from its highest point to its lowest and back again. In economic terms we might like to call this one cycle. However, if the shape of the curve is not the same as a sine or cosine function, the spectrum may show a major peak at

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[5] Loosely speaking, stationary implies that after removal of a trend in mean, the generating law of the residual does not change over time.

[6] The power spectrum is therefore, completely analogous to a probability density function.

the twelve month period plus a number of minor peaks at $12/2 = 6$ months, $12/3 = 4$ months, $12/4 = 3$ months and $12/5 = 2.4$ months, etc.[7] This results from the fact that "cycles" obtained from spectral techniques are the strictly mathematical cycles of sine and cosine functions. We recognize this as a problem for interpretation of results, but it does not affect the use of the technique for simple characterization and comparison of demographic or economic series from different places or times. In fact, for the results presented in this paper, we can show that the major peaks in the spectrum are not just an artifact of the shape of the time series. As will be shown below, the pattern of births does repeat over twelve months. For the rural data especially, an increase in births occurs both in February or March and again in August or September, where the increase is much smaller. Since February and August are six months apart, a peak in the spectrum at the six month frequency indicates that there is a cluster of births in August as well as February. If there were only a twelve month peak, this would imply that births were highest in February and lowest in August. In fact we do obtain the major peak at twelve months with a secondary but significant peak at six months. The twelve month peak arises since the six-month cycle cannot fully account for the higher level of births in February relative to August.[8]

In calculating the power spectrum diagram, one can only estimate a finite number of points at pre-selected (usually equal-spaced) intervals. Though the height of the spectrum at each point is only an estimate, meaningful comparisons of heights at different points may still be made. Most of our estimated spectra display two discernible peaks, as mentioned above; however, just comparing the heights of the major peaks or minor peaks is misleading since the overall heights are affected by the variability of the data. In fact, the area under the spectral density function decreases as the variance of the data decreases, and this decline in variance or standard deviation over time is obvious in our normalized rural data. It is important to remark here that nonstationery of the data arising from decreasing variance will not affect the spectrum as a measure of the relative strength of the various cycles.[9] That is, it will not affect the relative heights of peaks.

For our measure of the relative strength of the two cycles we calculate the ratio of the twelve-month peak to the six-month peak. From experience however, this does not seem to be the best measure. In the estimation of the spectrum it is possible that there is some escape of the effect of a cycle into the height of the spectrum in the frequencies adjoining the one of

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[7] See GRANGER and HATANAKA, *op. cit.*, p. 14.

[8] For example the spectrum of Belgian rural births for 9 villages in Veurne-Ambacht is shown in figure 1 below. A full discussion of results is found in subsequent sections.

[9] See GRANGER and HATANAKA, *op. cit.*, pp. 158-159.

interest.[10] The true measure would then be the area under the spectrum at these frequencies. However, as a close and simple approximation, we calculate the sum of the heights of the spectrum at the peak and its adjacent frequencies. For instance, to obtain the strength of the twelve-month cycle we use the sum of the three spectrum heights centered on the twelve-month frequency. We will call the ratio of this to a similar measure obtained for the six-month cycle our broader or three-frequencies measure of the relative strength of the cycles.

The decomposition approach becomes particularly useful when the objective is to analyze more than one series.

For the analysis of more than one series and the determination of the lag structure when investigating two series, cross-spectral techniques may be employed.[11]

The application of spectral methods is not without its many practical considerations, the first of which is whether or not a series of given length provides sufficient observations to estimate the spectrum. There is no agreed of minimum number of observations less than which the tool becomes inappropriate. A second consideration is the choice of the number of estimated components. This choice involves a trade off between the number of frequency intervals at which the spectral density is estimated and the accuracy of each point estimated. Again there exist only general guides resulting from varied experience. It is clear however, that decomposing a data series of given length has its advantages in terms of description. Oscillations with periods comparable in length with the length of the data series available are grouped with "trend" and cycles whose periods are shorter than the time-interval between successive observations cannot be extracted. In short, interpretation of such terms as "long run" and "rapid oscillations" depend upon the length of the available data and the time-interval between consecutive observations.

A fundamental problem for the practical application of spectral methods is the issue of non-stationarity. The theory and technique is developed under the requirement of "stationarity", essentially an assumption that the same "rules" have been in operation throughout the sample. This is likely to be strictly valid in few cases, hence some modification of the data, such as removal of trend, etc., may be necessary before estimates of spectra can be derived. While it is true that the study of non-stationary series is still in its early stages, techniques and methods do exist that minimize undesirable effects due to time-changing spectra under certain circumstances.[12]

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[10] This is known as leakage, see GRANGER and HATANAKA, *op. cit.*, p. 45.

