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James A. Brander, Barbara J. Spencer

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Strategic commitment with R&D: the symmetric case

James A. Brander*

and

Barbara J. Spencer**

When research and development take place before the associated output is produced, imperfectly competitive firms may use R&D for strategic purposes rather than simply to minimize costs. Using a simple symmetric two-stage Nash duopoly model, we show that such strategic use of R&D will increase the total amount of R&D undertaken, increase total output, and lower industry profit. However, the strategic use of R&D introduces inefficiency in that total costs are not minimized for the output chosen. Nevertheless, net welfare may rise, and certainly rises if products are homogeneous, marginal cost is non-decreasing, and demand is convex or linear.

1. Introduction

■ The relationship between the pattern of research and development (R&D) in an industry and the mature configuration of the industry is a matter of some subtlety. Even with cost-reducing R&D it seems likely that firms perceive strategic considerations beyond the simple desire to minimize total costs (including expenditures on R&D) for any level of output. Consider, for example, a market in which firms' market shares depend on their own and their rivals' marginal costs. (Presumably a lower own marginal cost leads to a higher market share, while lowered marginal costs for rivals would have the opposite effect.) If firms recognize this dependence of market share on marginal cost, and R&D expenditures occur before the associated output is produced, firms might be tempted to shift additional resources to the overhead or "sunk" category so as to reduce marginal costs and gain a strategic advantage in the imperfectly competitive output game. Total costs would not be minimized for the output produced.

This strategic (marginal) cost reduction is similar in concept to the recent "commitment" or "credible threat" models in oligopoly theory (Spence, 1977, 1979; Friedman, 1979; Dixit, 1980; Eaton and Lipsey, 1980, 1981) which emphasize the role of irreversible investments in establishing market power. These articles are concerned with entry so that established firms naturally have the advantage of being able to act first.¹ Our approach

* Queen's University.

** Boston College.

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¹ In the articles by Dixit (1980), Eaton and Lipsey (1980, 1981), and Friedman (1979), there is only one established firm. In Spence (1979), firms enter a new market in sequence. Flaherty (1980) examines a dynamic oligopoly model with cost-reducing R&D in which firms have equal opportunity.

differs in that we are concerned with industries in which related products are developed more or less simultaneously so that both firms are in a position to consider the strategic impact of their R&D decisions. This symmetric opportunity to use strategic R&D does seem appropriate in some industries such as automobiles in which new models are continually being developed. Under these conditions no firm has a first mover advantage.

We use what is perhaps the simplest model capable of capturing the essential characteristics. Despite the simplicity of the model, some suggestive results arise. Firms have incentives to undertake “too much” R&D in that total costs are not minimized. Also if one firm alone uses R&D strategically, it can increase both its output and profit at the expense of other firms. However, if all firms attempt to influence the final outcome, there is a tendency for output to be higher and profits and prices lower than in a corresponding industry without strategic R&D. We show the conditions under which strategic R&D leads to higher net welfare.

It should perhaps be emphasized that there is nothing in our model that formally distinguishes between cost-reducing R&D and investment in capital stock. We have chosen to focus on the R&D interpretation because we believe that the kind of process described in this article might be an important aspect of real R&D activity, and therefore, that the R&D interpretation deserves emphasis.²

Section 2 sets out the basic model. Section 3 is concerned with a comparison of the model with the corresponding model without strategic cost reduction, and Section 4 contains concluding remarks.

2. The model

■ The model with strategic R&D involves a two-step equilibrium. Firms choose R&D levels, these R&D levels are made known to each other, then output levels are determined. In deciding upon the amount of R&D to undertake, firms must have some expectation concerning how the output rivalry with other firms will be resolved. We assume, following Dixit (1980), that, for any given R&D levels, firms correctly “see through” to the (second-step) output equilibrium, which is resolved as a Nash quantity game. The “first-step” equilibrium is assumed to arise from a Nash game in R&D levels. This is an example of what is referred to as a “subgame perfect equilibrium.” It has the desirable property that, in equilibrium, expectations are confirmed.

The corresponding nonstrategic model is one in which R&D is used only to minimize costs for the output produced. Formally, the equilibrium here is a one-step Nash equilibrium with R&D levels and output set on the assumption that other firms are holding output fixed, which is a standard Cournot equilibrium with cost-minimization considered explicitly as in Dasgupta and Stiglitz (1980a). The nonstrategic model arises naturally if R&D and output are simultaneously determined.

