European Economic Review 36 (1992) 1601–1626. North-Holland

Pre-commitment and flexibility
Applications to oligopoly theory

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Received April 1989, final version received July 1991

This paper considers several related strategic duopoly settings in which uncertainty creates an 'option value' from retaining flexibility by delaying investment or output decisions until after uncertainty is resolved. This value of flexibility must be weighed against the strategic value of pre-commitment, yielding a trade-off between flexibility and pre-commitment. We obtain a simple characterization of the timing of output decisions as a consequence of the degree of uncertainty. Particular attention is paid to the implications for entry-deterrence by an incumbent firm and to the possible flexibility-enhancing effects of investment in capital.

1. Introduction

The role of pre-commitment in industrial organization has been a major research topic over the past decade. While there is no doubt the pre-commitment is an important part of real business strategy, many business people object that such strategies are too risky. For example, a firm that undertakes heavy capital investment so as to deter or manipulate entry, runs the risk of being overcommitted if demand for the product falls. There may be an advantage of flexibility which can offset the advantages of commitment. A firm that delays its investment plans until it learns more about the uncertain environment in which it operates may benefit from doing so. Our objective in this paper is to characterize directly the trade-off between pre-commitment and flexibility for three simple but interesting oligopoly models.

The first case we consider is probably the best-known example of pre-commitment: Stackelberg output leadership. We add a simple form of uncertainty to the Stackelberg duopoly model and analyze the trade-off between flexibility and commitment. One firm has an opportunity to move first. It must decide whether to pre-commit its output level before uncertainty

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*We are grateful to Peter Neary and two anonymous referees for very valuable comments. We also thank David Nickerson and Venkatraman Sadanand for helpful suggestions.

0014-2921/92/$05.00

about the level of demand is resolved, or whether to wait and enter a simultaneous output game with its rival. We obtain a direct algebraic representation of the cost of commitment for the case of linear demand, and show that high levels of demand uncertainty will induce the potential leader to forego pre-commitment. We also examine how the potential for entry-deterrence changes the trade-off between commitment and flexibility for the leader firm.

Our second case allows both firms the opportunity to move before uncertainty is resolved. We are therefore able to characterize the conditions under which a first mover arises endogenously as a response to the trade-off between flexibility and commitment. We show that asymmetries in firm-specific uncertainty provide a natural reason for one firm to act before another. The third case we consider focuses on the implications of the flexibility–commitment trade-off for an incumbent firm that can install its capital before or after the resolution of uncertainty. The uncertainty in this case concerns the magnitude of a rival firm's costs. We characterize the conditions under which an incumbent would prefer not to undertake pre-commitment, and we examine the general nature of entry deterrence and entry manipulation in this environment.

The value of flexibility has been studied in several contexts. Jones and Ostroy (1984) consider a flexibility-based demand for financial liquidity. The option value of delaying irreversible investment has been examined by Bernanke (1983), Brennan and Shwartz (1985), and McDonald and Siegel (1986), among others. These papers incorporate a general dynamic structure, but abstract from consideration of strategic rivalry between firms.

Papers that focus explicitly on the trade-off between pre-commitment and the informational advantages of flexibility in a strategic environment include Appelbaum and Lim (1985), Daughety and Reinganum (1990), Sadanand (1988) and Green and Sadanand (1992). In Appelbaum and Lim (1985), an incumbent firm facing entry by a competitive fringe weighs the cost and entry prevention advantages from immediate production against the informational advantage from waiting. In Daughety and Reinganum (1990), firms can purchase information about the value of an unknown demand parameter, as well as choose the timing of their production. Cournot or Stackelberg type outcomes emerge endogenously. Green and Sadanand (1992) are also concerned with the endogenous determination of Stackelberg leadership using a framework similar to our second case, but that differs in some important respects, as described later in the paper. Our third case is related to work by Vives (1986, 1989). Vives (1989) makes the important general point that pre-commitment and flexibility are not always substitutes. Specifically, a firm may consider a strategy of prior invesment in a highly flexible (multipurpose) plant design that would increase both flexibility and commitment.[1] Other more tangentially related papers include Boyer and Moreaux (1987) and Baldwin (1987).

Sections 2, 3 and 4 examine the three cases in turn, and section 5 contains concluding remarks.

2. Commitment versus flexibility: The Stackelberg case

2.1. The basic model

The Stackelberg duopoly equilibrium can be viewed as the Nash equilibrium of an output game in which one firm is able to commit its output level before the other. We introduce uncertainty in the form of an additive demand shock to the Stackelberg model and compare the advantages of moving first with the alternative advantages of flexibility. The two firms produce homogeneous products, with constant and identical marginal costs. We assume that demand is linear and we can, without loss of generality for our purposes, allow the slope of demand to equal 1. Denoting price by p, the inverse demand curve can be written as

$$p=a-(x+y)+u, \tag{2.1}$$

where x is the output of the (potential) leader, y is the output of the follower, and u is a random variable with mean 0 and variance σ^2. The density $f(u)$ is defined on the support $[\underline{u},\bar{u}]$. Unless otherwise stated, we assume that $\underline{u}$ is sufficiently large that both firms have positive outputs. We explicitly consider the possibility that one or both firms might shut down later in this section.

The leader has an exogenously given opportunity to commit its output in stage 1 before u is known, but may choose to defer this decision until stage 2, when u is known. In the latter case, the leader retains the flexibility to respond to the realization of u, but must choose its output in Cournot competition with its rival. In either case, the rival chooses output after uncertainty is resolved. Note that commitment is assumed to be an all or nothing decision: The potential leader must either pre-commit all of its output in stage 1, or remain fully flexible, producing all output in stage 2. This assumption makes the comparison between commitment and flexibility as sharp as possible. A reasonable alternative would be to allow the firm to add to its first stage production at the second stage, as in Appelbaum and Lim (1985), where an incumbent firm chooses output to affect the entry of competitive fringe firms. In a Cournot type model, this 'partial commitment'

[1]The idea that there is a trade-off between ex-post flexibility and up-front cost can be traced back to early work on 'putty-clay' models of investment and growth. See, in particular, Phelps (1963) and Inada (1964).

possibility can give rise to multiple equilibria as described in Saloner (1987). Our approach is typical of the literature concerned with endogenous timing decisions in oligopoly settings. [See Daughety and Reinganum (1990).]

