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ASYMMETRIC INFORMATION AND EXCESSIVE BUDGETS IN GOVERNMENT BUREAUCRACIES

A Principal and Agent Approach

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This paper develops an optimal sequential mechanism for setting the budget of a bureaucracy where the budget setter (the principal) has only partial information concerning the minimum required budget. Bureaucratic slack is constrained by the existence of managers who can replace the current manager but only at a cost due to the need to learn the job. The principal's initial information and the cost of turnover of managers is related to the expected bureaucratic slack. The principal gains information over the budget setting process, so that a previous budget level provides important information for current budget setting.

1. Introduction

In the current climate of public opinion the cost at which public services are provided is of particular interest to the taxpayers. The existing public economics literature generally assumes that the executive or legislative body responsible for allocating bureaucratic budgets, henceforth called the principal, has either full information or no information concerning the minimum cost at which the services could be provided. In the no information case [see Niskanen (1971, 1975)], a monopoly bureau is able to present all or nothing budget and output demands to the principal so as to obtain a budget which leaves the public with no net benefit from its services. At the other extreme is the traditional public finance model which assumes full information and least cost production. This paper develops a sequential budgetary mechanism based on rational maximization which incorporates the empirically relevant case of partial information as well as the extremes. With partial information bureaus generally produce with some slack but nevertheless provide a net benefit to the public.

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In practice there seems to be a wide variation in the level of excess cost. For example a highly technical defense program is likely to have more waste than a local garbage collection agency, although even the latter is not perfectly efficient [Spann (1977)]. In this paper, the budgetary mechanism which is shown to be best within a reasonably general class is used to specify conditions which determine the size of the expected slack. For example, if the minimum funds required to produce the services of a bureau are known with certainty by the principal, then the services are produced at minimum cost. At the other extreme, if uncertainty is great and costly to reduce, outcomes approaching those of Niskanen are obtained.

The prevalence of bureaucratic slack in both public organizations and private firms has been confirmed by the available studies. This is often manifested in excessive staff and low productivity.[1] Mechanisms which limit the level of slack, particularly in the private sector, include competition from other firms, takeover bids, and reward structures which provide an incentive for managers to increase productivity and reduce costs. Even given these mechanisms, the literature shows that continued slack is to be expected except under special circumstances [see, for example, Alchian and Demsetz (1972)]. Essentially as long as the body responsible for allocation of a budget has less than full information concerning the minimum funds needed for some task, a manager can exploit this to obtain some excess budget.

The budgetary mechanism developed in this paper reveals an additional constraint on bureaucratic slack which could be called sequential competition between managers. The mechanism is based on the existence of equally qualified managers who can replace the current manager of a bureau should he resign, but only at a cost to the principal imposed by the need of any new manager to learn the technology of the job. The principal can exploit this (availability of managers) by setting a budget below the maximum level of funds which, according to the principal's limited information, may be required for the job of the bureau. If the funds are insufficient, the manager resigns and moves to an alternative job so as to protect his reputation as a good manager. On the other hand if the budget is sufficient the principal has succeeded in reducing some of the slack within the bureau. In either case the principal gains more information concerning the minimum necessary funds. The budget offers can be chosen optimally by weighing the expected cost of changeover of bureau heads against the direct gains from budget cutting. Even if the bureau is the sole source of supply for its type of service, its monopoly power is limited by the probability (on the basis of the principal's information) and cost of turnover of the bureau's

[1]For example, see the survey articles by Orzechowski (1977) and Spann (1977). Leibenstein (1966) lists a number of cases where the costs of private firms have been reduced 20 to 30 percent by managerial reorganization. See Williamson (1964) for a model of a private firm where the management operates with an oversized staff.

management. Although such a turnover of bureau managers may not often be observed, this does not invalidate the model. Budget cutting which leads to the resignation of bureau heads is predicted not to be optimal once a program is well established (see section 8).

With this sequential budgetary mechanism, the principal observes only whether the budget offer at any stage is sufficient for the job. Information is updated by narrowing the range of possible minimum budgets. The budgetary mechanism is related to a well known sequential search procedure over prices [Rothschild (1974)]. Nevertheless, there are differences since in Rothschild's case actual prices are observed so that the appropriate updating of information (using Bayes' rule) differs.

The exploitation of the sequential availability of managers[2] also has implications for the distribution of expected slack within both public and private bureaucratic organizations. Since the budgetary mechanism is formulated in bilateral terms it also applies with appropriate modifications within a bureaucracy. Budget allocation decisions are made at each hierarchic level within the organization and each subhead, the agent, has more information concerning the minimum funds required to operate his section than his immediate superior, the principal.[3] Although this is not pursued further in this paper, one might expect slack to be greater in those areas of operation which are harder for an outsider to understand. For example, the computer section or research and development may have more slack than the typing pool, which can be easily monitored and where it is not difficult for a new manager to learn the particular characteristics of the job.

I have made some strong simplifying assumptions in developing a prototype model of the budgetary mechanism which nevertheless seems to capture the main aspects of the problem. Some of the assumptions allow a simple treatment of the main ideas and for this reason may be of interest in themselves. For example, the assumption of a deterministic budgetary mechanism, in which the managers are fully informed concerning the principal's reservation budget, results in substantial simplification relative to a randomized mechanism in which a manager faces a distribution of reservation budgets. Moreover the simple mechanism is shown to be superior (from the viewpoint of the principal) to the randomized mechanism under some circumstances.

Section 2 sets out the assumptions which determine the generality of the class of budgetary mechanisms to be considered. These assumptions are used

[2]A number of authors, for example, Alchian and Demsetz (1972) and Fama (1979), have recognized the importance of outside markets for managers as a disciplining mechanism to reduce slack. Fama discusses the conditions under which the outside labor market ensures that managers work within their contract. I am concerned with the effect of the outside labor market on the contract itself and the slack within the contract.

[3]See Albert Breton (1974, ch. 9), Gordon Tullock (1965) and Anthony Downs (1967, ch. XI) for analysis of the information flow through a bureaucracy.

to show the nature of the equilibrium budget. Section 3 deals with the full decision problem of the manager, and section 4 contains a brief comparison of the budgetary mechanism with some alternative bidding models. Section 5 is concerned with the existence and properties of the best budgetary mechanism from the viewpoint of the principal. One of the main simplifications of the class of mechanisms is relaxed so as to provide a comparison with a broader class of mechanisms. Some conditions under which the simple budgetary mechanism remains superior are derived. The remaining sections are devoted to results which follow from use of the best budgetary mechanism. In section 6, a simple comparison is made between the full information and extreme monopoly results obtainable from the optimal mechanism. Section 7 draws out some comparative static propositions regarding the level of expected excess cost and slack. The importance of previous budgets as a source of information in current budget setting is discussed in section 8, and section 9 contains some concluding remarks.

2. The model

Quite commonly it makes sense to view the services provided by a bureau, for example administrative services or perhaps a construction project as a single unit or job. The principle's choice of each bureau's level of output or job is assumed to be determined outside the model.[4] Also, abstracting from the well known problems of measurement of services, it is assumed that the principal observes whether or not each job is completed.

Assumption 1. A bureau is responsible for a job J with a fixed level of service which is costlessly observable by the principal and has value v to the principal.

The objective of the principal is to maximize the value from each job net of the expected budgetary cost, given the principal's limited information concerning the best practice cost of the job. Although the behavior of a legislative or executive body does not always conform to this ideal,[5] it allows

[4]Spencer (1980) shows that if the principal has only the minimal information that its demand function for a bureau's services intersects the bureau's best practice average cost curve, then the principal, rather than the bureau, is in a position to specify the level of provision of each service. The principal then knows that some per unit budget and output combination on its demand curve is feasible and that the bureau management cannot require a budget and output combination which extracts the area under the principal's demand curve as in the Niskanen (1971, 1975) models. The principal can then specify the level of output on the basis of per unit cost estimates possibly provided by the bureau.

[5]For example, members of the legislative body may improve their chances for re-election by allocating more funds than necessary for programs in their district, given that the tax cost is spread over all districts. Also the desire by legislators to obtain votes of bureaucrats may increase the budget allocation [Tullock (1974) and Chan (1979)].

a consideration of the excess cost arising from bureaucratic monopoly power per se.

The standard assumption that all the decision making power of a bureau is embodied in the bureau head or manager is made for ease of analysis. It is assumed that if a manager accepts the job he will complete it, since otherwise this would be observed (Assumption 1) and he will be marked as a bad manager.

Assumption 2. A manager's indirect utility if he accepts and completes job J is represented by the Von Neumann–Morgenstern utility index $u(b,\theta)$, where b represents the budget and $\theta\in[\underline{\theta},\bar{\theta}]$ is an index, which is not observed by the principal, of the best practice resource requirements of the job. $u(b,\theta)$ has partial derivatives, $u_b>0$ and $u_\theta<0$.

A larger value of θ is associated with greater resource requirements for the job. For example, for any given b, a larger θ might indicate that greater effort is required by the manager in co-ordination and supervision to complete the job so that the manager's utility is lower. Similarly for the same θ, $u_b>0$ since additional budget can be spent on extra staff, each of whom has to be supervised less closely to produce the same total output. Alternatively the management may have preference for extra staff for other reasons such as prestige or promotion which leads to a desire for additional budget and slack within the bureau.