[11] The authors are currently engaged in research employing cross-spectral methods to investigate the relationships between demographic and economic series. As these results are not reported in this paper, we will not be concerned to describe these techniques.

[12] See GRANGER and HATANAKA, *op. cit.*

To summarize, the spectral approach is an improvement over the traditional classification of a time series (trend, cycle, seasonal and error movements) in that it clearly separates time-varying phenomena according to the frequency of the underlying cycles. Spectral methods can have a wide range of applicability; from determining the empirical characteristics of a single series (such as the existence of particular cycles) to indicating the interrelationship between two series and estimating the lag structure. Most important, spectral analysis applied to historical data series may be used for specification and assistance in the development of causal models or alternatively, once a model has been given, spectral methods can provide a method of verifying the model's ability to explain actual behaviour.

DATA.

Our initial data sample was chosen from particular groups in the economy where in the light of previous empirical research one might expect to get different seasonal patterns of births. Using our spectral analysis we can then get an initial characterization of the seasonal pattern of births from each group and use these for preliminary comparisons.

As rural data we used the monthly data published by D. Dalle for the Veurne Ambacht (Métier de Furnes) and covering the period 1636-1795.[13] As well as using the aggregate data for this area broken into particular time periods (explained below) we analyze the data from the individual villages that make up this aggregate.

As urban data we use the monthly baptism figures for Liège between 1720 and 1794.[14] We also use the baptism for one parish (St. Jacobs) of Ghent for the periods 1700-1709 and 1770-1782.[15] For further comparison we use data from a smaller Italian city, Pavia, between 1577 and 1700.[16] Again, as well as studying the aggregate spectra we look for possible differences between the parishes comprising the old core of the town and the more recent outer or peripheral parishes. Some previous studies have indicated that the outer group of parishes are similar to the rural villages in birth patterns whereas the central core has a different pattern.[17]

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[13] D. DALLE, *De Bevolking van Veurne-Ambacht in the 17de en de 18de eeuw*, Verhandelingen van de Koninklijke Vlaamse Academie, Klasse de Letteren, vol. 49, Brussels 1963, pp. 236-286.

[14] E. HÉLIN, *La démographie de Liège aux XVII^e^ et XVIII^e^ siècles*, Académie Royale de Belgique, Mémoires, collection in 8°, vol. LVI, no. 4, Brussels 1963, pp. 256-258.

[15] Unpublished data.

[16] G. ALEATI, *La popolazione di Pavia durante il Dominio Spagnolo*, Università degli Studi di Pavia, Facoltà di Scienze Politiche, Istituto di Statistica, Pavia 1957, pp. 163-186.

[17] See F. BLOCKMANS, *De Bevolkingscijfers te Antwerpen in the XVIII^e^ Eeuw*, in « Antwerpen in de XVIII^e^ Eeuw », Antwerp 1952, pp. 397 et seq. Also P. DEPREZ, *Het Gents Bevolkingscijfer in de tweede helft van de 18de eeuw*, Handelingen der Maatschappij voor Geschiedenis and Oudheidkunde te Gent, new series, vol. XI, 1957, pp. 177-195.

RESULTS.

Our first aim is to obtain a typical pattern in our spectra for rural births using the data from the 9 vilages in Veurne-Ambacht. To find the pattern over time, we break the data into three periods, 1630-1699, 1697-1740, and 1741-1795. We justify this on the grounds of differences in the number of wars and epidemics and in the rates of population growth. These external factors would on a priori grounds be likely to affect the birth patterns.

The first period, 1636-1696, is characterized by wars, epidemics, food shortages and in general by a decrease in population. The second period, 1697-1740, is generally quieter in terms of wars and epidemics with just a small increase in population.[18] The third period 1741-1795 is a period of economic growth and has an appreciable population increase. The difference in spectra obtained between these periods using ten year intervals further justifies the breakdown in terms of relative uniformity of results (see below).

In general the estimated spectrum for any of the rural birth figures shows that the only peaks are at frequencies 0.083 and 0.167 cycles per month, which correspond to periods of twelve and six months respectively. As a further aid for interpretation we also calculated the ten-year aggregates of births by month for the period.[19] This aggregation, by smoothing out the minor fluctuations, makes it clear that the major number of births occur around February each year (sometimes March), indicating conception in spring (April or May), with a secondary cluster around August or perhaps September, indicating conception in November or December after the harvest. The six months between February and August would explain the peak at six months in the spectrum. The twelve month peak can also be explained since February births are greater than August births.[20] Thus the relative strengths of the twelve to six month peaks can indicate the importance of the secondary increase of births in August and September. Roughly, the greater the strength of the six-month peak the more important is the increase in births at this time of the year.