We examine the (expected) profit of the leader under the assumption that it pre-commits its output, then compare this with its profits if it opts for flexibility. This involves first setting out the decision of the flexible follower, whose maximand is $\pi^{f}=(a-c+u-(x+y))y$, where c represents marginal cost. Solving the associated first-order condition yields the following reaction function for y as a function of x and random shock u:

$$y=y(x,u)=(a-c+u)/2-x/2. \tag{2.2}$$

The leader sets output before uncertainty is resolved. We assume risk neutrality throughout the paper implying, in this case, that the leader maximizes expected profit. Using π^{c} to denote the committed leader's profits, and E to denote expected value, the leader maximizes

$$\mathrm{E}[\pi^{c}]=\mathrm{E}[a-c+u-x-y(x,u)]x, \tag{2.3}$$

where $\mathrm{E}[y(x,u)]=(a-c-x)/2$ [from (2.2)] and $\mathrm{E}[u]=0$. Solving the associated first-order condition for x, then substituting back into (2.2) yields the equilibrium output levels

$$x=(a-c)/2, \qquad y=(a-c+2u)/4. \tag{2.4}$$

As shown by (2.4), uncertainty has no effect on the output x chosen by the Stackelberg leader. Also, although the random variable u affects the leader's realized profit given by $\pi^{c}=(a-c+2u)(a-c)/8$, expected profit $\mathrm{E}[\pi^{c}]=(a-c)^{2}/8$ is unaffected by uncertainty. The profit of the follower, denoted π^{f}, is $(a-c+2u)^{2}/16$, and has expected value $(a-c)^{2}/16+\sigma^{2}/4$.

Uncertainty does affect the value of the leader's alternative: the simultaneous flexible Cournot equilibrium, where neither firm pre-commits. The solution for that case is obtained by taking eq. (2.2) and the corresponding reaction function for the leader, then solving the two linear equations for x and y. Using a superscript * to denote the flexible Cournot regime, we have

$$x^{*}=y^{*}=(a-c+u)/3. \tag{2.5}$$

Profit levels for each firm are $\pi^{*}=(a-c+u)^{2}/9$, which has the following expected value:

$$\mathrm{E}[\pi^{*}]=((a-c)^{2}+\sigma^{2})/9. \tag{2.6}$$

Eq. (2.6) indicates that the expected value of profit is increasing in the variance of demand.[2] In good states of the world firms can increase output

[2] We assume that variance is increased holding $[\underline{u},\bar{u}]$ constant. For example, assume density $f(u)=1/2(\bar{u}-\delta)$ for $u\in[-\bar{u},-\delta]$ and $u\in[\delta,\bar{u}]$ and $f(u)=0$ otherwise. An increase in δ will increase $\sigma^{2}=[\bar{u}^{2}+\delta^{2}+\bar{u}\delta]/3$ for $\delta\leqq\bar{u}$.

and in bad states they can reduce output. These adjustments allow them, on average, to do better than they could in the case of pure certainty with demand always at its expected level.[3]

The expected profit levels are easily compared, and imply that committed leadership is preferred to the flexible Cournot equilibrium if and only if condition (2.7) is satisfied.

$$\sigma^2 < (a-c)^2/8. \tag{2.7}$$

The meaning of (2.7) is clarified by relating uncertainty to the price cost margin. Specifically, commitment is preferred to flexibility if $\sigma < \sqrt{2}(\mathrm{E}[p^c - c])$: if the standard deviation of price is less than $\sqrt{2}$ times the expected price cost margin under commitment. If there is no uncertainty ($\sigma = 0$) then commitment is obviously worthwhile. With uncertainty, however, commitment makes the firm give up a valuable option. As uncertainty becomes more important, the relative value of pre-commitment falls, and eventually turns negative. We obtain this result with a linear demand function and risk neutrality. Risk aversion would increase the importance of this effect. Nonlinear demand or nonlinear randomness could introduce ambiguities, and could magnify or diminish the costs of pre-commitment, but would clearly not undermine the basic economic point that pre-commitment may have costs in an uncertain world.

2.2. The shutdown option

So far we have assumed that all states of the world are sufficiently favourable that firms will always produce positive outputs. We now explicitly consider 'shutdown' and entry deterrence as possibilities. The effect of a shutdown possibility depends on how it is introduced. A natural method of introduction is to consider widening the support $[\underline{u}, \bar{u}]$ of u, keeping the mean unchanged at 0, but raising the variance (i.e., through a mean-preserving spread), and leaving parameters a and c unchanged. We also assume for convenience that $\bar{u} = -\underline{u}$.

Supposing that the leader has pre-committed its output, the follower will choose to shut down (at the Stackelberg equilibrium) if the demand shock is sufficiently negative to make its profits negative. The boundary of this condition defines a critical value of u, denoted by u^{P}, at or below which the follower will not produce. We assume for this section that $\underline{u} < u^{\mathrm{P}}$, so that the probability of shutdown by the follower, denoted by $q \equiv \int_{\underline{u}}^{u^{\mathrm{P}}} f(u)\,\mathrm{d}u$ is strictly

[3]Allowing x and y to vary, the profit function is convex in u, implying that randomness in demand improves expected profit.

positive. From (2.2.), u^p depends on the leader's output according to the following relationship:

$$u^p(x) = x - (a - c). \tag{2.8}$$

Since $du^p/dx = 1$ from (2.8), it follows that $dq/dx = f(u^p) > 0$: The probability of shutdown by the follower is increasing in the leader's output.