The minimum budget necessary for the job, $m(\theta)$, is defined as the reservation budget at which a fully informed manager is just willing to accept the job rather than take his next best alternative at utility u_0,

$$m(\theta){:}\; u(m(\theta),\theta)=u_0, \tag{1}$$

where m is strictly increasing in θ, since $m'(\theta)=-u_b/u_\theta>0$ by Assumption 2 since $m'(\theta)$. Although $m(\theta)$ also depends on u_0, this is of no interest in the subsequent analysis.

Assumption 3. For any particular job J, the value of $m(\theta)$ is assumed to invariant across potential managers.

Assumption 3 is certainly satisfied if all potential managers are identical, but such a strong assumption is not necessary. For example if differences between managers are independently known to the principal, market pressure will lead to salary compensation so that $m(\theta)$ (with salary included) is the same for any manager offered the job. However, such independent knowledge of ability is also not realistic and only under special circumstances[6] do

[6]Managers could be ranked in ability according to the level of output they can produce from given inputs including their own effort. A manager is then 'better' than another manager if he can do the same job at the same budget but at less effort and therefore a higher utility to

(unknown) differences in ability not cause variations in the reservation budget. The complication of a budgetary requirement which varies both because of unknown technology and unknown managerial ability may better be considered in some further research.

Although the principal does not observe the minimum required budget m and the index θ for each job the principal can gain some information on the range and likelihood of possible minimum budgets. This initial information is less precise the greater the difference between the planned job and previous experience. There may be very little information indeed available regarding a totally new departure of high technical complexity as is common in defense or space research agencies. Information may also have been gained, at a cost, from various control devices such as investigation of previous jobs [see Brenton and Wintrobe (1975)]. The principal's information concerning job J is the probability, $F(m)$, that the minimum required budget for job J is less than the value m.

Assumption 4. The principal's information consists initially of an a priori probability distribution $F(m)$, which is assumed continuous on the closed interval $[\underline{b}, \bar{b}]$ where $\underline{b} = m(\underline{\theta})$ and $\bar{b} = m(\bar{\theta})$. The density of $F(m)$ denoted $F'(m)$, is assumed continuous and bounded away from zero on $M \equiv (\underline{b}, \bar{b}]$.

The remaining assumptions are concerned with the ground rules or class of the budgetary mechanism, and the manager's information at the various stages of the procedure.

Assumption 5. For each job J, the principal negotiates sequentially at a cost r per negotiation with candidates for manager picked (at random) from a set Ω_J of potential managers. Ω_J is assumed to be large relative to the number of managers the principal may possibly require in the process of sequential negotiation. If a manager picked from Ω_J does not accept the job, he is not returned to Ω_J. Managers are not returned to Ω_J to prevent the possibility that they may reject the job at a sufficient budget in the hope of being picked again from Ω_J at a higher budget offer.

Assumption 6. For each negotiation the principal determines a maximum budget (reservation budget) to be allocated to the job. The reservation budget is changed only if a new candidate for manager is picked from Ω_J. The process stops when a manager agrees to complete the job at or below the principal's reservation budget for that negotiation.

himself. If ability is not specific to the job it is possible that the 'better' manager may also enjoy just sufficient extra utility from his next best alternative job so as to leave his reservation budget at the same level as less able managers. Neither the manager's effort nor his utility level would be observable by the principal.

A further feature is that a manager picked by the principal knows the principal's reservation budget and learns $m(\theta)$ before he makes the final decision whether to accept the job.[7] Therefore a potential manager must first decide whether to study the job in order to have the opportunity to finally accept or reject the job. This first stage of the decision process is not necessary for the determination of the equilibrium budget and is set out in section 4.

Assumption 7. A candidate for manager knows $m(\theta)$ and the principal's reservation budget for that negotiation at the time of the budget decision.

Resource allocation with asymmetric information turns on the ability of the informed agent (the candidate for bureau head) to conceal the true minimum budget for his own benefit. A candidate for bureau head can credibly choose the outcome associated with any value of m in the principal's current information set. In particular the ith candidate can claim $m(\theta)=b_i$, where b_i is the principal's reservation budget at that stage. This is closely related to the self-selection constraint in the principal and agent literature [see, for example, Harris and Townsend (1981, p. 38)]. Also any equilibrium budget (at which the job is done) must be feasible.

Feasibility. b is a feasible budget if $u(b,\theta)\geqq u_0$ or alternatively if $m(\theta)\leqq b$.

Proposition 1. Given Assumptions 1 to 7, if an equilibrium budget exists it is the first feasible reservation budget in the principal's sequence of reservation budgets described in Assumption 6.

Proof. Let b_i be the principal's reservation budget for the ith negotiation. If b_i is not feasible, then by definition $u(b_i,\theta)<u_0$ and the fully informed manager (Assumption 7) rejects the job. Since by Assumption 6 the principal does not offer a higher b_i to the same manager, the manager takes his alternative job at utility u_0 and the job is not accepted at that negotiation. If b_i is feasible, the fully informed manager (Assumption 7) is in a position to insist $m(\theta)=b_i$ and will accept the job at the principal's reservation budget. This follows since by Assumption 2, a manager always prefers b_i to a lower budget and by Assumption 6, the manager does not receive a higher offer if he rejects the job. Therefore the first feasible reservation budget will be accepted and (by Assumption 6) the process then stops. Q.E.D.

[7] Harris and Raviv (1979) prove a number of propositions concerning optimal incentive contracts. The model described here differs from theirs since they assume the contract is negotiated before θ is known.

Proposition 1 shows that the analysis is greatly simplified by the Assumptions 6 and 7 that the principal's reservation budget is fixed for each negotiation and known to the manager at the time of the budget decision. Nevertheless, Assumption 6 may seem to be irrational since the principal could save the cost r of bringing in a new candidate to learn the job by re-opening negotiations after a manager has rejected a reservation budget as infeasible. However, if this were normal procedure, potential managers would soon believe that by rejecting the initial 'reservation' budget, they would receive a higher budget offer with probability one. In a sense the higher budget offer becomes the reservation budget known to the manager by Assumption 7. By Proposition 1 the initial 'reservation budget' would always be rejected whether or not it was feasible in favor of the true higher reservation budget. Assumption 6 is therefore really a consequence of Assumption 7. Thus the mechanism of budget offers that is best in an ex ante sense does not allow the bargainers to take advantage of this ex post opportunity for a Pareto improvement which alters the budgetary procedure and upsets the incentives after the decisions under the original procedure have been made.[8]

Also it is interesting that Proposition 1 implies that the preliminary process of negotiation is immaterial. However long and involved the bargaining process may appear to be, in the final analysis it resolves into the question of whether the candidate for agency head will accept the principal's reservation budget.

This of course does not rule out the possibility that the principal may gain from a mechanism in which both Assumptions 6 and 7 are relaxed to allow the principal's reservation budget to be raised with less than probability one after the manager has rejected the budget. This would no longer be strictly a case of asymmetric information since neither side is fully informed with respect to the other's reservation budget. Some conditions under which such a mechanism is inferior are proved in section 5 after the derivation of some of the properties of the best sequence of fixed reservation budgets.

3. The decision problem of the manager

The premise that the manager of the bureau has full information concerning the minimum budgetary requirements yet the principal has limited information is not realistic unless obtaining this information is costly. The cost and the large number of jobs for which the principal is responsible

[8]See for example Myerson (1979) and Harris and Townsend (1981) for exact definitions and some general theorems regarding the optimality of mechanisms under asymmetric information. An optimal mechanism is defined using the restrictions imposed by asymmetric information and therefore need not satisfy the *ex post* conditions for Pareto efficiency.

is assumed to preclude the principal from fully learning each job and necessitates delegation to the bureau manager.

Assumption 8. There is a strictly positive cost for any manager picked from Ω_j to learn $m(\theta)$. The principal's portion of this cost, which is paid as compensation to the manager, is included in $r>0$. The manager incurs a net loss of utility λ (which may be zero) after compensation in order to learn $m(\theta)$.

For simplicity a prospective manager is assumed to have the same information as the principal at the time he is picked from Ω_j. From Proposition 1, the principal and the ith pick for manager know $m\in(b_{i-1},\bar{b}]$ denoted M_{i-1} where b_{i-1} is the maximum of the $i-1$ reservation budgets that have previously been rejected. $F(m)$ is truncated from below at b_{i-1} and the distribution of m is updated by defining m conditional on $m\in M_{i-1}$. If $i=1$, $M_0=M$, the initial information.

Assumption 9. The ith pick for manager from Ω_j is assumed to know the principal's reservation budget for the ith negotiation and the distribution of m is $F(m\,|\,m\in M_{i-1})$.

If the fact that it takes time to learn $m(\theta)$ is conveniently ignored on the basis that the study period is short in relation to the time horizon of the job, from Assumptions 7, 8 and 9 the ith pick for manager knows that if he decides to study the job he can choose the $\max[u_0-\lambda, u(b_i,\theta)-\lambda]$ where b_i is the principal's reservation budget at the ith stage. Therefore the manager will study the job if his expected utility from the trial period exceeds his alternative utility or if

$$\mathrm{E}_i\{\max[u_0-\lambda, u(b_i,\theta)-\lambda]\}>u_0. \tag{2}$$

The expectation is taken with respect to the manager's knowledge of the distribution of m before he studies the job (Assumption 9).