We examine the duopoly case. Generalization to more firms and to conjectural variations other than zero is not a trivial exercise, but similar principles apply. The two firms are denoted 1 and 2. (We often use i and j to refer to the firms, and it is understood that if i denotes 1 in an expression, then j represents 2 and *vice versa*). Each firm i has output y^i , revenue R^i and cost C^i . Expenditure on R&D is denoted x_i , and this initial expenditure is converted to a flow by an implicit “rental” rate v^i . Thus the profit flow of firm i may be written

$$\pi^i(y^1, y^2; x_i) = R^i(y^1, y^2) - C^i(y^i; x_i) - v^i x_i. \quad (1)$$

The outputs y^1 and y^2 are not necessarily identical goods, but they are substitutes

² A relatively recent survey of the economics of R&D is Kamien and Schwartz (1975).

in the sense that increasing the output of good j decreases the total and marginal revenue of firm i . Using subscripts to denote derivatives,³ we then have

$$R_j^i < 0; \quad R_{ij}^i < 0. \quad (2)$$

C^i includes all costs except R&D and is regarded as a variable cost. The effect of having undertaken more cost-reducing R&D is, of course, to reduce C^i , given y^i . However, the rate of decrease is assumed to decline as x_i increases. Furthermore, marginal cost, $\partial C^i/\partial y^i$, is denoted c^i and is strictly positive and decreasing in x_i . Summarizing these conditions:

$$C_x^i < 0; \quad C_{xx}^i > 0; \quad c^i > 0; \quad c_x^i < 0. \quad (3)$$

Looking first at the strategic model, with R&D levels x_1 and x_2 already sunk, the output equilibrium occurs where each firm is maximizing its profit with respect to own output, given the output of its rival. This equilibrium is characterized by first-order conditions,

$$\pi_i^i \equiv \partial \pi^i / \partial y^i = R_i^i(y^1, y^2) - c^i(y^i; x_i) = 0, \quad (4)$$

and second-order conditions,

$$\pi_{ii}^i = R_{ii}^i - c_y^i < 0. \quad (5)$$

We also use the following condition:

$$A \equiv \pi_{11}^1 \pi_{22}^2 - \pi_{12}^1 \pi_{21}^2 = (R_{11}^1 - c_y^1)(R_{22}^2 - c_y^2) - R_{12}^1 R_{21}^2 > 0. \quad (6)$$

Condition (6) means that own effects of output on marginal profit exceed cross effects. It holds if own revenue effects exceed cross revenue effects, provided marginal cost is not too strongly decreasing, and it is closely related to uniqueness of the equilibrium and to reaction function stability.⁴

Condition (4) is the output reaction function for firm i in implicit form. The slope of the reaction function is found by total differentiation of (4): $\pi_{ii}^i dy^i + \pi_{ij}^i dy^j = 0$, from which, after noting that $\pi_{ij}^i = R_{ij}^i$, we have

$$dy^i/dy^j = -R_{ij}^i/\pi_{ii}^i < 0. \quad (7)$$

The solution to first-order conditions (4) depends on x_1 and x_2 . Therefore, y^1 and y^2 can be written as functions of x_1 and x_2 :

$$y^1 = q^1(x_1, x_2); \quad y^2 = q^2(x_1, x_2). \quad (8)$$

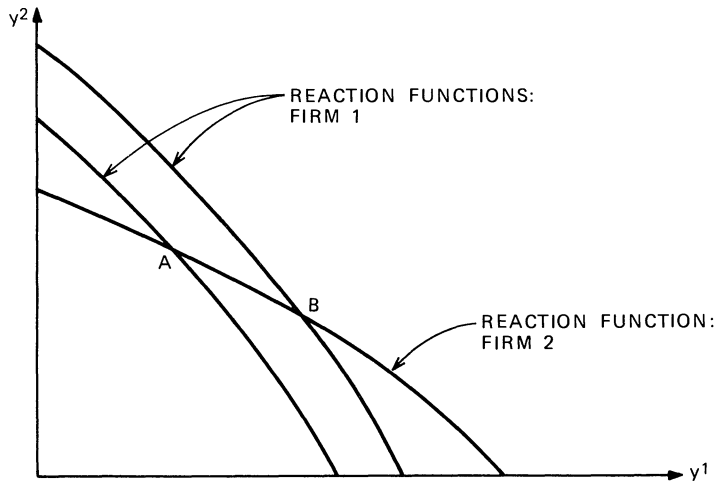
In effect, output (and market share) depend on marginal cost, which, in turn, depends on the amount of cost-reducing R&D that has been undertaken. An increase in R&D by firm 1 will lower its marginal cost c^1 , shift its reaction function outward, and increase its output and market share, provided the R&D level of firm 2 is held constant. This is illustrated in Figure 1. The equilibrium moves from A to B as the R&D level for firm 1 increases.