We refer to this effect of the leader's output on q as 'probabilistic entry deterrence'. As both firms have the same marginal cost, the leader will never deter entry with certainty. For the follower to be deterred with certainty, $\bar{u}$ (the best realization of u) would have to be less than the critical state u^p. This would occur only if the leader chose a value of x so large that its own expected profits would also be negative, and can therefore be ruled out.[4]

In order to understand how probabilistic entry deterrence affects the potential leader's choice between commitment and flexibility, we need to rewrite the experession for the expected value of the follower's output. In general, $E[y(x,u)] = \int_{\underline{u}}^{\bar{u}} y(x,u) f(u)\, du$. However, for values of u less than u^p, y is truncated at 0, whereas for values of $u \geqq u^p$, y is given by (2.2), yielding the following expression for $E[y]$:

$$E[y(x,u)] = \int_{u^p}^{\bar{u}} (a - c - x + u) f(u)\, du/2. \tag{2.9}$$

The effect of the leader's output x on $E(y)$ is given by the following derivative:

$$dE(y(x,u))/dx = (\partial E(y)/\partial u^p)(du^p/dx) + \partial E(y)/\partial x. \tag{2.10}$$

From (2.8) and (2.9), the first term of (2.10) is $-(a-c-x+u^p) f(u^p)/2 = 0$. In words, the marginal effect of x through the mechanism of increasing the range of entry deterrence (i.e., by increasing u^p) is 0 because, at the margin of entry deterrence, $y = 0$ anyway. The total effect of x on y is therefore given just by the second term of (2.10), leading to the following equation:

$$dE[y(x,u)]/dx = -(1-q)/2. \tag{2.11}$$

Not surprisingly, an increase in x lowers the expected output of the follower. Note, however, that the marginal reduction of the rival's output arising from an increase in x is actually reduced by increases in the probability of 'shutdown', q. This is because if q is large, the rival is likely to produce nothing anyway, so x has no marginal impact on y. We might have expected an additional term in (2.11) representing the effect on expected output of the

[4]Since $\bar{u} > 0$, the condition $\bar{u} < u^p(x)$, implies $x > a - c$ from (2.8) so that $E[\pi^c] < 0$ from (2.13).

higher probability of 'shutdown' associated with an increase in x but, as already indicated in the discussion of the first term of (2.10), this effect is zero.

Furthermore, introducing a shutdown option by widening the support of u serves to increase the expected output of the follower. Widening the support raises the follower's output for high realizations of u, but the follower's output is truncated at zero for low realizations of u. To show this result, we use (2.8) and $\mathrm{E}[u]=0$ to rearrange (2.9) into the form

$$\mathrm{E}[y(x,u)]=(a-c-x)/2+\left[\int_{\underline{u}}^{u^{\mathrm{p}}}(u^{\mathrm{p}}-u)f(u)\,\mathrm{d}u\right]/2. \tag{2.12}$$

If there is no possibility of shutdown (i.e., $\underline{u}>u^{\mathrm{p}}$), then the second term of (2.12) vanishes. However, if $\underline{u}<u^{\mathrm{p}}$, then the second term is positive and increasing in the support of u.

From (2.12) and (2.3), the expected profit of the Stackelberg leader can be written as

$$\mathrm{E}[\pi^{\mathrm{c}}]=[(a-c-x)-\int_{\underline{u}}^{u^{\mathrm{p}}}(u^{\mathrm{p}}-u)f(u)\,\mathrm{d}u](x/2). \tag{2.13}$$

The first term of (2.13) represents the leader's expected profit when there is no possibility of shutdown. The second term is negative reflecting the fact that the increase in the expected output of the follower associated with the shutdown option reduces the leader's expected profit. Differentiating (2.13) and using $q\equiv\int_{\underline{u}}^{u^{\mathrm{p}}}f(u)\,\mathrm{d}u$, the leader's output satisfies

$$\mathrm{dE}[\pi^{\mathrm{c}}]/\mathrm{d}x=(a-c-2x)/2-\left[\int_{\underline{u}}^{u^{\mathrm{p}}}(u^{\mathrm{p}}-u)f(u)\,\mathrm{d}u+qx\right]/2=0. \tag{2.14}$$

Condition (2.14) implies that the Stackelberg leader sets an output x^{c} below the level $(a-c)/2$ chosen when $q=0$. It then follows [using (2.13)] that the leader's expected profit under commitment is reduced below the level arising when $q=0$:

$$\mathrm{E}[\pi^{\mathrm{c}}]<(a-c)^2/8, \tag{2.15}$$

We now consider how the possibility of 'shutdown' by the follower affects the leader's comparison of pre-commitment versus flexibility. The value of flexibility to the potential leader depends on whether u can take on values low enough to induce shutdown at the flexible Cournot equilibrium. From

(2.5), this cannot occur[5] if $\underline{u} > -(a-c)$. In this case expression (2.6) still applies, implying that the value of flexibility is increased. Thus introducing a shutdown possibility by raising variance increases the value of flexibility and reduces the value of commitment. There is an increase in the range of cases in which flexibility is preferred to commitment. Modifying (2.7) to allow for a shutdown option, it still follows [from (2.6) and (2.15)], that the leader will prefer flexibility provided $\sigma^2 > (a-c)^2/8$, but the statement is no longer of the 'if and only if' form.

Suppose now that u can take on values low enough that both firms would shut down at the flexible Cournot equilibrium ($u < -(a-c)$). Since profit is truncated at zero for values of $u < -(a-c)$, the expression (2.6) for expected profit is modified as follows: $\mathrm{E}[\pi^*] = (\int_{-(a-c)}^{\bar{u}} (a-c+u)^2 f(u)\,\mathrm{d}u)/9$. As can be seen from this expression, a widening of the support of u increases the expected profits of both firms at the flexible Cournot equilibrium even with the associated increase in the probability that both firms shut down. The firms earn no profits when $u < -(a-c)$ but this is offset by the additional profits earned when $u > a-c$. The leader's expected profit under commitment[6] is reduced [as given by (2.13)], so that, as in the previous case, introducing a 'shutdown' option through a mean-preserving spread of u enhances the value of flexibility relative to the value of commitment.