The ith candidate will be willing to study the job if the compensation during the trial period is sufficient to offset any negative factors so that $\lambda=0$, provided b_i exceeds the previously rejected reservation budgets. The manager can then never do worse than his alternative. Even if $\lambda>0$, it is in the interest of both parties to ensure institutional arrangements such that the trial period is attractive or otherwise the job will certainly not be done. In a competitive market for managers the payment to each manager during the learning period, and hence λ, will be adjusted until each manager is indifferent between accepting the principal's offer to study the job and his alternative. The determination of the distribution of the learning cost between the principal and the manager is outside the scope of this paper.

4. Comparison with bidding models

The use of government operated agencies is of course not the only way to provide public services. Competitive bidding is one commonly suggested alternative [see, for example, Vickrey (1960), Demsetz (1968) and Holt (1979) for some competitive bidding models]. The fact that the technology is unknown and costly to learn means that competitive bidding is not necessarily better than the mechanism of this paper. If the uninformed bidder has to do the job within the bid price, the bids will include the expected cost of the job calculated on the basis of the partial information available to all parties plus the cost of learning the job and a premium for risk. Under the mechanism of this paper, the first budget offer is determined by the lowest estimate of the minimum budget and a cost which includes that part of the cost of learning the job borne by the principal. It does not include a premium for the bureau head's risk since he is fully informed at the time of the budget decision. Therefore this budget offer and others early in the sequence can be considerably less than the competitive bid and may in fact be less than the expectation of the best practice cost calculated from the principal's initial information.

Of course the premium for risk in a bidding model may be reduced by a contract which allows costs plus a normal profit, but the firm then has less incentive to reduce costs, leading to cost overruns. Another alternative is for all interested firms to undertake the costly investigation before they make their bid. This requirement that all firms make simultaneous investments is a disadvantage relative to sequential procedures in which the extent of search is chosen optimally and in which an additional search is made only if necessary. In the sequential model of this paper, each learning cost is incurred only if the previous budget offer is insufficient. Furthermore, if only a few firms undertake the investigation reducing the total learning cost, then there is no guarantee that the bid price is competitive. There are other problems with the bidding solutions, such as the specification of long term contracts under conditions of uncertainty and the enforcement of these contracts [see, for example, Williamson (1976)]. These kinds of considerations may eventually lead to an explanation of why some government jobs are put out to private contract and others are done in house.

5. Best budgetary mechanism

The principal's expected net return for any sequence of reservation budgets $\{b_t\}_1^n$, given initial information and offer cost r is represented by $\pi(\{b_t\}; M, r)$. Both the (possibly infinite) maximum number of offers, n, in the budget sequence and the elements of the sequence are to be chosen optimally.

Proposition 2, that any optimal sequence (if it exists) is strictly increasing, enables $v(\{b_i\}; M, r)$ to be expressed in terms of the distribution $F(m)$.

Proposition 2. If $\{b_i^*\}$ *maximizes* $v(\{b_i\}; M, r)$, *then* $b_{i+1}^* > b_i^*$ *for all* $i \in \{0, 1, 2, \ldots, n\}$ *where* $b_0 = \underline{b}$.

Proof. This follows immediately from the fact that if $b_{i+1} \leqq b_i$, then from Proposition 1, there is zero probability that b_{i+1} is accepted yet it costs the principal r to make the offer. Q.E.D.

Since the budget sequence is strictly increasing, the *a priori* probability the job is accepted over the entire sequence of n offers is $\Pr\{m \leqq b_n \mid m \in M\} = F(b_n)$. The ith offer, b_i, is made (at an offer cost r) if the previous $i-1$ offers are rejected. The previous $i-1$ offers are rejected with $\Pr\{m \in M_{i-1} \mid m \in M\} = 1 - F(b_{i-1})$ where $M_{i-1} = (b_{i-1}, \bar{b}]$. The offer, b_i, is accepted given that the previous offers are rejected with $\Pr\{m \leqq b_i \mid m \in M_{i-1}\} = \{F(b_i) - F(b_{i-1})\}/\{1 - F(b_{i-1})\}$. Therefore the *a priori* probability the offer b_i is made and accepted is $\Pr\{b_{i-1} < m \leqq b_i \mid m \in M\} = F(b_i) - F(b_{i-1})$. Hence the total expected cost is the sum of each offer, b_i, weighted by $F(b_i) - F(b_{i-1})$ plus the offer cost r weighted by $1 - F(b_{i-1})$. The expected net value of the job is

$$v(\{b_i\}; M, r) = vF(b_n) - \sum_{i=1}^{n} \{b_i[F(b_i) - F(b_{i-1})] + r[1 - F(b_{i-1})]\}. \tag{3}$$

All the best sequences of budget offers (if any exist) also have the useful property that the optimal number of offers in any such sequence is less than some number, $n(M, r, F)$ determined by the parameters M and r and the distribution F. Therefore in determining an optimal budget policy only sequences of length bounded by $n(M, r, F)$ need be considered. The proof of this Proposition 3 is established in appendix A. If $\{b_i\} \in B$, where B is the set of all infinite budget sequences, then it is convenient to write the principal's expected net return from $\{b_i\}$ as $v_\infty(\{b_i\}; M, r)$.

Proposition 3. Consider sequences of budget offers which terminate with probability one on or before the n*th trial. Let this set be denoted by* B_n. *Then for any budget sequence* $\{b_i\} \in B$ *there exists* $n(M, r, F)$ *such that*

$$\max_{\{b_i'\} \in B_n} v(\{b_i'\}; M, r) > v_\infty(\{b_i\}; M, r),$$

where $n = n(M, r, F)$.

The principal therefore chooses $\{b_i\}$ with a number of offers $n \leqq n(M, r, F)$ to maximize $v(\{b_i\}, M, r)$. Since F is continuous on $[\underline{b}, \bar{b}]$, $v(\{b_i\}, M, r)$ is also continuous on $[\underline{b}, \bar{b}]$ and Proposition 4, that an optimal budget sequence exists, follows almost immediately.

Proposition 4. An optimal budget sequence exists and all optimal sequences belong to B_n, where $n \leq n(M,r,F)$. The optimal sequence may not be unique.

Proof. Since $v(\{b_i\}, M, r)$ is a continuous function of a finite number of variables $\{b_i, i=1,2,\ldots,n\}$ defined over a compact set $[\underline{b}, \bar{b}]$, an optimal budget sequence exists by the Weierstrass theorem for any given number of offers n. Since from Proposition 3, the number of offers in an optimal sequence is bounded by $n(M,r,F)$, the maximum expected net value of the job exists and is the greatest of these maximum expected net returns conditional on $n \leq n(M,r,F)$. Since v is not necessarily concave in $\{b_i\}$ for all distributions F, the best sequence of budget offers exists but may not be unique. Q.E.D.

In any optimal budget policy, offers prior to the final offer b_n satisfy the first order conditions for an internal maximum of v. These are

$$F(b_i) - F(b_{i-1}) = (b_{i+1} - b_i + r)F'(b_i), \qquad i = 1,2,\ldots,n-1. \tag{4}$$

For example, if F is uniform on $[\underline{b}, \bar{b}]$, these reduce to

$$b_{i+1} - b_i = b_i - b_{i-1} - r, \qquad i = 1,2,\ldots,n-1, \tag{5}$$

so that the difference between the optimal offers reduces by a constant r each time.

If the final offer b_n is less than $\bar{b}$, then there is some probability, $1 - F(b_n)$, that the principal cancels the job because it has become too expensive. It is interesting to see the conditions under which this does not occur and the job is completed with certainty.

Proposition 5. If $v \geq \bar{b} + r$, then under any best sequence of budget offers, $b_n = \bar{b}$ and the offer is accepted with probability 1. If $v < \bar{b} + r$, $b_n < \bar{b}$ and there is a positive probability that the offer is rejected.

Proof. The expected net gain from the offer b_i, given that any previous offers have been rejected is the gain, $v - b_i$, multiplied by the probability the offer is accepted less the cost of the offer. This is $(v - b_i)\Pr\{m \leq b_i \mid m \in M_{i-1}\} - r$. The expected net gain from $\bar{b}$, since it is accepted with certainty, is $v - \bar{b} - r$. If $v \geq \bar{b} + r$, no matter how many offers have been rejected, the principal cannot lose from the final offer $\bar{b}$. Offers other than $\bar{b}$ are made only because they are expected to reduce costs. If $v < \bar{b} + r$, the expected net gain from the offer $\bar{b}$ is negative so that in any optimal sequence budget offers will cease at a budget, b_n, less than $\bar{b}$. The final offer $b_n < \bar{b}$ satisfies the condition that the expected net gain from b_n is positive or zero, and the expected net gain from b_{n+1} is negative. The probability that the job is not accepted is the probability that the final offer is rejected, $\Pr\{m > b_n \mid m \in M\}$, which is strictly positive. Q.E.D.