The analytics corresponding to Figure 1 come from totally differentiating (4) with

³ We assume as much differentiability as is convenient. In particular, the profit function is assumed to have continuous second partial derivatives. Also, the properties that are imposed on the profit function ((2), (5), (6), (13) and (14)) are assumed to hold over the entire region of interest.

⁴ Condition (6) means, given (5), that profit functions are strictly concave. It is clearly satisfied in the homogeneous product case if demand is linear or concave and marginal cost is nondecreasing and is the condition for the overall reaction mapping to be a contraction mapping and therefore implies uniqueness of equilibrium. (See Friedman (1977, ch. 7).) Alternatively, (6) allows direct application of the Gale-Nikaido global univalence result to prove uniqueness. Also (6) is the Routh-Hurwitz stability condition for the standard adjustment mechanism.

FIGURE 1
EFFECT OF AN R & D INCREASE ON OUTPUT EQUILIBRIUM



respect to y^1 , y^2 , and x_1 to yield the 2×2 simultaneous system:

$$\begin{aligned} \pi_{11}^1 dy^1 + \pi_{12}^1 dy^2 &= c_x^1 dx_1 \\ \pi_{21}^2 dy^1 + \pi_{22}^2 dy^2 &= 0. \end{aligned}$$

Then, using Cramer's rule, $q_1^1 \equiv \partial y^1 / \partial x_1 = c_x^1 \pi_{22}^2 / A$, where $A > 0$ is given by (6). Since π_{22}^2 is negative and c_x^1 is negative, q_1^1 is positive. Similarly, $q_1^2 = -c_x^1 \pi_{21}^2 / A < 0$. Symmetric results hold for the effect of changes in x_2 . Thus

$$q_i^i > 0; \quad q_i^j < 0. \tag{9}$$

The sum of output effects with respect to x_i is

$$q_i^i + q_i^j = c_x^i (\pi_{jj}^i - \pi_{ji}^i) / A. \tag{10}$$

At a symmetric equilibrium, condition (6) implies that $\pi_{jj}^i < \pi_{ji}^i$, so $q_i^i + q_i^j$ would certainly be positive.

Firms are aware of the dependence (via (8)) of output on R&D levels. Profit can be written directly as a function of x_1 and x_2 . Let g^i (for gain) represent this function:

$$g^i \equiv \pi^i(q^1(x_1, x_2), q^2(x_1, x_2); x_i). \tag{11}$$

The Nash equilibrium of the strategic R&D game occurs where each firm is maximizing its profit with respect to R&D, given the R&D level chosen by its rival. From (1), (4), and (11), the first-order condition for a profit maximum for firm i is

$$\begin{aligned} g^i &= \pi_i^i q_i^i + \pi_j^i q_j^i - C_x^i - v^i = 0 \\ &= R_j^i q_j^i - C_x^i - v^i = 0, \end{aligned} \tag{12}$$

since $\pi_i^i = 0$ and $\pi_j^i = R_j^i$. The second-order condition is

$$g_{ii}^i = R_j^i q_{ii}^i + q_j^i dR_j^i / dx_i - c_x^i q_i^i - C_{xx}^i < 0, \tag{13}$$

and we use the following condition for certain purposes:

$$|g_{ii}^i| > |g_{ij}^i|, \tag{14}$$

where

$$g_{ij}^i = R_j^i q_{ij}^i + q_j^i dR_j^i / dx_j - c_x^i q_j^i.$$

Condition (14) states that own effects of R&D on marginal profit dominate cross effects. However (14) implies (and, under perfect symmetry is equivalent to)

$$g_{11}^1 g_{22}^2 - g_{12}^1 g_{21}^2 > 0, \quad (15)$$

which is the Routh-Hurwitz condition for reaction function stability of the R&D game. If (13) and (14) hold globally, then uniqueness of the R&D equilibrium follows as a direct application of a Gale-Nikaido univalence theorem (Nikaido, 1968, ch. 7, p. 371).

Inspection of the expressions for g_{ii}^i and g_{ij}^i , in (13) and (14) respectively, shows that existence and uniqueness of equilibrium can be a problem in two stage models. At least one of the terms of g_{ii}^i is positive,⁵ thereby making it difficult to ensure that the second-order condition for an interior profit maximum holds. Nevertheless, (13) and (14) will hold if the marginal cost-reducing effect of R&D is strongly diminishing so that C_{xx}^i is relatively large and positive. We restrict attention to cases which satisfy (13) and (14).

Proposition 1. Strategic behavior induces each firm to use more R&D than required to minimize the cost of the output it produces.