3. The symmetric case

In the model of section 2, one firm has an exogenously given opportunity to act first. A natural extension is to allow both firms the option of choosing output before uncertainty is resolved. We examine that case here. To save space while retaining as much clarity as possible, we abstract from the shutdown option by assuming that $\underline{u}$ is high enough that both firms would always produce positive outputs. The only question is whether to pre-commit or wait.

There are, effectively, three stages in the game. In stage 1, each firm decides whether to commit its output before uncertainty is resolved, or whether to retain the flexibility to make its output decision after u is revealed. The outcome of this 'timing' decision is observed by both firms. In the next stage, if either firm has decided to pre-commit, it then chooses its output level. Stage 3 occurs after uncertainty has been resolved. If both firms

[5]Note that there are values of u, including the range $[-(a-c), -(a-c)/2]$, where shutdown by the follower occurs at the Stackelberg equilibrium, but neither firm shuts down in the flexible regime.

[6]Once committed to a positive level of output, a Stackelberg leader would be unable to shut down and, as can be seen from (2.13), would make actual losses for realizations of u below $-(a-c)$. However, this is at least partially offset by the additional profit associated with values of u above $a-c$.

have already made production decisions, then nothing can be changed at this stage. If only one firm has pre-committed its output, then the other produces at this stage and chooses its follower output. If neither firm pre-committed its output level, then each produces its ex post Cournot output level.

An alternative specification is contained in Green and Sadanand (1991), where firms play a two-stage game. In stage 1, before uncertainty is resolved, both firms have an opportunity to pre-commit output, but whether the other firm is pre-committing is not yet common knowledge. In stage 2, after uncertainty is resolved, if either firm has not yet chosen its output level, it then does so. This structure compresses the decision to commit and the choice as to the level of output to produce under commitment into a single stage. The commitment–flexibility trade-off is thus affected by uncertainty as to the nature of the rival's action as well as by the random shock itself. In our model, whether the rival is pre-committing output is common knowledge before output levels are actually decided on. One could justify this on the basis that the decision whether or not to produce early could be transmitted to rivals through preliminary actions such as orders to outside suppliers. Furthermore, consideration of the random shock as the only source of uncertainty gives rise to a simpler and more transparent formulation of the implications of the random shock itself.

It follows from our specification that the sequentially rational equilibrium will imply a Nash equilibrium in the timing game. Each firm makes its profit-maximizing timing decision, taking the timing decision of the other as given at the equilibrium value. If both firms pre-commit to output before uncertainty is resolved, then the output equilibrium will be of the Cournot type. We refer to this as the 'committed Cournot' regime. If one firm commits its output before u is revealed, while its rival chooses output after u is revealed, then sequential rationality implies that the output equilibrium will be of the Stackelberg leader–follower type and, as in section 2, the firms are referred to as the committed leader and the flexible follower. Each firm confronts four possibilities: It might be a committed leader, a committed Cournot firm, a flexible Cournot firm, or a flexible follower. The relative rankings of these alternatives by each firm will determine the equilibrium order of moves and market structure.

As before, variables associated with a committed leader are denoted by a superscript c. Variables associated with a flexible follower are denoted by a superscript f, variables associated with the flexible Cournot regime are identified by a superscript *, and variables associated with a committed Cournot firm will be indicated by the superscript cc. The expected profit of a committed Cournot firm is the certainty value of the profit under Cournot duopoly, $(a-c)^2/9$. This expected profit, and the expected profits for the other regimes (from section 2) are listed below.

$$E[\pi^{c}]=(a-c)^2/8, \quad E[\pi^{cc}]=(a-c)^2/9,$$

$$E[\pi^{*}]=(a-c)^2/9+\sigma^2/9, \quad E[\pi^{f}]=(a-c)^2/16+\sigma^2/4.$$

A comparison of these expected values indicates that there are four possible relative rankings of the roles firms might take on, depending on the size of σ^2, as reported in table 1.

Table 1 shows that if, for example, σ^2 were very small, then a firm would prefer to be a committed leader, would like the flexible Cournot regime next, the committed Cournot regime third, and flexible follower role least. On the other hand, if uncertainty were very significant, then the flexible Cournot regime would be most attractive, followed, in order, by the flexible follower role, the committed leader role, and finally, by the committed Cournot regime.

The role of each firm is determined in part by the action of its rival. A firm might want to be a committed leader, but unless its rival acquiesces by opting for the role of a flexible follower, it cannot achieve this desire. Our solution concept is the non-cooperative Nash equilibrium, which restricts attention to outcomes in which the firms are not 'surprised'. This is reasonable if firms are rational enough to understand the nature of the game they are playing, leading them to choose only those strategies that are optimal given reciprocal optimizing behaviour by the rival.

Proposition 1 (*i*) *If* $\sigma^2<(a-c)^2/8$, *then the unique Nash equilibrium is the committed Cournot regime. This is also a dominant strategy equilibrium in the order of move game.*

(*ii*) *If* $(a-c)^2/8<\sigma^2<7(a-c)^2/36$, *then both the committed Cournot and flexible Cournot regimes are Nash equilibria.*

(*iii*) *If* $\sigma^2>7(a-c)^2/36$, *then the flexible Cournot regime is the unique Nash equilibrium and is a dominant strategy equilibrium.*

Proof. The results follow by inspection from table 1. We go through case (i)

Table 1

Regime rankings.

	Committed leader	Committed Cournot	Flexible Cournot	Flexible follower
$\sigma^2<(a-c)^2/8$	1	3	2	4
$(a-c)^2/8<\sigma^2<7(a-c)^2/36$	2	3	1	4
$7(a-c)^2/36<\sigma^2<(a-c)^2/4$	2	4	1	3
$\sigma^2>(a-c)^2/4$	3	4	1	2

Table 2

Ordinal pay-off matrix.