We have already noted that the shape of the spectrum is not affected by the variance of the data so relative heights of peaks are still meaningful. In Table 1 below, the standard deviation of the normalized data is shown

[18] Based on the data given by D. DALLE, *op. cit.*, p. 38, the population of the 9 villages increased by approximately 11 per cent between 1697 and 1740 or at an average annual rate of .3 per cent and between 1740 and 1794 by approximately 24 per cent or at an average annual rate of increase of 0.5 per cent.

[19] Our calculations are not given in this paper. It can also be pointed out that the aggregation procedure is a very standard one and has been used in most (if not all) historical demographical studies dealing with seasonal fluctuations.

[20] See methodology section for more explanation.

as decreasing from 49.1 in 1636-1696 to 41.8 in 1741-1795. We also remarked that because of "leakage" effects, a three-frequencies measure of the spectrum height should be employed in order to compare relative strengths of cycles. The results of these calculations for rural births for the three periods are also set out in Table 1.

TABLE 1

STRENGTH OF TWELVE MONTH PEAK RELATIVE TO SIX-MONTH PEAK OBTAINED FROM SPECTRA OF RURAL BIRTHS FROM VEURNE-AMBACHT

		Ratio 12/6 Single Frequency	Ratio 12/6 3 Frequencies	Standard Deviation (Normalized Data)
Period I:	1636-1696	3.34	3.08	49.1
Period II:	1697-1740	1.71	1.59	45.4
Period III:	1741-1795	2.29	2.09	41.8

As can be seen from the results in Table 1 the relative importance of the twelve-month to the six-month peak declines between periods I and II, and rises slightly again in period III. Thus, overall, between periods I and III the importance of the six-month peak declines relative to the twelve-month peak. This was true for both the single frequency and the three-frequency measures. This would suggest that generally there was a larger increase in births in August and September in the second period than in the other periods.

VARIATION OVER TIME.

In order to see how much variation there was within our periods we also calculate the spectra for ten-year intervals from 1636-1795 (see Table 2).[21] It is notable that both measures of the twelve to six month ratios calculated for the 10 year intervals between 1636 and 1695 are all greater than the corresponding ratios between 1696 and 1735, providing a partial confirmation of our choice of time periods. One problem, however, arises. The individual ratios in the third period seem higher than that indicated by the ratio of the aggregated data. In Table 3 is shown the averages of the ratios from the 10 year data.

[21] See Table 2 in appendix. Periods of less than 10 years (= 120 observations) could be dangerous to estimate: $\frac{n}{m} = \frac{120}{24} = 5$ while Granger and Hatanaka recommend that at least $\frac{n}{m} = 3$.

TABLE 2

STRENGTH OF TWELVE-MONTH CYCLE RELATIVE TO SIX-MONTH CYCLE OBTAINED FROM SPECTRA OF RURAL BIRTHS FROM VEURNE-AMBACHT BY 10 YEAR PERIODS 1636-1795

Periods	Ratio 12/6 Single Frequency	Ratio 12/6 3 Frequencies	S. D. (Norm)
1636-1645	3.30	2.85	31.06
1646-1655	5.12	4.72	39.84
1656-1665	2.25	2.33	37.84
1666-1675	2.56	2.48	28.26
1676-1685	3.37	2.97	28.20
1686-1695	4.46	3.86	32.83
1696-1705	1.05	1.00	23.34
1706-1715	1.89	1.77	31.44
1716-1725	1.64	1.34	25.03
1726-1735	1.72	1.68	28.01
1736-1745	2.78	2.44	25.61
1746-1755	1.07	1.04	27.70
1756-1765	2.76	2.35	24.07
1766-1775	3.16	2.77	23.71
1776-1785	3.50	3.02	24.39
1786-1795	5.85	3.95	23.71

One possible explanation is that in the third period some of the strength of the six-month peak has been masked by aggregation. This may be because the proportion of the number of births in August, September and October can vary over time and is thus partially lost by aggregation through an averaging or smoothing process. This many occur less for the twelve-month peak since for the 10 year aggregations, and the third period, the maximum of births is commonly about equally spread between February and March. (In earlier periods February is much stronger). That masking of this sort can take place is shown by the fact that there are a number of small peaks in the ten year data.

TABLE 3

AVERAGE OF RATIOS FROM 10 YEAR SPECTRUM

	12/6 Single Frequency	12/6 3 Frequencies
1636-1695	3.51	3.20
1696-1735	1.57	1.45
1746-1795	3.19	2.76

The exact position of these minor peaks varies among the 10 year periods so that in the further aggregated data they have all but disappeared. We shall again come to this problem of masking by too much aggregation when shortly we examine the spectra from the 9 villages individually. The data for the individual villages is set out in Table 4.