Proof. The total cost of producing output flow y^i is $C^i(y^i; x_i) + v^i x_i$. This cost is minimized for a given y^i when $C_x^i + v = 0$ (with second-order condition $C_{xx}^i > 0$). If, in contrast, firms undertake strategic cost reduction, condition (12) implies $C_x^i + v^i = R^i q_i^i > 0$ (by (2) and (9)). This means, since $C_{xx}^i > 0$, that more R&D than required to minimize total cost is undertaken.⁶ *Q.E.D.*

This result can be regarded formally as an extension of Dixit (1980) to the case of firms with equal opportunity, provided x_i is interpreted as capital rather than R&D. Our concern now is to compare the strategic R&D equilibrium with the corresponding equilibrium when R&D does not have a strategic role.

3. R&D, output, profit, and welfare comparisons

■ The nonstrategic Nash equilibrium arises from maximization of profit function (1) with respect to y^i and x_i , taking y^j as given:

$$\partial \pi^i / \partial x_i = -C_x^i - v^i = 0 \quad (16)$$

$$\partial \pi^i / \partial y^i = R^i(y^1, y^2) - c^i(y^i; x_i) = 0. \quad (17)$$

Condition (16) is the condition for cost minimization, so these nonstrategic firms minimize costs. Condition (17) is identical in form to the first-order conditions (4) for strategic firms. Therefore, the Nash equilibrium quantities y^1 and y^2 are related to x_1 and x_2 in the same way as before: $y^i = q^i(x_1, x_2)$ as in equation (8).

In the strategic case firms know these relationships and take them into account in choosing R&D levels. In the nonstrategic case these relationships need not be known by firms, but just emerge as a consequence of simultaneous selection of y^i and x_i in a noncooperative setting.

The difficulty in comparing the strategic model with the nonstrategic (Cournot) model is that the comparison is not simple comparative statics, in which an exogenous variable changes endogenous values in a given model, but involves comparing the equilibria of two different models. Thus, a different methodology is indicated. The comparisons can

⁵ The term $-c_x^i q_i^i$ of g_{ii}^i is positive from (3) and (9). Also, letting $i = 1, j = 2$, the term $q_2^2 dR_1^1/dx_1 = q_2^2 (R_{21}^1 q_1^1 + R_{22}^1 q_2^1)$ will be positive (from (2) and (9)) if $R_{22}^1 \geq 0$.

⁶ If an increase in the output of firm 1 were to increase the marginal revenue of firm 2 ($R_{21}^2 > 0$), violating (2), we would get exactly the opposite result here. q_1^1 would be positive so $C_x^1 + v^1$ would be negative, implying an underuse of R&D.

be made using mean value theorem methods. A statement of the relevant version of the mean value theorem and proofs of the propositions are in the Appendix.

Proposition 2. (i) The strategic equilibrium involves more total R&D than the corresponding Cournot equilibrium. (ii) Under perfect symmetry each firm undertakes more R&D in the strategic model than in the Cournot model.

The intuition of Proposition 2 is that, at Cournot levels of R&D, the perceived marginal profit of extra R&D in the strategic regime is positive ($g_i^i(x_1^c, x_2^c) = R_i^i q_i^i > 0$), indicating that R&D should be increased from each firm's point of view. Geometrically, R&D reaction functions are shifted out in the strategic regime, which leads to Proposition 3.

Proposition 3. Under perfect symmetry, at the strategic equilibrium

- (i) each firm produces greater output
- (ii) prices are lower
- (iii) each firm earns less profit

than at the corresponding Cournot equilibrium.

By perfect symmetry we mean that firms face symmetric demands and costs and have identical levels of R&D, output, marginal cost, and interaction effects in equilibrium. A look at the proof (in the Appendix) indicates that small asymmetries will not undermine the results of Proposition 3, since total R&D still rises. The results may not hold with large asymmetries.

Firms clearly move further away from the joint profit maximum through this R&D rivalry. They are trapped by the incentive structure of the environment. If one firm ignores the possibility of strategic use of R&D while the other firm does undertake strategic R&D, the first firm loses, while the second gains relative to the pure Cournot rules. The situation is a classic prisoner's dilemma, and firms have an incentive to collude. Each firm then has the usual incentive to violate any agreements, especially in situations in which there is a "once and for all" aspect to the rivalry, as is often the case with R&D.

Strategic R&D might reasonably be interpreted as predatory and therefore in violation of antitrust laws. A natural question concerns whether strategic use of R&D is against the public interest. Strategic firms use excessive R&D and do not minimize costs, which tends to reduce welfare, but there is also a tendency for strategic behavior to increase output, which is socially desirable since price exceeds marginal cost.