		Firm 2	
		Commit	Flexible
Firm 1	Commit	(3,3)	(1,4)
	Flexible	(4,1)	(2,2)

here. We construct an ordinal pay-off matrix indicating how each firm ranks each possible timing combination. Firm 1 can commit its output or remain flexible, as can firm 2, yielding table 2. (The first entry in each cell is firm 1's ranking of that cell; the second entry is firm 2's ranking.)

If firm 1 were to commit output before uncertainty, then firm 2 would like to commit its output also, because the committed Cournot regime is better than being a flexible follower. In addition, if firm 1 were to remain flexible, then firm 2 would like to commit, because being a committed leader is better than the flexible Cournot regime. Thus, moving before uncertainty is a dominant strategy for firm 2, and, by parallel reasoning, for firm 1 also. Anticipating that each will move before uncertainty is resolved, the firms settle on the Nash equilibrium outputs prior to the resolution of uncertainty. Reasoning for cases (ii) and (iii) is similar. □

The model described here incorporates a timing decision and an output decision. Since the timing decision is predicated on Nash output equilibria, the entire equilibrium is sequentially rational. Looked at by itself, the timing decision yields dominant strategy equilibria for high and low levels of uncertainty. Since these dominant strategy equilibria in the timing game incorporate non-dominant strategy output equilibria, however, the overall equilibrium cannot be described as a dominant strategy equilibrium.

The results are intuitively plausible in that high levels of uncertainty lead firms to delay their output decisions until after uncertainty is resolved, low levels of uncertainty lead firms to commit output before uncertainty is resolved, and intermediate levels of uncertainty may lead to either symmetric outcome. It is, however, noteworthy that if uncertainty is low but positive, then firms are trapped in a low level prisoner's dilemma. They would prefer the flexible Cournot regime, but instead are trapped in the committed Cournot regime. Consumers are also made worse off by this low level equilibrium, since they too would be better off if the firms could respond flexibly to uncertainty. We therefore have a 'rat race' phenomenon in which social inefficiency arises through competitive incentives to obtain first mover advantages.

Provided that firms are initially symmetric, the only (pure strategy) equilibria are symmetric. There are no pure strategy equilibria in which one firm acts before uncertainty and the other after. This result contrasts with Green and Sadanand (1992) in which the Stackelberg leader–follower equilibrium can arise with symmetric firms.[7] There are several types of asymmetry that might be introduced. The easiest way of introducing asymmetry is to eliminate demand uncertainty altogether and assume that firms 1 and 2 face firm specific marginal cost uncertainty, represented by random variables u_1 and u_2, with corresponding variances σ_1^2 and σ_2^2, and covariance σ_{12}. Let u_1 and u_2 be negative shocks to marginal cost (so that high values are 'good news' to the firm in question). If these disturbances are identical and perfectly correlated, then we have exactly the same model as already analyzed.

This kind of asymmetry can generate endogenous Stackelberg leadership. The calculations delimiting the various cases are similar to those presented previously. The conditions for endogenous Stackelberg leadership are as follows:

$$4\sigma_1^2+\sigma_2^2-4\sigma_{12}<(a-c)^2/8, \tag{3.1}$$

$$7(a-c)^2/36<\sigma_2^2. \tag{3.2}$$

Expression (3.1) implies that firm 1 would rather be a leader than a flexible Cournot firm, while expression (3.2) implies that firm 2 would prefer to be a flexible follower rather than a committed Cournot firm. If both conditions hold simultaneously, then it is a Nash equilibrium for firm 1 to be a committed leader while firm 2 is a flexible follower. It is helpful to consider a specific example. Let $u_1=\alpha u_2$ where $0<\alpha<1$. In this case $\sigma_1^2=\alpha^2\sigma_2^2$ and $\sigma_{12}=\alpha\sigma_2^2$. Conditions (3.1) and (3.2) are then satisfied if, for example, $\alpha=1/2$, while σ_2^2 is set at any value satisfying (3.2).

The emergence of Stackelberg leadership is not, however, completely straightforward. It is not the case that simply allowing high variance for one firm and low variance for the other will lead to commitment by the low variance firm and flexibility by the high variance firm. In particular, if $\alpha=0$, implying $\sigma_1^2=0$ and therefore that firm 1 faces no randomness in its costs, and σ_2^2 is large enough to satisfy (3.2), then firm 1 does not choose to become a committed leader, even though firm 2 would remain flexible and take the follower role. This can be seen from (3.1) which, given $\sigma_{12}=0$, requires that $\sigma_2^2<(a-c)^2/8$ if firm 1 is to prefer commitment. This requirement conflicts with (3.2), implying that firm 1 cannot be in a position to prefer commitment if firm 2 is willing to be a follower.

[7]This suggests that this Green and Sadanand (1992) result is a consequence of their assumption of uncertainty concerning the nature of the rival's actions.

If $\alpha=0$ and σ_2^2 is large, then the equilibrium outcome is the flexible Cournot regime. Firm 1 gains from its ability to adjust its output (in accordance with Cournot rules) in response to the realization of firm 2's uncertain marginal cost. This is better for firm 1 than taking the risk of being trapped with high output when firm 2's marginal cost is low and its output is correspondingly high. The output of firm 1 in the flexible Cournot regime is $x=(a-c-u_2+2u_1)/3$. With $u_1=0$ (as implied by $\alpha=0$), we see that firm 1 reduces its equilibrium output as u_2 becomes larger. If, however, $\alpha=1/2$, it follows that $u_1=u_2/2$, and x is independent of the realization of uncertainty. Firm 1's desire to contract in response to a reduction in its rival's costs is just offset by its inclination to expand due to its own (smaller) reduction in costs. In this case, flexibility is of no value to firm 1, so it would prefer to be a Stackelberg leader. Provided σ_2^2 is large enough, firm 2 will accept the follower role.

While these results arise from a very specific case, some general statements can be made. First, asymmetric uncertainty can give rise to the endogenous emergence of a leader–follower structure. Secondly, however, the value of flexibility to a firm depends not just on the distribution of its own random shock, but also on the distribution of its rival's random shock.