TABLE 4

STRENGTH OF TWELVE-MONTH CYCLE RELATIVE TO SIX-MONTH CYCLE: INDIVIDUAL VILLAGES OF VEURNE-AMBACHT

	Period I		Period II		Period III		Population		
	Ratio 12/6 (*a*)	S. D. (Norm) (*b*)	Ratio 12/6 (*a*)	S. D. (Norm) (*b*)	Ratio 12/6 (*a*)	S. D. (Norm) (*b*)	Census 1668	Land Tax Register 1668	Census 1794
Haringe	2.09	53.29	0.91	44.30	1.74	47.74	1403	1140	1245
Leisele	2.60	60.33	1.15	49.56	1.48	47.02	1221	1060	1684
Beveren	1.71	60.90	1.41	54.28	1.67	53.02	1022	889	1670
Pollinkhove	2.41	57.05	1.51	52.86	1.42	47.95	1009	899	1520
Stavele	1.39	63.08	1.75	59.32	0.97	59.19	824	—	1150
Krombeke	1.22	73.28	1.09	67.60	1.27	65.69	511 (*c*)	444	438 (*c*)
Hoogstade	1.52	79.18	0.71	79.01	1.16	71.48	459	400	593
Gijverinkhove	1.76	89.54	0.90	81.33	1.24	75.26	429	391	615
Avekapelle	1.57	100.20	1.15	97.30	1.32	86.30	294	261	390

Sources: 1688. D. DALLE, *De Volkstellingen te Veurne en in Veurne-Ambacht op het einde van de zeventiende eeuw*, « Bulletin de la Commission royale d'Histoire de Belgique », vol. CXX, 1955, pp. 20-21.

1794. D. DALLE, *De Bevolking van Veurne-Ambacht in de 17de en de 18de eeuw*, Verhandelingen van de Koninklijke Vlaamse Academie, Klasse de Letteren, vol. 49, Brussels, 1963, p. 227.

Notes: (*a*) This is a measure of the strength of the twelve-month cycle relative to the six-month cycle as explained in the text. The 3 frequency measure gained by adding the heights of the spectra around the frequency of interest is used here. Because the smallness of some of the villages increases the likelihood of leakage, we consider it a more appropriate measure than the single frequency ratio.

(*b*) This is the standard deviation of the normalized births.

(*c*) D. Dalle stated populations 135, and 149 for Krombeke in 1688 and 1794 respectively. These are probably just the figures for the old part of village since they are less than the population in the land tax register. The population figures 511 and 438 are obtained by using the proportions found in the land taxation register between Stavele and Krombeke and Krombeke and Hoogstade.

VARIATION AND SOIL CONDITIONS.

Our study of the individual villages is motivated by two main considerations. The first is to examine, however imperfectly, whether differences in birth patterns can be attributed to differences in soil conditions which can indicate socio-economic differences.[22] We admit that our sample of nine villages with seven of one-soil type and two of another is inadequate insofar that two is too small a number. Nevertheless, our results should suggest whether further study along these lines is useful. The second motive is to

22 P. DEPREZ, *The demographic Development of Flanders in the eighteenth century*, in D. V. GLASS and D. E. C. EVERSLEY, *Population in History*, London 1965, pp. 608-630.

see if the same pattern of births can be obtained even from the smallest village and also to find out the effect of aggregation on the results.

From Table 4 it is clear that many of the birth patterns are the same for all villages. The villages in the Table are listed in order of population size in 1688. The number of people listed in the land taxation register of 1688 is also shown. The population figures for Krombeke given in the original list would seem to apply to the old village core only as they are less than the corresponding figures in the land taxation register. In Table 4 we have adjusted them upwards using the proportions from the land taxation register. That Krombeke lies between Stavele and Hoogstade in size (from the land taxation figures) is confirmed by the figures for the standard deviations of the normalized data. Our table shows that the standard deviations with very few exceptions increase as the size of population decreases. In each period the standard deviation for Krombeke is greater than that for Stavele but less than that for Hoogstade. We also calculate the coefficient of variation [23] from the non-normalized data for each village (see Table 5).[24] These were strikingly close in value to the standard deviation figures from the normalized data showing that the latter figures are a reliable measure of variability.