To analyze the relative strength of these two partially offsetting effects, we assume a partial equilibrium framework in which utility can be approximated by $U = u(y^1, y^2) + z$, where z is expenditure on a competitively supplied *numeraire* good. Thus the marginal utility of income is constant and equal to one and welfare, as measured by surplus, reduces to

$$W(x_1, x_2) = u(y^1, y^2) - \sum_{k=1}^2 (C^k(y^k; x^k) + v^k x_k), \tag{18}$$

where $y^k = q^k(x_1, x_2)$. Using $p^i = \partial u / \partial y^i$ and assuming perfect symmetry so that superscripts can be dropped from the common values of p, c, C_x , and v , differentiation of (18) with respect to x_i yields

$$\partial W / \partial x_i = (p - c)(q_i^1 + q_i^2) - (C_x + v). \tag{19}$$

From (10), the sum of the output effects, $q_i^1 + q_i^2$ is positive. Therefore $\partial W / \partial x_i$ is positive at the (symmetric) nonstrategic equilibrium where, by (16), $C_x + v = 0$. A small increase in R&D by either firm then increases welfare.

We can also show that in some cases the move from the nonstrategic equilibrium to the strategic equilibrium is welfare improving as described in Proposition 4, which

follows. Proposition 4(i) involves the condition that $p_i^i + y^i p_{ii}^i \leq p_j^i + y^i p_{ij}^i \equiv R_{ij}^i$, where $R_{ij}^i < 0$ by (2) and $p_{ij}^i = \partial^2 p^i / \partial y^i \partial y^j$. This relationship will hold if the effect of own output on price, p_i^i , is sufficiently larger (in absolute value) and more negative than the cross price effect p_j^i . In addition, Proposition 4(i) assumes that $p_{ij}^i \geq 0$, i.e., that an increase in the output of firm j does not make the slope of the demand curve for the i th firm more negative. Both these conditions hold in the case of linear demand (arising from a quadratic utility function in y^1 and y^2) since then $p_{ij}^i = p_{ii}^i = 0$. If outputs y^1 and y^2 are homogeneous, these conditions simplify nicely to the requirement that demand be linear or convex ($p'' \equiv d^2 p / dY^2 \geq 0$, where $Y = y^1 + y^2$).

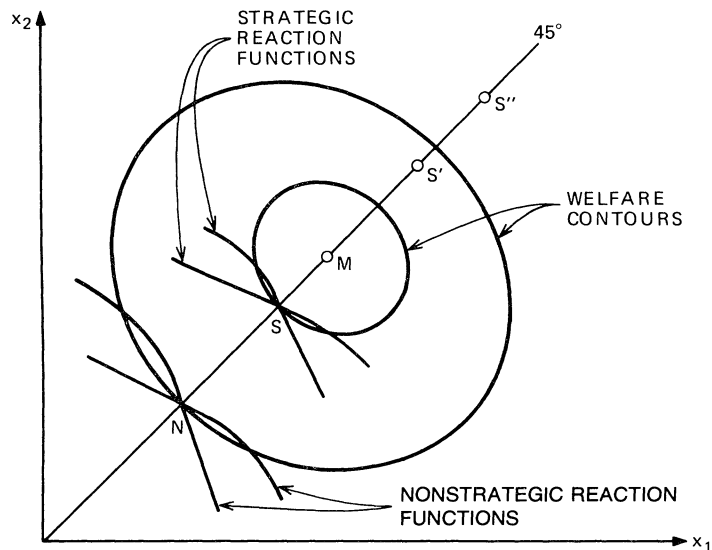
Proposition 4. (i) Assuming symmetry (and provided that $p_i^i \leq p_j^i < 0$), strategic R&D is welfare improving if $c_y \geq 0$, $p_{ij}^i \geq 0$, and $p_i^i + y^i p_{ii}^i \leq p_j^i + y^i p_{ij}^i$.

(ii) Assuming symmetry, if y^1 and y^2 are homogeneous, strategic R&D is welfare improving if marginal cost is nondecreasing and demand is (weakly) convex.

The intuition associated with Proposition 4(ii) is that, for a given increase in R&D and corresponding fall in marginal cost, the welfare increasing output effect is larger as demand is more convex.