4. Commitment of capital versus flexibility

4.1. Model structure

Sections 2 and 3 demonstrate the trade-off between flexibility and commitment for models in which output itself is the only variable that can be pre-committed. A related structure is one in which a firm can pre-commit some factors of production but not all. In this section we introduce uncertainty into such a model, then analyze the trade-off between the value of capital pre-commitment and capital flexibility.

We consider an industry with one incumbent firm (firm 1) and one potential entrant (firm 2). The model has two stages. In the first stage, the incumbent may install capital, and in the second stage the incumbent produces output. If the incumbent does not install capital in the first stage, it may do so in the second stage. The entrant chooses capital and output in stage 2. Equilibrium outputs are determined by a Nash quantity game. The modelling innovation is to assume that in stage 1 there is uncertainty about the cost structure of firm 2, but no uncertainty concerning the incumbent's costs. This specification is chosen largely so as to introduce uncertainty in the simplest possible way consistent with analyzing the trade-off between flexibility and commitment. Also, the idea that an entrant's cost function has substantial uncertainty associated with it is attractive on empirical grounds and, as suggested by section 3, it is natural that the firm with less firm specific uncertainty should be the first mover.

Uncertainty is resolved between stages 1 and 2, so if the incumbent waits until stage 2 before undertaking capital investment, it can respond with flexibility in its choice of capital to the realization of firm 2's cost conditions. Following the approach taken in the previous two sections, we assume that firm 1 cannot add capital in stage 2 if it has invested in stage 1. In other words, firm 1 can invest in stage 1, or in stage 2, but not in both stages. Nevertheless, this allows more flexibility in the commitment regime than does the simple Stackelberg model of section 2, because the committed leader is still free to vary other (non-capital) factors of production after uncertainty is resolved.[8]

The incumbent's problem subdivides into two parts. One sub-problem involves deciding on the optimal capital stock, given that the firm has decided to invest in stage 1. We call this the commitment regime. The second subproblem involves choosing the optimum strategy, given that the firm defers capital investment until the second stage. This is the flexible regime. The overall optimum is then the better of these two alternatives. In the terminology of section 3, firm 1 may choose to be either a committed leader or to be part of a flexible Cournot regime. The entrant, correspondingly, will be either a flexible follower or a flexible Cournot firm. Unlike section 3, however, flexibility and commitment refer here to capital rather than to output.

4.2. The committed leadership regime

Firms 1 and 2 produce homogeneous outputs x and y. Firm 1 can install capital k at cost v per unit. Such investments would naturally shift down the firm's second stage marginal cost curve. More generally, investment might also affect the slope of marginal cost, with steeper marginal cost curves representing less flexible technologies, as first described by Stigler (1939). These features are incorporated into the following quadratic total cost function for firm 1:

$$C(x;k)=c(k)x+\gamma(k)x^2/2+vk, \tag{4.1}$$

where $c'(k)<0$ and $\gamma(k)\geqq 0$. This total cost function gives rise to a linear marginal cost function with intercept $c(k)$ and slope $\gamma(k)$. As for the entrant, it sets its capital in stage 2 simultaneously with output, so investment is

[8]As far as descriptive accuracy is concerned, the idea that capital can neither be augmented nor reduced in the second stage has some appeal. A firm may, ex ante, be able to choose any size of factory it likes. Once the factory is built, however, it is very costly to slightly augment the size of the factory. Many investment processes have this kind of ex post indivisibility associated with them. It is also true, of course, that some types of capital can be easily augmented.

always at the cost minimizing level given the revealed value of $u \in [\underline{u}, \bar{u}]$. With this choice of capital, the entrant's total cost function is

$$C^{e}(y,u)=(c^{e}-u)y+\gamma^{e}y^{2}, \tag{4.2}$$

where $c^{e}>0$ and $\gamma^{e}\geqq 0$ are constants. Since u (defined in the same way as before) is a negative shock to cost, high values of u are good states for firm 2 and bad states for firm 1. We assume that $\bar{u}$ is not so large that the entrant's cost becomes negative. The profit functions of the incumbent and entrant are, respectively,

$$\pi(x,y,k)=xp(x+y)-C(x,k), \qquad \pi^{e}(x,y,u)=yp(x+y)-C^{e}(y,u). \tag{4.3}$$

Proceeding by backwards induction, we first analyze stage 2, contingent upon the resolution of uncertainty and upon the stage 1 decisions of the incumbent. Until we specifically deal with entry deterrence (section 4.5), we assume that both firms produce a positive level of output.

Stage 2. The output game

In stage 2, u is known, and each firm maximizes its stage 2 profit with respect to output, leading to the following first order conditions (subscripts denote partial derivatives):

$$\pi_{x}=p+xp'-(c(k)+\gamma(k)x)=0, \tag{4.4}$$

$$\pi^{e}_{y}=p+yp'-(c^{e}+\gamma^{e}y-u)=0. \tag{4.5}$$

We assume that outputs are strategic substitutes (i.e., that reaction functions slope downward):

$$\pi_{xy}\equiv p'+xp''<0, \qquad \pi^{e}_{yx}\equiv p'+yp''<0. \tag{4.6}$$

Conditions (4.6) can be violated by reasonable demand structures, but can be taken as the 'standard case' for output rivalry. They are helpful in signing comparative static effects, and ensure the following second-order and regularity conditions, which in turn are sufficient for existence and uniqueness of the output equilibrium:

$$\pi_{xx}<0, \quad \pi^{e}_{yy}<0, \quad D=\pi_{xx}\pi^{e}_{yy}-\pi_{xy}\pi^{e}_{yx}>0. \tag{4.7}$$