TABLE 5

COMPARISON OF STANDARD DEVIATIONS FROM NORMALIZED FIGURES WITH THE COEFFICIENTS OF VARIATION OF THE NON-NORMALIZED DATA FOR THE INDIVIDUAL VILLAGES IN VEURNE-AMBACHT

	Period I		Period II		Period III		1688	1759	1794
	S. D. (Norm) (*a*)	S. D. / μ (*b*)	S. D. (Norm) (*a*)	S. D. / μ (*b*)	S. D. (Norm) (*a*)	S. D. / μ (*b*)			
Haringe	53.29	55.17	44.93	46.74	47.74	48.46	1403	1665	1245
Leisele	60.33	57.09	49.56	49.99	47.02	48.54	1221	1372	1684
Beveren	60.90	63.55	54.28	56.32	53.02	55.08	1022	1273	1670
Poltinkhove	57.05	58.99	52.86	54.76	47.95	49.78	1009	1205	1520
Stavele	63.08	63.90	59.32	59.45	59.19	63.01	824	883	1150
Krombeke	73.28	76.61	67.60	69.73	65.69	68.98	511 (*c*)	(*c*)	438 (*c*)
Hoogstade	79.18	80.93	79.01	81.45	71.48	72.40	459	559	593
Gijverinkhove	89.54	88.86	81.33	83.16	75.26	76.15	429	471	615
Avekapelle	100.20	100.13	97.30	97.48	86.30	86.97	294	330	390

Source: See Table 4.

Notes: (*a*) This is the standard deviation of the normalized figures.

(*b*) This is the coefficient of variation (standard deviation devided by mean) calculated from the non normalized data.

(*c*) The figures for Krombeke, 1688 and 1795 (see Sources, Table 5) have been corrected upwards using the land taxation figures as explained in footnote (*c*) of Table 4. The figure for 1759 is left out as it also is incomplete in D. DALLE, *De Bevolking van Veurne . . .*, p. 219-233.

23 The coefficient of variation is the standard deviation divided by the mean.

24 See appendix.

Only for the three smallest villages, Hoogstade, Gijverinkhove and Avekapelle (population figures in 168, 5080 or less) were the peaks in the spectrum not exactly centred at the frequencies corresponding to our twelve or six month peak. In each case the height of the spectrum at twelve or six months was very close to the peak height (always at an adjacent frequency). In all cases it was clear that the twelve and six-month peak were the only important ones.

As mentioned above, the nine villages can be broken into two groups according to soil conditions. Two of the villages, Stavele and Avekapelle have predominantly fertile soil. The other seven villages are less fertile since for each, roughly fifty per cent of the land has soil of a loamy, sandy type rather than alluvial soil. This difference in soil conditions often means that a greater proportion of the work force in the less fertile villages is involved in cottage industries. Under these circumstances we should not rule out the possibility of a diferent birth pattern in the two types of villages.

For the seven less fertile villages, the relative importance of the twelve to six-month peak declines between periods I and II, and rises again in period III. In all but one, Krombeke, which has a rather small population of around 500, the third-period ratio is less than the first-period ratio. This is the same result as was found for the aggregate data. It is of interest to note that results consistent with similar villages of the same soil type can be obtained even from quite small villages.[25]

Variations and Village Size.

The villages also fall into two groups according to size. The five largest have in 1688 populations of over 800. The rest have populations of 511 down to 294. The four largest villages (Haring, Leisele, Beveren and Pollinkhove) have relative peak heights (using the 3 frequencies measure) varying between 1.71 and 2.60 for the first period, 1636-1696. Stavele which is the fifth of these larger villages has a twelve-month to six-month peak ratio in the first period of 1.39. This ratio for Stavele, one of the more fertile villages, is rather lower than the ratios for the four less fertile villages just given above even though Stavele's population is in the same range. Avekapelle, which is also a more fertile village has a relative peak ratio of 1.57 which is also low relative to the larger villages but not low

[25] The authors are at present in the process of studying the extent to which data can be disaggregated without affecting the results. It is obvious that as villages become smaller, the variability of the data increases. It is of interest to find the population level after which this increase in the variability becomes faster. We are also studying the effect of increased time periods on the variance.

relative to the three smaller less fertile villages; Krombeke (ratio 1.22), Hoogstade (ratio 1.52) and Gijverinkhove (ratio 1.76). Plainly data from more villages would be of assistance here. Another difference between Stavele and the other villages is that for Stavele the major to minor peak ratio actually increases between periods I and II then decrease to below the period I level in period III. The changes in all the less fertile villages were in just the opposite directions. On the other hand Avekapelle seems to follow the pattern of the rest of the villages although its second period ratio is high relative to the other small villages. Thus our data do not rule out the possibility that the fertile villages have a different birth pattern from the non fertile villages but further study is obviously needed. The smallness of Avekapelle (population 294 in 1688) relative to Stavele (population 824 in 1688) may also be confounding our results.

The examination of the above aggregate rural data showed that aggregation of the ten-year data into longer periods seemed to mask some of the results. This is even more obvious when the aggregate rural data are split into their individual villages. From Table 1, the twelve to six-month peak ratio using the three frequency measure is 3.08 in period I for the aggregate data. The highest value for this ratio for the individual villages is 2.6, ranging all the way down to 1.22 (see Table 4). In the other periods it is also clear that aggregation reduces the importance of the six-month peak. This seems to occur because the position of the six-month peak may vary among villages at each point in time and thus become partially averaged out in the aggregation process.