It is interesting to relax Assumptions 6 and 7 and consider a more complicated class of budgetary mechanism in which renegotiation occurs with some positive probability whenever the candidate for manager rejects the job. Under Assumption 10, the budgetary mechanism is randomized since for any negotiation the principal has a distribution of reservation budgets.

Assumption 10. For the ith negotiation the principal determines an initial maximum budget b_i^* to be allocated to the job. If b_i^* is rejected by the candidate for bureau head, and if $i < n$, the principal reopens negotiations with probability π_i where $0 < \pi_i < 1$ and raises the budget offer to b_{i+1}^*, which is renegotiable with probability π_{i+1}. If b_i^* is rejected, the principal brings in a new manager with probability $(1-\pi_i)$ at an initial maximum budget offer of b_{i+1}^*. The process stops when a manager agrees to complete the job at a budget the principal is willing to pay. At the time of the budget decision, the ith prospective manager is assumed to know $m(\theta)$, π_j and b_{j+1} for all $j = i, i+1, \ldots, n$.

There is no loss of generality in assuming that the new candidate is given the same offer as the existing candidate would receive with renegotiation. The principal has the same information concerning the minimum acceptable budget in either case and the cost r of bringing in a new candidate acts as a fixed cost with respect to the choice of further budget offers.

Unfortunately a powerful theorem which restricts optimal mechanisms to a simple type of direct mechanism [Harris and Townsend (1981, p. 47)] does not apply to sequential mechanisms with learning. It is therefore possible that a randomized budgetary mechanism of the type defined by Assumption 10 is optimal. Proposition 6 shows some special circumstances under which this is not the case. See appendix A for the proof.

Proposition 6. Consider any sequence of renegotiable offers $\{b_i^*\}^n$ where $n > 1$, which satisfied Assumptions 1–5 and 10 and is not inferior in the sense that the principal's expected net return $z_n(\{b_i^*\}; M, r)$ strictly exceeds the expected net return if one offer is omitted from the sequence. If the managers in Ω_1 are risk neutral $(u_{bb} = 0)$, and if $u_{b\theta} \geqq 0$ then there exists a sequence, $\{b_i\}_1^n$, of non-negotiable offers which satisfies Assumptions 1–7 and for which $v(\{b_i\}; M, r) > z_n(\{b_i^*\}; M, r)$.

Proposition 6 shows that if managers are risk neutral the principal should choose the simple budgetary mechanism characterized by Assumptions 1–9 rather than follow a more complicated procedure in which there is uncertainty concerning the principal's reservation budget. With the possibility of renegotiation, there is some probability that a manager will reject a feasible reservation budget in the hope of receiving a higher budget offer. The principal's expected loss from this behavior exceeds the expected saving in the offer cost from renegotiation. If managers are risk averse the probability that a manager will reject a feasible budget is lower but strictly positive so that the simple budgetary mechanism may continue to be superior depending on the distribution F and value of r. In any case the comparative static results of the remainder of this paper are not likely to be affected by the use of this more complicated mechanism and it seems best to show these results in the simplest manner using the mechanism described by Assumptions 1–9.

6. Comparison of solution with the public finance and monetary monopoly bureau results

One of the themes of this paper is that partial information leads to costs somewhere in between the extremes obtained with full information on the one hand and a monopoly bureau on the other.

The public finance solution where the job is done at minimum feasible cost occurs only if the minimum budget, $m(\theta)$, is known initially or if the offer cost is zero so that it is costless for the principal to find this minimum cost. If r is zero, by starting with a budget offer near $\bar{b}$ and increasing the offer by small amounts until the job is accepted or becomes too expensive, the principal can ensure that any offer that is accepted is arbitrarily close to the minimum cost and there is no slack.

A useful concept is the expected budgetary slack $s(M,r)$ for an optimal budgetary policy if the job is accepted. This is the expected excess of the best budget offers over the minimum budget cost, based on initial information M and offer cost r. The amount of each excess budget is weighted by the probability it occurs.

$$s(M,r)=\sum_{i=1}^{n}\int_{b_{i-1}}^{b_i}[b_i-m]\,\mathrm{d}F(m\,|\,m\leqq b_n) \quad \text{where} \quad b_0=\underline{b}. \tag{6}$$

If $v\geqq\bar{b}+r$, so that $b_n=\bar{b}$, then $F(m\,|\,m\leqq b_n)=F(m)$.

If the principal has only partial information on costs, the expected slack, $s(M,r)$ is strictly positive, even if the best budgetary procedure is followed. This shows that on average there is excess budget within the bureau. The expected slack is zero only if the principal has full information on costs.

Extreme monopoly in this context means that the job must be done by the bureau and its initially chosen head or not at all. The cost r of bringing in a new manager is prohibitive so that it is optimal to have only one offer. From Proposition 5, if $v > \bar{b}+r$, then the best single offer is $\bar{b}$. The bureau extracts the maximum budget that might be required given the principal's information. The expected slack is then at its maximum $s(M,r) = \int_{\underline{b}}^{\bar{b}} [\bar{b}_l - m]\,\mathrm{d}F(m)$. The expected slack at the single offer $\bar{b}$ increases with $\bar{b}-\underline{b}$, the range of initial information M. This range is likely to be large if similar jobs have been done under similarly adverse circumstances so that the principal must rely mostly on limited technical information concerning the job.

The Niskanen (1975) model assumes that the principal has 'no information' concerning the cost of the job and that the monopoly bureau can extract a budget equal to the entire value of the job to the principal. This holds for the model of this paper only in the special case where the minimum cost budget sequence is the single offer $\bar{b}$ and $v = \bar{b}+r$. If $v > \bar{b}+r$, then from Proposition 5, the final offer is $\bar{b}$ and the principal's expected net return from the optimum sequence of offers is at least $v-(\bar{b}+r)$ which is strictly positive. If $v < \bar{b}+r$, the expected net gain from the final offer b_n, $(v-b_n)\Pr\{m \leq b_n \mid m \in M_{n-1}\} - r$, is non-negative. The probability that the offer b_n is accepted is strictly less than one ensuring that v is strictly greater than $b_n + r$. The expected surplus if the job is accepted is at least $v-(b_n+r)$, which is strictly positive. This holds for all positive integer values of n, including a value of one. Even in the extreme case where there is only one budget offer in the optimal sequence, the bureau is in general unable to extract the entire value of the job to the principal.

7. Excess cost and slack: Some comparative statics

For simplicity it is assumed throughout this section that $v \geq \bar{b}+r$, so that the job is accepted with certainty. In this case, any budget sequence which maximizes the expected net return also minimizes the expected cost of the job.

The minimum expected cost of the job under Assumptions 1–9 and $v \geq \bar{b}+r$ is denoted $w(M,r)$ and is the sum[9] of three components — the expected slack, $s(M,r)$ [see (5)], the expected search cost, $rq(M,r)$, and the expected

[9]From expression (3),

$$w(M,r) = \sum_{i=1}^{n} \{b_i[F(b_i) - F(b_{i-1})] + r[1 - F(b_{i-1})]\}$$

$$= \sum_{i=1}^{n} \int_{b_{i-1}}^{b_i} [b_i - m]\,\mathrm{d}F(m) + \int_{\underline{b}}^{\bar{b}} m\,\mathrm{d}F(m) + \sum_{i=1}^{n} r[1 - F(b_{i-1})]$$

$$= s(M,r) + \mathrm{E}(m) + rq(M,r).$$

budget cost with efficient operation, $E(m)=\int_{\underline{b}}^{\bar{b}} m\,\mathrm{d}F(m)$. The expected search cost is the offer cost multiplied by the expected number of offers, $q(M,r)$, in the optimal sequence. Since the ith offer is made with the probability that b_{i-1} is rejected, $q(M,r)=\sum_{i=1}^{n}[1-F(b_{i-1})]$, where $\{b_i\}_{i=1}^{n}$ is the best sequence of budget offers. The expected excess cost of the job over the expected cost with efficient operation is $s(M,r)+rq(M,r)$.

Proposition 7 contains the comparative static effects of a change in the negotiation cost r. The proofs of this proposition and all the remaining propositions of this paper are in appendix B.

Proposition 7. Let n be the number of offers in the optimal sequence with offer cost r satisfying Assumptions 1–9. If $n>1$, $r'>r$ and $v\geq\bar{b}+r'$ then

(*i*) $q(M,r)>q(M,r')$,

(*ii*) $s(M,r)<s(M,r')$, *and*

(*iii*) $s(M,r)+rq(M,r)<s(M,r')+r'q(M,r')$.

The simple results when $n=1$ were discussed in section 6. If $n>1$, an increase in r makes it optimal for the principal to increase the reservation budget at each stage of the sequential search procedure. This reduces the probability that any offer is rejected with the consequence that the expected number of offers before the job is accepted is reduced [see Proposition 7(i)] and the expected slack is increased [see Proposition 7(ii)]. As discussed in section 6, the expected slack is zero if there is no cost of obtaining information ($r=0$) and at its maximum when there is at most one offer in the optimal sequence. Proposition 7(ii) shows that this result is sufficiently robust to hold for small variations in r. Although $q(M,r)$ decreases with r, this is more than offset by the rise in $s(M,r)$ and r itself, so that the expected excess cost, $s(M,r)+rq(M,r)$ is also increasing in r [see Proposition 7(iii)].