First-order conditions (4.4) and (4.5) then define the equilibrium outputs as functions of the predetermined variables:

$$x = x(k, u), \qquad y = y(k, u). \tag{4.8}$$

We obtain the comparative static effect of changes in stage 1 capital on equilibrium outputs by totally differentiating first-order conditions (4.4) and (4.5) with respect to x, y, and k, applying Cramer's rule, and making use of (4.6) and (4.7):

$$x_k(k, u) = (c'(k) + \gamma'(k)x)\pi^e_{yy}/D > 0,$$

$$y_k(k, u) = -(c'(k) + \gamma'(k)x)\pi^e_{yx}/D < 0. \tag{4.9}$$

We assume that any flexibility-reducing effect of capital (i.e., $\gamma'(k) > 0$) is not so large that investment would increase marginal cost at the equilibrium level of output. Then, from (4.9), increases in the incumbent's first-stage capital increase its own equilibrium output and reduce the equilibrium output of the entrant. Similarly, the comparative static effects of the entrant's cost shock are as follows:

$$x_u(k, u) = \pi_{xy}/D < 0, \qquad y_u(k, u) = -\pi_{xx}/D > 0. \tag{4.10}$$

A low realization of marginal cost for the entrant will raise its equilibrium output and lower the equilibrium output of firm 1. Expression (4.10) shows that both outputs are subject to uncertainty, even though only firm 2 feels a direct impact of uncertainty on costs.

Stage 1. Pre-committed capital

We now examine the decision by firm 1 to invest capital in stage 1. This decision is sequentially rational (or closed loop) in the sense that the firm correctly accounts for the effect of its stage 1 investment on the outcome of the second-stage output game. Firm 1 sets capital so as to maximize its expected profit, given the second-stage, equilibrium outputs as expressed by (4.8). Noting that $d\pi/dk \equiv \pi_x x_k + \pi_y y_k + \partial\pi/\partial k$, and that π_x will be set to zero by second-stage maximization of profit, the committed level of capital (denoted k^c) satisfies the first-order condition[9]

$$dE[\pi]/dk \equiv E[d\pi/dk] = E[\pi_y y_k] - E[C_k(x^c, k^c)] = 0. \tag{4.11}$$

Observing that $\pi_y = xp' < 0$, and using (4.9), it follows that the first term of

[9]We assume an internal equilibrium with $k > 0$. Otherwise there would be no investment in either the committed or flexible regimes.

(4.11) is positive. This term represents the strategic effect of adding first stage capital. Additional capital lowers the equilibrium output of the rival because, by lowering marginal cost, the extra capital commits the incumbent to a more aggressive reaction function. Since the first term is positive, the second term must be negative. Assuming that the second order condition for stage 1 maximization is satisfied ($d^2E[\pi]/dk^2<0$), it follows that the level of committed capital k^c exceeds the level necessary to minimize expected costs.[10] From (4.1) this latter level of capital, denoted by k^0, satisfies

$$E[C_k(x,k^0)]=c'(k^0)E[x(k^0,u)]+\gamma'(k^0)E[x^2]/2+v=0. \tag{4.12}$$

Capital k^0 represents the level of capital that would be chosen in an open loop model in which the incumbent takes the second-stage output of its rival as given, but has to invest prior to the resolution of uncertainty.

Proposition 2 shows that the effect of uncertainty on capital commitment depends on whether capital plays a role in affecting the flexibility of production technology. Proposition 2 applies in the linear demand case. Allowing for nonlinear demand will affect the desired capital stock, but is not likely to introduce any systematic changes in the direction of the response of the capital stock to increased uncertainty.

Proposition 2. With linear demand, an increase in the variance of the shock to marginal cost decreases the amount of pre-committed capital if $\gamma'(k)>0$, leaves it unchanged if $\gamma'(k)=0$ and increases it if $\gamma'(k)<0$.

Proof. Letting $p=a-b(x+y)$, we first solve for the stage 2 equilibrium outputs using (4.1), (4.2), (4.4) and (4.5):

$$x=[(a-c(k))(2b+\gamma^e)-b(a-c^e+u)]/D,$$

$$y=[(a-c^e+u)(2b+\gamma(k))-b(a-c(k))]/D, \tag{4.13}$$

where from (4.7) , $D=(2b+\gamma(k))(2b+\gamma^e)-b^2>0$. This implies that for any given k,

$$E[x(k,u)]=x(k,0) \quad \text{and} \quad E[y(k,u)]=y(k,0) \tag{4.14}$$

are independent of the variance or higher moments of u. Now using $\pi_y=-bx$, $y_k=c'(k)b/D$ [from (4.3) and (4.9)], (4.12) and (4.14) in (4.11), k^c satifies

[10]This result is standard in the certainty case. See Dixit (1980) and Brander and Spencer (1983).

$$\mathrm{d}\mathrm{E}[\pi]/\mathrm{d}k = -x(k^{c},0)c'(k^{c})(1+b^2/D) - v - \gamma'(k^{c})\mathrm{E}[(x^{c})^2](1/2+b^2/D) = 0. \tag{4.15}$$

Since $\mathrm{E}[(x^{c})^2] = (x(k,0))^2 + (b^2/D^2)\sigma^2$ is increasing in σ^2, the result follows from (4.15). □

If capital investment simply shifts down the marginal cost curve without affecting its slope and demand is linear, proposition 2 has the striking implication that uncertainty as to the rival's output has no effect on the quantity of capital in the commitment regime; an increase in variance does not make lower levels of capital more attractive than higher levels. Greater variance decreases investment only when investment would reduce the flexibility of technology by making the marginal cost curve steeper. In the converse case where capital investment increases the flexibility of technology, greater uncertainty would increase the level of committed capital. However, as we shall show, even if greater uncertainty has no effect on the level of committed capital, it can make it more attractive for the firm to forego commitment altogether.