In addition to comparing the strategic and nonstrategic equilibria, it is also of interest to consider the second-best optimum, which arises from maximizing welfare subject to the imperfectly competitive output rivalry described by $y^i = q^i(x_1, x_2)$. (The first-best optimum requires prices equal to marginal cost and total cost to be minimized for the output chosen). Figure 2 shows the second-best welfare contours in x_1, x_2 space. Given some conditions⁷ on demand and cost, the welfare contours are convex (as drawn), and welfare achieves a unique maximum at point M where $\partial W / \partial x_1 = \partial W / \partial x_2 = 0$. Strategic

FIGURE 2
WELFARE AND EQUILIBRIA IN R & D SPACE



⁷ The welfare contours are as drawn if welfare is quasi-concave in x_1 and x_2 . This quasi concavity follows from the natural quasi concavity of utility in consumption of y^1 and y^2 and is reinforced by the tendency of output to be concave in own R&D. Failure of quasi concavity cannot be ruled out, and might arise, for example, if marginal cost were strongly downward sloping ($c_y < 0$). Existence of a unique (second-best) welfare maximum depends on concavity of net surplus in R&D.

and nonstrategic equilibria lie at the intersections of the appropriate R&D reaction functions.

The reaction functions in R&D space for the strategic case are given, in implicit form, by (12). For the nonstrategic case, the R&D reaction functions are the translations, via cost minimization, of the output reaction functions. Formally the reaction function for firm 1 is derived using expression (16) for $i = 1$ and 2, and (17) for $i = 1$, and has slope $dx_1/dx_2 = -C_{xx}^2\pi_{12}c_x^1/C_{xx}^1\pi_{11}c_x^2 < 0$. Under symmetry, candidate equilibria must lie on the 45° line. As implied by (19), the nonstrategic equilibrium, N , lies below M . Also, strategic reaction functions are shifted out beyond the nonstrategic reaction functions. The strategic equilibrium may lie at points like S , S' , S'' , or at M itself, depending on the nature of demand and cost. Generally speaking, these reaction functions are further shifted out if marginal cost is decreasing ($c_{yy} < 0$) or if second-order cross effects of output on price are negative ($p_{ij}^i < 0$). The case of homogeneous products and constant marginal cost is particularly clear cut.

Proposition 5. (Homogeneous products and $c_y = 0$.) The strategic equilibrium has more R&D (and output) than the second-best optimum, M , if demand is concave (points S' and S''), less R&D if demand is convex (point S), and coincides with the second-best optimum if demand is linear.

It is worth emphasizing that the second-best optimum does not involve cost minimization, which is a striking second-best result: given the distortion of imperfectly competitive output rivalry, cost minimization is no longer optimal.

Proposition 4 corresponds to cases like S , in which the strategic equilibrium does not overshoot the optimum. Even with overshooting, it may still be welfare superior to the nonstrategic equilibrium, as illustrated by point S' . Thus, overall welfare improvement seems the more likely result of strategic cost reduction through R&D.

One might contrast this analysis with Dasgupta and Stiglitz (1980a) where cost-reducing R&D is examined in a (one-stage) Cournot model. In that article, free entry can induce socially excessive R&D (compared with the first-best solution) despite individual cost minimization by firms, provided demand is sufficiently inelastic.

5. Concluding remarks

■ In the strategic setting firms use more R&D, they do not minimize the cost of producing output, and there is a tendency for the total output of each firm to be larger. Firms earn lower profits, but net welfare, as measured by the sum of consumer surplus and profit, is likely to rise.

One important assumption in the strategic model is that R&D levels are observable. Even if R&D levels are private information, however, it is possible, at least in a repeated game, that they might be inferred. If each firm knows that its rival will be on the output reaction function corresponding to its actual R&D level, then its R&D level can be deduced from the output equilibrium,⁸ provided information about demand and technology is complete. If the game is played only once, R&D levels cannot be revised once the output phase is in progress, but in a repeated R&D game the two-step Nash equilibrium would be the steady-state equilibrium. Alternatively, however, it is conceivable that firms might, if they understand the incentive structure in which they operate, present false output reaction functions. If firms base their actions only on observables (in this case, output) and assume that rivals optimize with respect to unobservables (in this case choose the cost-minimizing level of R&D), then the Cournot equilibrium is appropriate.

⁸ From (4), y^1 is increasing in x^1 , holding y^2 fixed: a greater level of R&D is associated with a unique larger profit-maximizing value of y_1 for any value of y_2 . Each point in output space is consistent with only one R&D level for each firm.

The phenomenon described here as strategic cost reduction is essentially the same as the phenomenon described as “commitment” in Dixit (1980), Spence (1977, 1979), and Eaton and Lipsey (1980, 1981), although the results obtained are new and the R&D setting is different. Strategic cost reduction is actually only one form that commitment might take, in which firms “commit” themselves to certain output reaction functions. Firms might also commit themselves to certain product types or locations (as in Prescott and Visscher (1977)) so as to manipulate the industry equilibrium.