VARIATION AND URBAN ACTIVITIES.

To provide a rural-urban comparison the spectrum of normalized births for Liège was calculated for the period 1741-1794 (if 1795 had been included it would exactly correspond to our period III) (see Table 6).[26] As with the rural births the only significant peaks are at frequency levels 0.083 and 0.167. The six-month peak, however, is of much less importance. The ratio of the relative peaks is now 4.35 as compared with 2.29 for the same period in the rural data. The broader measure using the heights of the spectra around the peak frequency is also significantly larger at 3,59 (ocmpared with 2.09). That the concentration of births around September is reduced is also clear from the original data. See Figure 1 for a graph showing the difference between the urban and rural results.

Another notable feature is that the overall variability of the normalized data is much lower (the s.d. is 12.55 as compared with 24.74 for the aggregate rural data in the same period). To check that these conclusions are not

TABLE 6

LIÈGE

STRENGTH OF TWELVE-MONTH CYCLE RELATIVE TO SIX-MONTH CYCLE OBTAINED FROM SPECTRA OF LIÈGE BIRTHS BY 10 YEAR PERIODS 1726-1794

Periods	Ratio 12/6 Single Frequency	Ratio 12/6 3 Frequencies	S. D. (Norm) (*a*)
1726-1735	5.46	4.48	11.97
1736-1745	7.17	5.42	12.92
1745-1755	2.98	2.71	12.37
1756-1765	3.44	3.01	13.01
1766-1775	2.67	2.29	11.29
1776-1785	5.37	4.03	12.65
1786-1794	11.07	6.18	12.77
Total 1720-1794	4.26	3.54	12.50
Total 1741-1794	4.35	3.59	12.55

Source: E. HÉLIN, *La démographie de Liège* . . ., pp. 256-258.
Note: (*a*) This is the standard deviation of the normalized data.

affected by our choice of time period the spectra and standard deviations were also calculated for ten-year periods between 1736 and 1795 (see Table 6). For all of these periods the ratio of the twelve to six-month peak was more than the rural average. Also for corresponding ten-year periods, using both the single and three frequency measures the twelve-month cycle with but one exception (1766-1775) was relatively stronger for Liège than for the rural data. It is notable that in the period 1786-1794 there was no actual peak in the spectrum at the six-month frequency nor at an adjoing frequency. This may be explained by the fact that most of this period was characterized by the turmoil of the Liège revolution. A further fact of interest is that the standard deviations for the ten-year periods of Liège varied only from 11.29 to 12.92. The smallest standard deviation from the rural data was 22.59 (1786-1795). Thus not only is the variance less in the Liège data but it is relatively constant. From the rural data broken into ten-year periods the standard deviation varied from 27.70 in 1746-1755 to 23.71 in 1786-1795. The explanation for this may partly be in the larger size of Liège (55,000 in 1790) relative to nine rural villages in Veurne-Ambacht (9,305 in 1794). Generally, decreasing variance is related to increasing size (see footnote 25).

VARIATION AND LOCATION.

Having looked at the aggregate urban data for Liège it seems appropriate to discover if there was any difference in the pattern of births between central

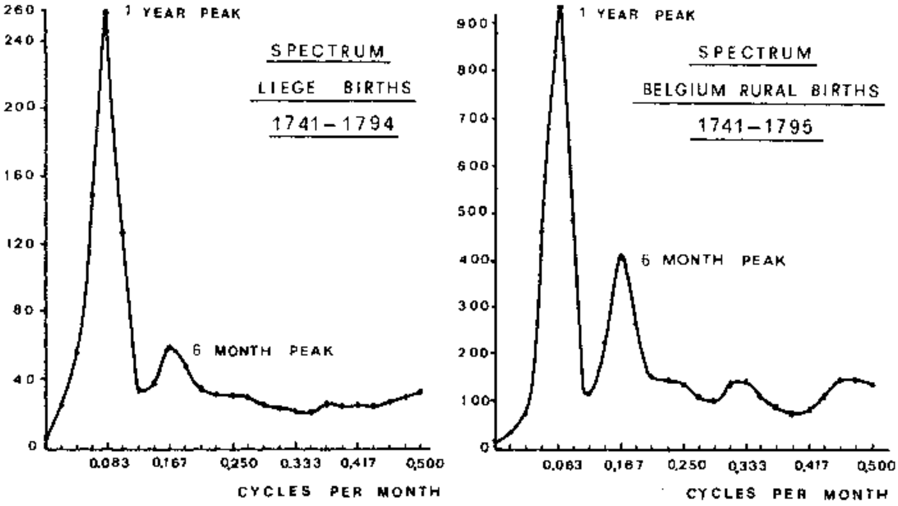

FIGURE 1. — The above graphs illustrate the difference between the spectra of an urban centre Liege, and the aggregate of 9 rural villages in Veurne-Ambacht for the same time period. The one-year peaks have been drawn to the same height to make the comparison easier. It is clear that the 6-month peak is much smaller for the urban than for the rural data. This implies that the proportion of the area under the one-year peak is greater for the urban than for the rural data. Since the area under the spectrum can be considered to correspond to the total variability of the data, this means that more of the seasonal variability is explained by a one-year cycle (maximum births in February), for urban than for rural data.