Information is less precise if the same distribution $F(m)$ is defined conditional on a broader range M while maintaining the same mean $\mathrm{E}(m)$.[10] Lemma 1 ensures that the expected excess cost and its components $s(M,r)$ and $rq(M,r)$ are unaffected by a transformation $m'=m+\alpha$ of $F(m)$, which changes the mean to $\mathrm{F}(m')=\mathrm{E}(m)+\alpha$. Therefore there is no loss of generality in setting $\alpha=-\underline{b}$ so as to measure the range of initial information from $\underline{b}'=0$. Less precise information can then be characterized by a simple transformation $m'=\gamma m$, where $\gamma>1$, which stretches the domain of F from $m\in M=(0,\bar{b}]$ to $m\in M'=(0,\gamma\bar{b}]$. Although $\mathrm{E}(m')\neq\mathrm{E}(m)$, this does not affect the components of interest (by Lemma 1).

[10]The transformation $m^*=\gamma(m-\mathrm{E}(m))+\mathrm{E}(m)$, where $\gamma>1$ represents less precise information by a factor γ. $\mathrm{E}(m^*)=\mathrm{E}(m)$ and $\mathrm{var}(m^*)=\gamma^2\,\mathrm{var}(m)$.

Lemma 1. If $v \geqq \bar{b}+r$ and $m' = m+\alpha$, where $m \in M$ and $m' \in M'$, then $s(M',r) = s(M,r)$ and $q(M',r) = q(M,r)$.

Lemma 2 is a homogeneity property of the components of cost which is critical for the proof of the comparative static effects of changes in the range of the principal's initial information M. If the initial information is less precise by a factor α and if the offer cost is increased by the same factor, then the expected number of offers is unaffected and the expected slack and expected excess cost increase by a factor γ.

Lemma 2. If $v \geqq \gamma\bar{b}+\gamma r$, $m' = \gamma m$ and $r' = \gamma r$, where $\underline{b}=0$, $m \in M$ and $m' \in M'$, then $s(M',\gamma r) = \gamma s(M,r)$, $q(M',\gamma r) = q(M,r)$ and $s(M',\gamma r) + \gamma r q(M',\gamma r) = \gamma[s(M,r) + rq(M,r)]$. Also $w(M',\gamma r) = \gamma w(M,r)$.

Proposition 8. If $v \geqq \gamma\bar{b}+r$, $m' = \gamma m$ where $m \in M$, $m' \in M'$, $\underline{b}=0$ and $\gamma > 1$ then

(i) $q(M',r) > q(M,r)$, provided $q(M',r) > 1$, and
(ii) $s(M',r) + rq(M',r) > s(M,r) + rq(M,r)$.

Proposition 8(i) implies that the expected number of offers and therefore the expected search cost increases as the principal's information becomes less precise.[11] In general not much can be said concerning the direction of change of expected slack. From the first order conditions (4), the difference between offers may change in an irregular way either increasing or decreasing depending upon both F and F' as the distribution is stretched over a wider range. Nevertheless the expected excess cost is increased [Proposition 8(ii)] so that the additional search cost must more than outweigh any reduction in expected slack. The *a priori* expected slack $s(M,r)$ should not be confused with the expected slack after an offer is accepted. Information is then updated so that F is conditional on $m \in (b_{k-1}, b_k]$.

8. The informational value of previous budgets

So far I have restricted the discussion to a one time job which has not been done previously. Many government jobs do in fact repeat, especially those which are mainly administrative in nature. Bureaus responsible for administering a welfare program or maintaining working safety standards are examples of this. For simplicity I shall analyse the case of a job which has been done under a former administration or government but for which the

[11]This is similar to the Rothschild (1974) result that under a sequential search procedure with a cost per observation, increased price dispersion increases the intensity of search. The problem studied here is not quite the same, since Rothschild defines increased price dispersion by adding more weight to the tails of the distribution but maintaining the domain over which the distribution is defined.

budget is determined only once during the current administration. The principal is interested only in minimizing the current expected cost of the job with no desire to take into account the fact that the job may be repeated in future during a new administration. In these circumstances, the principal's maximization problem is unchanged from section 5 except that there may be no cost of making the initial budget offer to the fully informed existing manager. The absence of this fixed cost does not affect the optimal budget sequence. The budget, b_k, at which the job has been completed previously provides information that the job can be done for that budget or less. The upper limit on the funds which may be required for the job, $\bar{b}$, is therefore equal to b_k. This is one of the most important processes which determine the initial information held by the principal on the range of possible minimum budgets for each job.

As a polar case, it is possible that the information from former budgets does no more than confirm the originally known upper limit for costs based on technical conditions. This could occur if, from the inception of the job, the possible saving from setting a budget below the originally known upper limit is not sufficient to offset the risk and cost of rejection of a budget offer that is too low. Each year it would then be optimal to offer the budget based on the worst known technical conditions. This offer would be accepted with certainty. The expected budgetary slack would not be reduced over time with the repetition of the job.

Alternatively if at any time a budget offer is made which would not cover costs under the worst known technical circumstances then the principal gains information on the true minimum cost whether or not the offer is accepted. If the offer is rejected as explained previously, this increases the known lower limit of cost and truncates the distribution $F(m)$ from below. Similarly, if the budget offer is accepted, the distribution $F(m)$ is truncated from above, reducing the expected cost of a future repetition of the job. Once the offer is accepted, any new information on the range of minimum budgets can be used by the next administration to determine the best offer or offers for the next budgetary period. If the bureau head accepted a budget the previous period, which lowered the known maximum cost of the job, it may be optimal for the principal to make an even lower budget offer this period. The condition under which this is the case are stated in Proposition 9. (See appendix B for the proof.) One reason why administrative heads everywhere rush to spend their full budget before the new budgetary process begins is that any sign that the funds are not needed is an invitation to the principal to cut the next period's allocation.

Proposition 9. *Suppose an optimum sequence of budget offers is followed for a one time job based in $m \in M = (\underline{b}, \bar{b}]$ and $b_k < \bar{b}$ is accepted, which indicates $m \in M' = (b_{k-1}, b_k]$. If the job is repeated by the next administration, an offer b*

where $b_{k-1} < b < b_k$ *is optimal provided*

$$r \Pr\{m > b \mid m \in M'\} < (b_k - b) \Pr\{m \leq b \mid m \in M'\}.$$

Otherwise it is optimal to repeat the same offer b_k.

Nevertheless as the known range of minimum costs narrows, the possible gain from attempting to reduce the budget of a repeating job diminishes The difference between the budget, b_k, accepted the previous period and any budget b less than b_k becomes closer to zero. At some stage (depending on the offer cost r) the optimum strategy for a repeating job is simply the offer of the budget of the previous year (adjusted for inflation). The expected slack then remains constant for future repetitions of the job.

9. Conclusion

Since programs, such as road construction and police protection, occur in many jurisdictions and have usually been done before in the same jurisdiction, the relevant principal has a rather more precise estimate of costs for these programs than for a new project such as a large irrigation scheme. Although the latter type of project may have been done elsewhere, a particular cost problem, such as prevention of environmental damage, may be unique to the location. Since less precise information is generally available concerning the minimum necessary cost, the model predicts a higher expected excess cost in such a new program [Proposition 8(ii)]. Also if the offer cost is constant over projects, the expected number of offers is greater for the large irrigation scheme than for routine road construction [Proposition 8(i)].

On the other hand the offer cost may be greater in the new scheme, which by itself tends to increase the expected excess cost [Proposition 7(iii)] and the expected slack [Proposition 7(ii)]. The specialized knowledge required by a non-routine project may mean that replacing the manager is likely to impose high costs in terms of less productivity during the new manager's training period. Those projects which combine very imprecise information on budgetary requirements with high cost of replacement of the management are those that the model predicts are associated with the greatest expected slack Such a combination is most likely to be found in the management of a program involving sophisticated technology within an area such as defense or space research.

In the initial years of a new program, the model predicts that the budget will often fall in real terms. Only if a budget offer is rejected will future offers rise, but not above a previously accepted budget. The existing empirical studies such as Gist (1977) do not directly test this, since budget cuts are not corrected for reductions in the level of service, which often occur at the same time. Nevertheless the fact that disproportionate increments in uncontrollable expenditure in the US Federal Budget have been partly offset by budget reductions in the controllable items [Gist (1977)] provides some support for the idea that there is at least some effort to cut bureaucratic slack. The controllable portion of the base, actually declined by over 16 billion, from 114.0 billion in 1968 to 97.7 billion in 1974. Uncontrollable expenditure consists of costs which the executive and legislative body cannot change without statutory revision and is mostly composed of transfer payments which are excluded from the analysis of this paper.