It is quite possible that in certain industries advertising and marketing could play the role attributed here to R&D. Consider an industry in which a significant part of the cost of producing and selling an item is the actual cost of sales effort, as for example, with insurance. The sales effort required per unit can be lowered if extensive advertising and marketing are undertaken. In this case, advertising and marketing costs become overhead costs and lower actual marginal cost. This conforms exactly to the structure described in this article. The implication would be that increases in advertising and marketing might be associated with price reductions rather than price increases, which contrasts sharply with the usual idea that advertising shifts out a firm’s demand curve, thereby increasing both quantity and price.

Schmalensee (1982) develops a model in which customers are informed by advertising, which shifts out the demand curve. He obtains the surprising result that strategic advertising for the purpose of entry deterrence by an incumbent monopolist can lead to less advertising than if entry were not possible. However, this seemingly paradoxical result can be explained within our framework. The essential point is that in Schmalensee (1982) advertising by the incumbent has an externality in that it increases the equilibrium output and profit of the entrant at the Cournot equilibrium. In particular, this means (in our notation) that $q_j^i(x_1, x_2)$ is positive. It follows from our footnote 6 that in this case a strategic firm would then use less advertising than a nonstrategic firm. This conforms with the Schmalensee result. Similar positive externalities might also arise in the R&D case if, for example, R&D were not fully appropriable and lowered the marginal cost of one’s rival.

The possibility for strategic cost reducing advertising and marketing is no doubt a relatively minor aspect of total advertising and marketing behavior. With cost reducing R&D, on the other hand, one suspects that strategic considerations are very important. The actual strategic interactions are undoubtedly complex and subtle. We have used a simple model to illustrate one form such interaction might take.⁹

Appendix

Mean value theorem

■ Let $f(x)$ be a continuously differentiable real-valued function defined on a convex subset of Euclidean n -space. Let X^c and X^s be two vectors in this subset. Then there exists a point X^* such that

$$\Delta f \equiv f(X^s) - f(X^c) = \nabla f(X^*) \cdot (X^s - X^c), \quad (A1)$$

where $\nabla f(X^*)$ is the gradient of f evaluated at X^* , and $X^* = X^c + \theta(X^s - X^c)$ for some $\theta \in (0, 1)$. (X^* is said to be “between” X^c and X^s .) Expression (A1) is sometimes referred to as an exact linear Taylor’s series (Rosenlicht, 1968, p. 205).

□ **Notation.** $X^c = (x_1^c, x_2^c)$; $X^s = (x_1^s, x_2^s)$. “ Δ ” refers to any difference between the strategic and Cournot regimes. For example, $\Delta x_1 = x_1^s - x_1^c$, $\Delta g_i^j = g_i^j(X^s) - g_i^j(X^c)$, etc.

⁹ There are, of course, other aspects of cost-reducing R&D activity. One important idea is that R&D expenditure determines (possibly with some uncertainty) the time when the new process is discovered. Firms then compete over timing, as in Dasgupta and Stiglitz (1980b).

Proof of Proposition 2. Applying (A1) to $g_i^i(x_1, x_2)$ yields

$$\Delta g_i^i = g_{i1}^i \Delta x_1 + g_{i2}^i \Delta x_2, \quad (\text{A2})$$

where g_{i1}^i and g_{i2}^i are evaluated at some point between X^c and X^s . Letting $i = 1$, then $i = 2$ in (A2) yields a two-equation simultaneous system in unknowns x_1 and x_2 which can be solved, using Cramer's rule, to yield

$$\Delta x_1 = (\Delta g_1^1 g_{22}^2 - \Delta g_2^2 g_{12}^1) / D \quad (\text{A3})$$

$$\Delta x_2 = (\Delta g_2^2 g_{11}^1 - \Delta g_1^1 g_{21}^2) / D, \quad (\text{A4})$$

where $D = g_{11}^1 g_{22}^2 - g_{12}^1 g_{21}^2 > 0$ by (15). Adding (A3) and (A4) yields

$$\Delta x_1 + \Delta x_2 = ((g_{22}^2 - g_{21}^2) \Delta g_1^1 + (g_{11}^1 - g_{12}^1) \Delta g_2^2) / D. \quad (\text{A5})$$

By (13) and (14) $g_{ii}^i - g_{ij}^i < 0$. The remaining problem is to sign Δg_i^i : $g_i^i(X^s) = 0$ by (12) and $g_i^i(X^c) = R_i^i q_i^i > 0$ by (2), (9), (12), and (16). Therefore, $\Delta g_i^i < 0$. It follows that $\Delta x_1 + \Delta x_2 > 0$. (ii) follows immediately, since $\Delta x_1 = \Delta x_2$. *Q.E.D.*

Proof of Proposition 3. (i) Applying (A1) to $y^i = q^i(x_1, x_2)$ yields

$$\Delta y^i = q_1^i \Delta x_1 + q_2^i \Delta x_2, \quad (\text{A6})$$

where q_1^i and q_2^i are evaluated at some point between X^s and X^c . Under perfect symmetry $\Delta x_1 = \Delta x_2 \equiv \Delta x$, which must be positive by Proposition 2. Also by symmetry $q_1^i = q_2^i$. It then follows from (10) that Δy^i is positive.