parishes and peripheral parishes in Liège or in a city of similar size. Unfortunately the only data we could obtain for Liège giving monthly figures was already aggregated over twenty-four years.[26] As we have already mentioned, much can be hidden in this aggregation. One main advantage of the spectral technique is that it uses the original data without this initial smoothing. However, looking at Hélin's data we can agree with him that the seasonal fluctuations seem more marked in the peripheral parishes and closer to the rural pattern than the central parishes which have less of a peak in births in September. As a proxy for a central parish of Liège we obtained data from 1700-1709 and 1770-1782 for St. Jacobs which is a central parish of Ghent. This is an appropriate comparison since both Liège (55,000 in 1790) and Ghent (51,000 in 1784) are of comparable size. Despite the fact that

26 E. HÉLIN, *op. cit.*, pp. 168-169.

St. Jacobs was a large parish of some 5,500 inhabitants in 1741,[27] the results were very erratic and no interpretation of a cyclical nature can be made. For both periods, although there is still a peak at one year, this is lower than the other "peaks". For 1770 to 1782 there is no peak at "6 months", the closest peak is at 5.4 months. A look at the original data confirms the erratic nature of the births. The aggregated and normalized data is shown in footnote.[28] The results from St. Jacobs thus seem obviously different from the situation described by Hélin [29] for the central parishes of Liège, although it must be remembered that this data as published is an aggregation of several parishes.

As a third comparison we examine data for Pavia, Italy. For some parishes we have figures from 1577 to 1700 and others from 1599 to 1700. Pavia in the 16th century had declined from the political and administrative importance it had in the 15th century. In the 16th century it was mainly a supply centre to the countryside and was dependent on the incomes earned in the surrounding rural areas.[30] The population was 13,000 in 1576, 18,000 in 1597, between 15 and 16,000 in 1638 and about 20,000 in 1700.[31] Thus it was much smaller than Ghent or Liège. Also its population structure was very similar to the one observed for the rural villages, whereas this was not true for Ghent or Liège. For instance in Pavia in 1660 and 1700 about 33% and 30% of the population was less than 14 years of age respectively.[32] This is close to the rural figures. In Liège the percentage of the population under 14 is much smaller (about 26% around 1750).[33] This is also true of Ghent (22%) and St. Jacobs (23%) between 1741 and 1784.[34]

Given the close integration of Pavia with the surrounding countryside it is not surprising that birth figures seem close to the rural pattern. The births in August and September retain their importance together with the February or March births. For Pavia we have data on only ten parishes.[35] For the

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27 P. Deprez, *Het Gents Bevolkingscijfer* ..., p. 178.

28 Normalized figures for births St. Jacobs, Ghent.

Months	J	F	M	A	M	J	J	A	S	O	N	D
1700-1709	114	106	99	77	88	100	100	104	105	96	105	106
1770-1782	113	102	97	111	94	85	90	93	100	103	100	112

29 E. Hélin, *op. cit.*, pp. 165-171.

30 G. Aleati, *op. cit.*, pp. 7-24.

31 *Ibid.*, pp. 15, 20, 24-25.

32 *Ibid.*, p. 28.

33 E. Hélin, *op. cit.*, p. 72.

34 P. Deprez, *Het Gents Bevolkingscijfer* ..., p. 192.

35 For Pavia we have monthly births for only about 18 per cent of the total population. Our data covers only a population of 3,376. The parishes for which monthly data are available are listed in footnote 37.

TABLE 7

PAVIA

STRENGTH OF TWELVE-MONTH CYCLE RELATIVE TO SIX-MONTH CYCLE OBTAINED FROM SPECTRA OF PAVIA BIRTHS BROKEN INTO CENTRAL, PERIPHERAL AND OTHER PARISHES, FOR 1599-1700