The main impact of Gist's (1974, 1977) work is to show the weakness of the empirical support for the incrementalist theory of budgeting provided in the well known paper by Davis, Dempster and Wildavsky (1966).[12] Although the simple incremental decision rules are not supported by the evidence, this does not contradict the importance of last year's budget as a source of information and base from which to determine this year's budget. A major implication of this paper is that setting the current budget on the basis of last year's budget is a *rational* process in a situation characterized by asymmetric information. Even if the body in charge of budget setting is capable of comprehensive attention to the costs of all programs, and there is some doubt that this is the case [see, for example, Simon (1978) and Winter (1981)], it is optimal for it to concentrate attention on budget setting for new or recently introduced programs. Information is best gained by attempting to cut the budget of a program in its initial years of operation. After the program has been in place for some time the expected cost of gaining further information outweighs the expected benefits, so that the previous year's budget becomes the best offer for this year. The program then becomes part of the base so that only a change in the program will produce a change in the budget. Incrementalism in this sense is an efficient budgetary decision process under asymmetric information. As Wildavsky (1974, p. 216) writes: 'Budgeting is incremental, not comprehensive. The beginning of wisdom about an agency budget is that it is almost never actively reviewed as a whole every year, in the sense of reconsidering the value of all existing programs as compared to all possible alternatives. Instead it is based on last year's budget with special attention given to a narrow range of increases or decreases.'

[12] Davis, Dempster and Wildavsky (1966), with US data at the Federal level for the years 1947–1967, fit a simple linear scheme relating each year's congressional appropriation to the bureau of the budget's request and relating the bureau of the budget's request to last year's budget. Gist (1974) shows that the Davis, Dempster and Wildavsky results are highly sensitive to their use of aggregated data which included both controllable and uncontrollable expenditures and to their failure to correct for a time trend. For example, Gist (1974) finds that from 30 to 50 percent of the defense budget — that portion going to procurement and research and development — is not determined by the decision rules implied by the David, Dempster and Wildavsky scheme.

Appendix A

Proposition 3. Consider sequences of budget offers which terminate with probability one on or before the nth trial. Let this set be denoted B_n. The set of all infinite budget sequences is denoted B. Then for any budget sequence $\{b_i\} \in B$, there exists $n(M,r,F)$ such that

$$\max_{\{b_i'\} \in B_n} v(\{b_i'\}; M, r) > v_\infty(\{b_i\}; M, r),$$

where $n = n(M,r,F)$.

Proof. The proof of this proposition is separated in two lemmas.

Lemma 1. There exists $\{b_i'\} \in B_{n'}$, such that $v(\{b_i'\}; M, r) > v_\infty(\{b_i\}; M, r)$ for any budget sequence $\{b_i\} \in B$ and for some n' depending on $\{b_i\}$.

Proof. Suppose there exists $\{b_i\} \in B$, such that $v_\infty(\{b_i\}; M, r) \geqq v(\{b_i'\}; M, r)$ for all $\{b_i'\} \in B_{n'}$ and all n'.

Since $b_i \leqq \bar{b}$ and $b_{i+1} > b_i$ for all i (Proposition 1), there exists $b_u \leqq \bar{b}$, such that $\lim_{n\to\infty} b_n = b_u$.

Since in addition F is continuous on $[\underline{b}, \bar{b}]$, for any $\varepsilon > 0$, $\delta > 0$, there exists an N such that for all $n \geqq N$,

$$|b_u - b_n| < \delta \quad \text{and} \quad |F(b_u) - F(b_n)| < \varepsilon. \tag{A.1}$$

Consider the net advantage of $\{b_i\} \in B$ after N offers relative to $\{b_i'\} \in B_{N+1}$ where $b_i' = b_i$ for $i \in \{1, 2, \ldots, N\}$ and $b_{N+1} = b_u$.

The probability that any of the offers $\{b_{N+1}, b_{N+2}, \ldots\}$ of $\{b_i\}$ are accepted is strictly less than $F(b_u) - F(b_N)$, which is the probability $b_{N+1} = b_u$ is accepted in the sequence $\{b_i'\}$. Hence ignoring the cost of making each offer the expected relative gain from $\{b_i\} \in B$ over $\{b_i'\} \in B_{N+1}$ is strictly less than $(b_u - b_{N+1})[F(b_u) - F(b_N)]$.

Since more than one offer is made in the sequence $\{b_{N+1}, b_{N+2}, \ldots\}$ with probability greater than $1 - F(b_{N+1})$, the expected additional offer cost of $\{b_i\} \in B$ over $\{b_i'\} \in B_{N+1}$ exceeds $r[1 - F(b_{N+1})]$.

Hence, since by assumption $\{b_i\} \in B$ is superior to $\{b_i'\} \in B_{N+1}$,

$$(b_u - b_{N+1})[F(b_u) - F(b_N)] - r[1 - F(b_{N+1})] > 0. \tag{A.2}$$

Since $1 - F(b_u) \geqq 0$, (A.1) and (A.2) imply

$$r[F(b_u) - F(b_{N+1})]/[b_u - b_{N+1}] < F(b_u) - F(b_N) < \varepsilon. \tag{A.3}$$

Also

$$\lim_{N\to\infty} r[F(b_n)-F(b_{N+1})]/[b_n-b_{N+1}]=rF'(b_n)\geqq r\underline{\rho}>0,$$

where the density of F on M is bounded away from zero by $\underline{\rho}$.

Hence for $\varepsilon<r\rho$, $r[F(b_n)-F(b_{N+1})]/[b_n-b_{N+1}]>\varepsilon$, which is a contradiction of (A.3). There exists $\{b'_i\}\in B_{N+1}$, such that $v_{N+1}(\{b'_i\};M,r)>v_\infty(\{b_i\};M,r)$ for some N and any $\{b_i\}\in B$. Q.E.D.

Lemma 2. Suppose $\{b'_i\}$ with n' elements is a best sequence in B_n with $m\in M$ and offer cost r. Then there exists $n(M,r,F)$ such that $n'\leqq n(M,r,F)$.

Proof. Since $\{b'_i\}$ is optimal in B_n, the sequence defined by eliminating b'_j from $\{b'_i\}$, where $j=1,2,\ldots,n'$ has a lower expected net return to the principal. The expected loss for higher budgets is greater than the expected gain from a reduction in the search cost arising from the elimination of b'_j. I.e.,

$$(b'_{i+1}-b'_i)\Pr\{b'_{i-1}<m<b'_i\,|\,m\in M\}\geqq r\Pr\{m>b'_i\,|\,m\in M\},\qquad i=1,2,\ldots,n'-1. \tag{A.4}$$

By assumption the density of F, conditional on M, is continuous and bounded away from zero by $\rho>0$ for all $m\in M$. Let the upper bound for the density of F conditional on $\bar{M}$ be $\bar{\rho}$. Note that $F(\bar{b})=1$, $F(\underline{b})=0$ and that $\underline{\rho}$ and $\bar{\rho}$ depend on F.

Then for any $\{b_i\}\in B_n$ with $m\in M$,

$$\bar{\rho}(b_i-b_{i-1})\geqq F(b_i)-F(b_{i-1})\quad\text{and}\quad 1-F(b_i)\geqq\underline{\rho}(\bar{b}-b_i),\qquad i=1,\ldots,n'.$$

Since in particular this holds for $\{b'_i\}$, and from (A.4),

$$(b'_{i+1}-b'_i)\bar{\rho}(b'_i-b'_{i-1})\geqq r\underline{\rho}(\bar{b}-b'_i),\qquad i=1,2,\ldots,n'-1.$$

Therefore

$$\bar{\rho}(\bar{b}-\underline{b})>\bar{\rho}\sum_{i=1}^{n'-1}(b'_i-b'_{i-1})\geqq r\underline{\rho}\sum_{i=1}^{n'-1}(\bar{b}-b'_i)/(b'_{i+1}-b'_i)\geqq r\underline{\rho}(n'-1).$$

Take

$$n(M,r,F)=[(\bar{b}-\underline{b})\bar{\rho}/r\underline{\rho}]+1,$$

then

$$n'<n(M,r,F).$$ Q.E.D.

Proposition 6. *Consider any sequence of renegotiable offers $\{b_i^*\}_1^n$ where $n>1$, which satisfies Assumptions 1–5 and 10 and is not inferior in the sense that the principal's expected net return $z_n(\{b_i^*\};M,r)$ strictly exceeds the expected net return if one offer is omitted from the sequence. If the managers in Ω_j are risk neutral ($u_{bb}=0$) and if $u_{b\theta}\geqq 0$, then there exists a sequence $\{b_i\}_1^n$ of non-negotiable offers which satisfies Assumptions 1–7 and for which $v(\{b_i\};M,r)>z_n(\{b_i^*\};M,r)$.*

Proof. For any $\pi_i\in(0,1)$ and $i<n$, let $\theta_i\in(\underline{\theta},\bar{\theta})$ (if it exists) satisfy

$$u(b_i^*,\theta_i)=\pi_i u^*(\theta_i)+(1-\pi_i)u_0, \tag{A.5}$$

so that a prospective agency head, knowing $m(\theta_i)$, is indifferent between b_i^* with certainty and the uncertain prospect of $u^*(\theta_i)$ if the principal renegotiates and u_0 otherwise. $u^*(\theta_i)\geqq u(b_{i+1}^*,\theta_i)$, since the manager then has the option of choosing b_{i+1}^* with certainty. Assuming that $u_{\theta b}\geqq 0$, so that a decrease in θ does not increase u more at high budgets than at low budgets, then for all $\theta<\theta_i$ (if θ_i exists), $u(b_i^*,\theta)>\pi_i u^*(\theta)+(1-\pi_i)u_0$. Therefore given that the manager takes the sure prospect when he is indifferent, the manager accepts b_i^* for all $m(\theta)\leqq b_i$, where $b_i\equiv m(\theta_i)$, and rejects b_i^* otherwise.