(ii) Given that demand is downward sloping and that x_1 and x_2 are substitutes, if outputs rise, prices must fall.

(iii) Applying (A1) to profit, g^1 (which, under symmetry, equals g^2) yields

$$\Delta g^1 = g_1^1 \Delta x_1 + g_2^1 \Delta x_2 = (g_1^1 + g_2^1) \Delta x. \quad (\text{A7})$$

g_1^1 is given by (12) and $g_2^1 = \pi_1^1 q_2^1 + \pi_2^1 q_2^2 = R_2^1 q_2^2$. Therefore,

$$g_1^1 + g_2^1 = R_2^1 (q_1^1 + q_2^2) - C_x^1 - v^1. \quad (\text{A8})$$

$R_2^1 < 0$ by (2), $q_1^1 + q_2^2 > 0$ by (10), and $C_x^1 + v^1$ is positive at points between X^s and X^c . Since $\Delta x > 0$ by Proposition 2, it follows that $\Delta g^1 (= \Delta g^2) < 0$. *Q.E.D.*

Proof of Proposition 4. Applying (A1) to welfare, $W(x_1, x_2)$ yields

$$\Delta W = W_1 \Delta x_1 + W_2 \Delta x_2, \quad (\text{A9})$$

where $W_1 (\equiv \partial W / \partial x_1)$ and $W_2 (\equiv \partial W / \partial x_2)$ are evaluated at some point $X^* = X^c + \theta(X^s - X^c)$ for some θ between 0 and 1. From Proposition 2, under symmetry $\Delta x_1 = \Delta x_2 > 0$ and $W_1 = W_2$. Therefore, welfare rises if $W_1 > 0$ at X^* .

(i) From (19) $W_1 = (p^1 - c^1)(q_1^1 + q_2^1) - (C_x + v)$. By (12) and since $g_{11}^1 < 0$ (globally), it follows that $C_x + v < R_2^1 q_1^2$ at X^* . Therefore, $W_1 > 0$ if $(p^1 - c^1)(q_1^1 + q_2^1) - R_2^1 q_1^2 > 0$. Substituting for $q_1^1 + q_2^1$ from (10), and using $R_2^1 = y^1 p_2^1$, $(p^1 - c^1) = -y^1 p_1^1$ (from 4) and $q_1^2 = -c_x^1 \pi_{21}^2 / A$ yields

$$W_1 > -(c_x^1 / A) y^1 (p_1^1 (\pi_{22}^2 - \pi_{21}^2) - p_2^1 \pi_{21}^2). \quad (\text{A10})$$

(W_1 equals the right-hand side at $\theta = 1$.) Noting that $\pi_{22}^2 = 2p_2^2 + y^2 p_{22}^2 - c_y^2$ and $\pi_{21}^2 = p_1^2 + y^2 p_{21}^2$ and using $p_2^2 + y^2 p_{22}^2 \leq p_1^2 + y^2 p_{21}^2$, it follows that $p_1^1 (\pi_{22}^2 - \pi_{21}^2) \geq p_1^1 (p_2^2 - c_y^2)$. Therefore, $W_1 > 0$ if $p_1^1 (p_2^2 - c_y^2) \geq p_2^1 (p_1^2 + y^2 p_{21}^2)$. Since $p_1^1 p_2^2 > p_2^1 p_1^2$, $c_y^2 \geq 0$, $p_{21}^2 \geq 0$, $p_1^1 < 0$, and $p_2^2 < 0$, the result follows.

(ii) With homogeneous products, (A10) becomes

$$W_1 > (c_x/A)yp'(c_y + yp''). \quad (\text{A11})$$

The result follows with $c_y \geq 0$ and $p'' \geq 0$. *Q.E.D.*

Proof of Proposition 5. Proposition 5 requires signing W_i ($\equiv \partial W / \partial x_i$) at the strategic equilibrium. Starting with (19) and $C_x^i + v^i = R_j^i q_j^i$ (from (12)) yields $W_i = (p - c) \times (q_i^i + q_j^i) - R_j^i q_j^i$. Using substitutions similar to those used in deriving (11a) and noting that $c_y = 0$, it follows that $W_i = c_x^i (y^i)^2 p' p'' / A$ at the strategic equilibrium. Then $W_i \geq 0$ as $p'' \geq 0$. *Q.E.D.*

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