		Ratio 12/6 Single Frequency	Ratio 12/6 3 Frequency	S. D. (Norm) (a)	Population c. 1660
	(b)				
Central Parishes:					
Group A:	1581-1650	1.28	1.30	37.6	1419
	1651-1700	2.38	1.97	39.1	
Group B:	1581-1650	1.54	1.37	50.4	1027
	1651-1700	2.04	1.70	50.9	
	(c)				
Peripheral Parishes:					
	1599-1650	2.03	1.69	51.2	311
	1651-1700	2.36	1.96	55.8	
	(d)				
Other Parishes:					
	1599-1650	1.13	1.03	63.8	619
	1651-1700	1.15	1.01	70.1	
Total Ten Parishes:					
	1599-1650	1.99	1.87	29.6	3376
	1651-1700	3.75	3.11	29.3	

Source: G. ALEATI, *La popolazione di Pavia* . . ., pp. 14, 39, 163-186.
Notes: (*a*) This is the standard deviation of the normalized data.
(*b*) The central parishes, are broken into two groups, A, and B. Group A is Parrocchia della Cattedrale, Par. di San Michele. Group B is Parrocchia di San Bartolomeo da Ponte, Par. di San Pietro in Vincoli, Par. di San Marino.
(*c*) The peripheral parishes are Parrocchia di San Giovanni in Borgo, Par. di San Giorgio in Montefalcone.
(*d*) The parishes which were unclear as to category were Parrocchia di San Romano, Par. di Sant'Eusebio, Par. di Santa Maria Corte Cremona.

aggregate of these ten parishes the 3 frequency measure of the ratio of the twelve-month to six-month peak was 1.87 for the period 1599-1650 and 3.11 for 1651 to 1700. It was 3.03 for 1636 to 1696 (i.e., for the period I of our rural analysis using Belgian data. The rural result for that period was 3.08).

In order to compare the central and peripheral parishes in Pavia, we disaggregate the data into three groups; central parishes, peripheral parishes and some parishes for which the category is uncertain.[36]

[36] Central parishes: Parrocchia della Cattedrale, Par. di San Michele, Par. di San Marino, Par. di San Pietro in Vincoli, Par. di San Bartolomeo da Ponte.

Peripheral Parishes: Parrocchia di San Giorgio in Montefalcone, Par. di San Giovanni in Borgo.

Unclear: Parrocchia di San Romano, Par. di Sant'Eusebio, Par. di Santa Maria Corte Cremona.

There seems little difference between the results (see Table 7), confirming the belief in the carry over of the rural pattern in this city. A few population figures may clarify this observation. The central parishes which we analyze account for 2,446 inhabitants (about ½ the total central urban population). The central parish of Ghent, St. Jacobs that we analyzed, had by itself 5,500 inhabitants with a total population of the central parishes of 28, 300 in 1741. Pavia is simply a much smaller, more rural city.

Conclusion.

We do not deny that the results we have obtained by this first attempt are only very tentative. This is due to the fact that the data on which we base our spectral analysis are not only very sparse but also very scattered These words of caution should not prevent us however, from acknowledging the positive results from the present paper.

First of all, clear differences between urban and rural spectra appear, such as between the villages of Veurne-Ambacht and the city of Liège. For Liège the six-minth peak is clearly of less significance than for Veurne-Ambacht although for both, the twelve-month peak withe the majority of births at February or March it still of importance. In fact, from an overall look at our results we would like to tentatively put forward the hypothesis that an increase in urban economic activities and a reduction in agricultural activities causes the births to smooth out in the autumn months so that there is less of an increase in August and September. As an extreme case of urbanization we can take the St. Jacobs parish of Ghent. In this instance all seasonal pattern in births has disappeared: there is no cycle of particular importance.

The position of Liège, as a whole, is less extreme. Unfortunately in this case we were unable to examine the disaggregated central parish data. Without further data we cannot rule the possibility that some central parish in Liège has a birth pattern just as erratic as St. Jacobs. The work done by Hélin does indicate, however (and this once again supports our hypothesis), that the births in August and September may be less numerous in the central parishes than in the peripheral ones which are closer to the rural situation. Pavia, with very strong ties with the surrounding countryside, has a seasonal pattern that is very close to the one observed for the villages of Veurne-Ambacht. Furthermore, the breakdown of the nine villages of the Veurne-Ambacht into two groups (one with highly if not predominantly agricultural villages and one where the likelihood of non-agricultural economic activities is much greater) yields results which do not disprove our hypothesis. For the villages in the fertile areas, there seems to be some tendency for the six-month peak to be more prominent than in the less villages. Admittedly we have only two villages in our total sample.

We express the hope here that in future work, by broadening our empirical base we will be able to strengthen our hypothesis. These conclusions are, as mentioned, very tentative; but in spite of their shortcomings, they give a very clear indication that spectral analysis will allow an investigator to assess in an objective way the relative significance of observed seasonal fluctuations. In this paper, we have been able to characterize, compare, and assess the differing patterns of seasonality due to a number of factors, including size, nature of economic activity, location, etc., that might well escape even the most fastidious inspector of meticolously constructed charts.