The fact that, for any non-inferior sequence $\{b_i^*\}_1^n,\theta_i$ and $b_i\equiv m(\theta_i)$ exist for all $i<n$ and that $b_{i-1}<b_i$ for all $i\leqq n$, where $b_n\equiv b_n^*$ is proved by contradiction. Suppose $b_j=m(\theta_j)$ does not exist or that $\{b_i\}_1^n$ exists but $b_j<b_{j-1}$ for some $j\leqq n$. In either case whenever b_{j-1}^* is rejected, b_j^* is made and rejected with probability one. If b_j^* were eliminated from the sequence, the principal would gain by the expected cost $r(1-\pi_j)$ of bringing in a new candidate when b_j^* is rejected. This implies $\{b_i^*\}_1^n$ is inferior, which is a contradiction. Therefore there is a strictly positive probability that b_i^* is made and accepted (i.e., that $m\in(b_{i-1},b_i]$) for all $i\leqq n$. Also $u^*(\theta_i)=u(b_{i+1}^*,\theta_i)$, since $m(\theta_i)\equiv b_i<b_{i+1}$, so that b_{i+1}^* would be accepted (if offered) when $\theta=\theta_i$. From (A.5), this implies θ_i satisfies

$$\pi_i=(u(b_i^*,\theta_i)-u_0)/(u(b_{i+1}^*,\theta_i)-u_0). \tag{A.6}$$

To find the condition under which $\{b_i^*\}_1^n$ is not inferior to $\{b_i^*\}_2^n$, consider the expected net return $z_{n-1}(\{b_i^*\};M,r)$ from $\{b_i^*\}_2^n$ relative to $z_n(\{b_i^*\};M,r)$ from $\{b_i^*\}_1^n$. Since b_2^* is made with probability $F(b_2)$ in the sequence $\{b_i^*\}_2^n$ and with probability $F(b_2)-F(b_1)$ in the sequence $\{b_i^*\}_1^n$ and adding in the expected net return from the offer b_1^*,

$$\begin{aligned}z_n(\{b_i^*\};M,r)&=z_{n-1}(\{b_i^*\};M,r)-(v-b_2^*)F(b_2)+(v-b_2^*)(F(b_2)-F(b_1))\\&\quad+(v-b_1^*)F(b_1)-(1-F(b_1))(1-\pi_1)r_1\\&=z_{n-1}(\{b_i^*\};M,r)+(b_2^*-b_1^*)F(b_1)-(1-F(b_1))(1-\pi_1)r.\end{aligned}$$

If the manager is risk neutral ($u_{bb}=0$), from (A.6), $\pi_i=(b_i^*-b_i)/(b_{i+1}^*-b_i)$, which implies $b_2^*-b_1^*=(1-\pi_1)(b_2^*-b_1)$, so that

$$z_n-z_{n-1}=(1-\pi_1)[(b_2^*-b_1)F(b_1)-r(1-F(b_1))].$$

Therefore $\{b_i^*\}_1^n$ is not inferior only if

$$(b_2^*-b_1)F(b_1)-r(1-F(b_1))>0. \tag{A.7}$$

The expression for $z_n(\{b_i^*\},M,r)$ is shortened when convenient and $z_{n-j}(M_j)$ denotes the principal's expected net return from the sequence $\{b_i^*\}_{j+1}^n$ when the distribution F is conditional on $m\in M_j=(b_j,\bar{b}]$.

$$\begin{aligned}z_n(M)&=(v-b_1^*)F(b_1)-r+(1-F(b_1))[\pi_1(z_{n-1}(M_1)+r)+(1-\pi_1)z_{n-1}(M_1)]\\&=(v-b_1^*)F(b_1)-r+(1-F(b_1))(z_{n-1}(M_1)+\pi_1 r).\end{aligned} \tag{A.8}$$

Consider the non-negotiable sequence $\{b_i\}_1^n$, where $b_i\equiv m(\theta_i)$ for $i<n$ and $b_n\equiv b_n^*$. The expected net return $v_{n-j}(M_j)$ is defined analogously to $z_{n-j}(M_j)$ and $v_n(M)\equiv v(\{b_i\};M,r)$. Using a similar expansion as in (A.8),

$$v_n(M)=(v-b_1)F(b_1)-r+(1-F(b_1))v_{n-1}(M_1). \tag{A.9}$$

The proposition is now proved by induction. Since $b_n\equiv b_n^*$ and b_n^* is not renegotiable, the expected net return from the last offer $z_1(M_{n-1})=v_1(M_{n-1})$, where $m\in M_{n-1}$ anf $j=n-1$. When only two offers remain and $j=n-2$, from (A.8) and (A.9) and from $z_1(M_{n-1})=v_1(M_{n-1})$,

$$\begin{aligned}z_2(M_{n-2})&=(v-b_{n-1}^*)F(b_1)-r+(1-F(b_{n-1}))(v_1(M_{n-1})+\pi_{n-1}r)\\&=v_2(M_{n-2})-(b_{n-1}^*-b_{n-1})F(b_{n-1})+\pi_{n-1}r(1-F(b_{n-1})).\end{aligned}$$

If the manager is risk neutral, from (A.6), $\pi_{n-1}=(b_{n-1}^*-b_{n-1})/(b_n^*-b_{n-1})$ and

$$z_2(M_{n-2})=v_2(M_{n-2})-\pi_{n-1}[(b_n^*-b_{n-1})F(b_{n-1})-r(1-F(b_{n-1}))].$$

Assuming b_{n-1}^*, b_n^* is not inferior to the single offer b_n^*, from (A.7), $z_2(M_{n-2})<v_2(M_{n-2})$. Now consider the sequences $\{b_i^*\}_2^n$ and $\{b_i\}_2^n$ when $n-1$ offers remain, so that $j=1$. Suppose $z_{n-1}(M_1)<v_{n-1}(M_1)$. From (A.8) and (A.9), $z_n(M)<v_n(M)-(b_1^*-b_1)F(b_1)+\pi_1 r(1-F(b_1))$. Hence by the same argument as when $j=n-2$, if the manager is risk neutral and $\{b_i^*\}_1^n$ is not inferior, $z_n(M)<v_n(M)$. Q.E.D.

Appendix B

Since in this section it is assumed $v \geq \bar{b} + r$ so that the job is completed with certainty, the expected cost $w(\{b_i\}, M, r)$ of the sequence $\{b_i\}_1^n$, where $b_n = \bar{b}$, is

$$w(\{b_i\}, M, r) = \sum_{i=1}^{n} \{b_i[F(b_i) - F(b_{i-1})] + r[1 - F(b_{i-1})]\}. \tag{B.1}$$

Also for the purpose of the proof of Proposition 7, let $\{b_i'\}_1^{n'}$ and $\{b_i\}_1^n$ represent best budget sequences when $m \in M$ and the negotiation costs are r' and r respectively.

Proposition 7. Let n be the number of offers in the optimal sequence with offer cost r satisfying Assumptions 1–9. If $n > 1$, $r' > r$ and $v \geq \bar{b} + r'$ then

(i) $q(M, r) > q(M, r')$,

(ii) $s(M, r) < s(M, r')$, *and*

(iii) $s(M, r) + rq(M, r) < s(M, r') + r'q(M, r')$.

Proof. (i) If r is replaced by $r' > r$ in the first order condition (4) of the text, $\{b_i\}_1^{n-1}$, where $n > 1$ no longer satisfies (4). Similarly, $\{b_i'\}_1^n$, where $b_n' = \bar{b}$ and $n > 1$ is not an optimal budget sequence with offer cost r. If $n = 1$, since $v \geq \bar{b} + r'$, $b_1' = b_1 = \bar{b}$ and the optimum sequence is unaffected. Therefore if $n > 1$, $w(\{b_i'\}; M, r) - w(M, r) > w(M, r') - w(\{b_i\}; M, r')$. With rearrangement, $w(\{b_i\}; M, r') - w(M, r) > w(M, r') - w(\{b_i'\}; M, r)$. By expansion of each expected cost into its components $(r' - r)q(M, r) > (r' - r)q(M, r')$. Since $r' > r$, if $n > 1$, $q(M, r) > q(M, r')$.

(ii) By the definition of $w(M, r)$, $w(\{b_i'\}; M, r) \geq w(M, r)$. From expansion of the expected costs into their component costs,

$$s(M, r') + rq(M, r') + \mathrm{E}(m) \geq s(M, r) + rq(M, r) + \mathrm{E}(m).$$

Since by Proposition 7(i), $q(M, r') < q(M, r)$, then $s(M, r') > s(M, r)$.

(iii) Since a lower offer cost reduces costs for the same sequence of budget offers,

$$w(\{b_i'\}; M, r) < w(M, r').$$

But by definition $w(M, r) \leq w(\{b_i'\}; M, r)$. Therefore $w(M, r) < w(M, r')$ and $s(M, r) + rq(M, r) < s(M, r') + r'q(M, r')$. Q.E.D.

Lemma 1. If $v \geq \bar{b} + r$ and $m' = m + \alpha$, where $m \in M$ and $m' \in M'$, then $s(M', r) = s(M, r)$ and $q(M', r) = q(M, r)$.

Proof. Let G be the distribution of $m' \in M' = (\underline{b}+\alpha, \bar{b}+\alpha]$. Then $G(m') = F(m)$. Suppose $\{b_i^*\}_1^n$ is a best budget sequence based on $m \in M$ and r. For $m' \in M'$, let $b_i' = b_i^* + \alpha$ for all $i = 1, 2, \ldots, n$. Then using $G(m') = F(m)$ and the definition (B.1) of w,

$$w(M,r) + \alpha = w(\{b_i^*\}; M, r) + \alpha = w(\{b_i'\}; M', r). \tag{B.2}$$

Since $w(M,r)$ is the minimum expected cost of the job with $m \in M$ and r, from (B.2)

$$w(\{b_i'\}; M', r) \leqq w(\{b_i\}, M, r) + \alpha \quad \text{for all } \{b_i\} \text{ and } n,$$

$$\leqq w(\{b_i + \alpha\}; M', r) \quad \text{for all } \{b_i + \alpha\} \text{ and } n.$$

Therefore $b_i' = \{b_i^* + \alpha\}_1^n$ is an optimum sequence with $m \in M'$ and from (B.2), $w(M,r) + \alpha = w(M', r)$. It then follows from the definitions of $q(M', r)$ and $s(M', r)$ that $q(M', r) = q(M, r)$ and $s(M', r) = s(M, r)$. Q.E.D.

Lemma 2. If $v \geqq \gamma\bar{b} + \gamma r, m' = \gamma m$ and $r' = \gamma r$, where $\underline{b} = 0$, $m \in M$ and $m' \in M'$, then $s(M', \gamma r) = \gamma s(M, r)$, $q(M', \gamma r) = q(M, r)$ and $s(M', \gamma r) + \gamma r q(M', \gamma r) = \gamma[s(M, r) + rq(M, r)]$. Also $w(M', \gamma r) = \gamma w(M, r)$.

Proof. The proof is very similar in structure to the proof of Lemma 1 and is omitted.

Proposition 8. If $v \geqq \gamma\bar{b} + r, m' = \gamma m$, where $m \in M$, $m' \in M'$, $\underline{b} = 0$ and $\gamma > 1$, then

(i) $q(M', r) > q(M, r)$, *provided* $q(M', r) > 1$, *and*
(ii) $s(M', r) + rq(M', r) > s(M, r) + rq(M, r)$.

Proof. (i) If $q(M', r) > 1$, then $n' > 1$ and from Proposition 7(i), $q(M', r) > q(M', r')$, if $r' > r$. Since from Lemma 2, $q(M', r') = q(M, r)$, this implies $q(M', r) > q(M, r)$.

(ii) Suppose $s(M', r) + rq(M', r) \leqq s(M, r) + rq(M, r)$.
From Lemma 2,

$$\gamma[s(M, r) + rq(M, r)] = s(M', \gamma r) + \gamma r q(M', \gamma r).$$

Therefore

$$\gamma[s(M', r) + rq(M', r)] \leqq s(M', r') + r'q(M', r'), \tag{B.3}$$

where $r' = \gamma r$.

Let $\{b_i^*\}_{i=1}^{n}$ be an optimum budget sequence with $m' \in M'$ and offer cost r. Then, since with the same sequence of offers, $s(M',r) = s(M',r')$ and $q(M',r) = q(M',r')$,

$$\begin{aligned} w(\{b_i^*\}; M', r') &= s(M',r) + r'q(M',r) + \mathrm{E}(m') \\ &< s(M',r') + r'q(M',r') + \mathrm{E}(m') \quad \text{from (B.3)} \\ &< w(M',r'). \end{aligned}$$

This contradicts the definition of $w(M',r')$. Hence

$$s(M',r) + rq(M',r) > s(M,r) + rq(M,r). \qquad \text{Q.E.D.}$$

Proposition 9. Suppose an optimum sequence of budget offers is followed for a one time job based on information $m \in M = (b, \bar{b}]$ and $b_k < \bar{b}$ is accepted, indicating $m \in M' = (b_{k-1}, b_k]$. If the job is repeated by the next administration, an offer b where $b_{k-1} < b < b_k$ is optimal, provided

$$r \Pr\{m > b \mid m \in M'\} < (b_k - b) \Pr\{m \leq b \mid m \in M'\}.$$

Otherwise it is optimal to repeat the same offer b_k.

Proof. If with initial information $m \in M$, offer b_{k-1} is rejected, the principal then knows $m \in M_{k-1} = (b_{k-1}, \bar{b}]$. Since the next offer in the optimal sequence is b_k, an offer b, where $b_{k-1} < b < b_k$, is not optimal. The expected cost of the additional offer b must exceed the expected gain for all b where $b_{k-1} < b < b_k$.

$$r \Pr\{m > b \mid m \in M_{k-1}\} \geq (b_k - b) \Pr\{m \leq b \mid m \in M_{k-1}\}$$

If b_k is accepted, then the principal knows $m \in M'$. An offer $b < b_k$ is then optimal if

$$r \Pr\{m > b \mid m \in M'\} < (b_k - b) \Pr\{m \leq b \mid m \in M'\}.$$

This may hold since the probability the offer b is rejected, $\Pr\{m > b \mid m \in M'\}$, is less when it is known $b < b_k$ rather than $b < \bar{b}$. The probability the offer b is accepted is correspondingly higher. Q.E.D.

References

Alchian, Armen A. and Harold Demsetz, 1972, Production, information costs, and economic organization, American Economic Review 62, Dec., 777–795.

Breton, Albert and Ronald Wintrobe, 1975, The equilibrium size of a budget maximizing bureau: A note on Niskanen's theory of democracy, Journal of Political Economy 83, Feb., 195–207.

Chan, Kenneth Shun-Yuen, 1979, A behavioural model of bureaucracy, Southern Economic Journal 45, April, 1188–1194.

Davis, Otto A., M.A.H. Dempster and Aaron Wildavsky, 1966, A theory of the budgetary process, American Political Science Review 60, Sept., 529–547.

Demsetz, Harold, 1968, Why regulate utilities, Journal of Law and Economics 11, April, 55–65.

Downs, Anthony, 1967, Inside bureaucracy (Little, Brown and Company, Boston, MA).

Fama, Eugene F., 1980, Agency problems and the theory of the firm, Journal of Political Economy 88, April, 228–307.

Gist, John R., 1974, Mandatory expenditures and the defense sector: Theory of budgetary incrementalism, Sage publications no. 04020 (Sage, Pacific Palisades, CA).

Gist, John R., 1977, 'Increment' and 'base' in the congressional appropriations process, American Journal of Political Science 21, May, 341–352.

Harris, Milton and Artur Raviv, 1979, Optimal incentive contracts with imperfect information, Journal of Economic Theory 20, April, 231–259.

Harris, Milton and Robert M. Townsend, 1981, Resource allocation under asymmetric information, Econometrica 49, Jan., 33–64.

Holt, Jr., Charles A., 1979, Uncertainty and bidding for incentive contracts, American Economic Review 69, Sept., 687–705.

Leibenstein, Harvey, 1966, Allocative efficiency vs. 'X-efficiency', American Economic Review 56, 392–449.

Myerson, Roger B., 1979, Incentive compatibility and the bargaining problem, Econometrica 47, Jan., 61–74.

Niskanen, William A., 1971, Bureaucracy and representative government (Aldine–Atherton, Chicago, IL).

Niskanen, William A., 1975, Bureaucrats and politicians, Journal of Law and Economics 18, Dec., 617–743.

Orzechowski, William, 1977, Economic models of bureaucracy: Survey extensions and evidence, in: Thomas E. Borcherding, ed., Budgets and bureaucrats: The sources of government growth (Duke University Press, Durham, NC) 229–259.

Rothschild, Michael, 1974, Searching for the lowest price when the distribution of prices is unknown, Journal of Political Economy 82, July–Aug., 689–712.

Simon, Herbert A., 1978, Rationality as process and as product of thought, American Economic Review 68, May, 1–16.

Spann, Robert M., 1977, Public versus private provision of governmental services, in: Thomas E. Borcherding, ed., Budgets and bureaucrats: The sources of government growth (Duke University Press, Durham, NC) 71–89.

Spencer, Barbara, 1980, Outside information and the degree of monopoly power of a public bureau, Southern Economic Journal 47, July, 228–233.

Tullock, Gordon, 1974, Dynamic hypothesis in bureaucracy, Public Choice 19, Fall, 127–131.

Vickrey, William, 1962, Counterspeculation, auctions, and competitive sealed tenders, Journal of Finance 16, May, 8–37.

Wildavsky, Aaron, 1974, The politics of the budgetary process (Little, Brown and Company, Boston, MA).

Williamson, Oliver E., 1967, Hierarchical control and optimum firm size, Journal of Political Economy 15, April, 123–138.

Williamson, Oliver E., 1976, Franchise bidding for natural monopolies — In general and with respect to CATV, Bell Journal of Economics 7, Spring, 73–104.

Winter, Sidney G., 1981, Attention allocation and input proportions, Journal of Economic Behavior and Organization 2, 31–46.