Proposition 2 is related to a result in Vives (1989), in which firms can choose their plant design so as to increase flexibility (make the marginal cost curve flatter) at the expense of shifting up the intercept of the marginal cost curve. An increase in variance (arising from a demand shock in this case) tends to lead firms to choose a more flexible technology. This is analogous to our result that committed capital will fall in response to an increase in variance when a reduction in capital investment makes technology more flexible (i.e., $\gamma'(k) > 0$).[11]

4.3. *The flexible regime*

We now consider the flexible regime in which the incumbent waits until after the resolution of uncertainty to install its capital, implying that the equilibrium is of the flexible Cournot type. As before, supercripts * are used to denote values associated with the flexible Cournot regime. Thus k^* is the incumbent's second stage capital choice, and x^* and y^* are the corresponding output levels of the incumbent and entrant, respectively. Variables associated with the commitment regime are denoted by the superscript c, so we write committed capital as k^{c}, etc. A flexible incumbent is in essentially the same position as the entrant. The (second stage) objective of the flexible

[11]In Vives (1989) the total up-front cost of plant design is assumed independent of the flexibility of the plant. Our model differs because the total up-front cost vk depends on k, the instrument used to affect the flexibility of the plant.

incumbent is $\pi(x^*,y^*,k^*)=(p-c(k^*))x^*-vk^*$. The first-order condition for the incumbent's choice of capital is[12]

$$\pi_{k^*}=-(c'(k^*)x^*+\gamma'(k^*)(x^*)^2/2+v)=0, \tag{4.16}$$

which implicitly defines k^* as a function of x: $k^*=k^*(x)$. The incumbent's first-order condition with respect to output is as given by (4.4) with marginal cost $c(k^*(x))$. These first-order conditions implicitly define

$$x^*=x(k^*,u), \qquad y^*=y(k^*,u), \qquad k^*=k^*(x^*). \tag{4.17}$$

The functional relationships for x and y in (4.17) are identical to the corresponding relationships in (4.8) obtained for the commitment regime. The meaning of (4.8) differs from (4.17), however, for k is predetermined in (4.8), while in (4.17) it is endogenous. The total effects of a change in u on x^* and y^* differ from the commitment regime precisely because there are induced changes in k^* under the flexible regime.

4.4. *Commitment versus flexibility*

We wish to compare the flexibility advantage to the incumbent from delaying investment with the strategic gain from commitment. We first decompose the expected difference in profit between the two regimes into a commitment effect and a flexible timing effect:

$$\mathrm{E}[\pi^c-\pi^*]=\mathrm{E}[\pi^c-\pi^0]-\mathrm{E}[\pi^*-\pi^0], \tag{4.18}$$

where π^0 represents the incumbent's profit at $k=k^0$. Recalling that k^0 represents the level of capital (chosen prior to the resolution of uncertainty) that minimizes total costs for the expected level of output, the first term of (4.18) represents the strategic gain from commitment. Proposition 3 shows that this term has positive expected value, even in the presence of uncertainty.

Proposition 3. $\mathrm{E}[\pi^c-\pi^0]>0$.

Proof. This result follows from a revealed preference argument. The firm could choose $k=k^0$ in the commitment regime giving rise to an expected profit $\mathrm{E}[\pi^0]$, but chooses $k^c>k^0$ achieving expected profit $\mathrm{E}[\pi^c]$ instead. □

[12]Since output is assumed positive in this section, we assume an internal equilibrium.

Now consider the second term of (4.18) representing the flexible timing effect. It captures the expected gain from installing the cost minimizing level of capital based on the actual output of the firm rather than on the expected level of output. If there were no uncertainty, there would be no benefit from flexibility and the term vanishes. It is convenient to break this term into two further parts. Letting $\pi' \equiv \pi(x^*, y^0, k^*)$, where $x(k^0, u) \equiv x^0$ and $y(k^0, u) \equiv y^0$, it follows that

$$\mathrm{E}[\pi^* - \pi^0] = \mathrm{E}[(\pi^* - \pi') + (\pi' - \pi^0)]. \tag{4.19}$$

The first term of (4.19) represents the effect on the incumbent's profit of a change in the entrant's output from y^0 to y^*, holding the incumbent's output at x^* and investment at k^*. The second term measures the gain from flexibility holding the entrant's output at y^0. By not allowing the entrant's output to vary with the incumbent's choice of capital, this latter term captures the pure effect of the flexible choice of capital on the cost of production. Not surprisingly it is positive for all non zero realizations of u.

Proposition 4. $\pi' - \pi^0 > 0$ *for all* $u \neq 0$.

Proof. From the mean value theorem using $\pi_x = 0$, we obtain $\pi' - \pi^0 = \pi_k(k^* - k^0)$ where π_k is evaluated at some $k \in [k^0, k^*]$. Since $\pi_{k^*} = 0$ from (4.16), it follows that $\pi_k > 0$ if $k < k^*$ and vice versa. Hence $\pi_k(k^* - k^0) > 0$ for all $u \neq 0$.

The first term of (4.19) can be usefully expressed in the form

$$\mathrm{E}[\pi(x^*, y^*, k^*) - \pi(x^*, y^0, k^*)] = \mathrm{E}[x^* p'(y^* - y^0)], \tag{4.20}$$

where $p' = p'(x^* + y)$ is evaluated at some $y' \in [y^*, y^0]$. Since y^* differs from y^0 only because k^* differs from k^0, this term represents the effect on the incumbent's profit of flexible choice of capital acting through induced changes in the output of the entrant. When the entrant's marginal cost is low ($u > 0$), lower investment by the incumbent in the flexible regime increases the output of the entrant reducing the incumbent's profits in the flexible regime and vice versa. Thus the term inside the expectation operator is negative when the output of the entrant is high and positive when the output of the entrant is low. This effect of investment by the incumbent on the entrant's output is not taken into account by the incumbent because of our assumption that each firm takes the output of the other firm as given in its second stage decisions.

In proposition 5, we develop an example in which the term $\mathrm{E}[\pi^* - \pi']$ is negative. For this example, we assume that $c(k)$ takes the form

$$c(k) = \alpha_0 + \alpha_1 e^{-\lambda k}. \tag{4.21}$$

Proposition 5. Assuming that the distribution of u is symmetric about $u=0$, $c(k)$ is given by (4.21), $\gamma(k)=\gamma^e=0$ and demand is linear, then $E[\pi^* - \pi'] < 0$.

Proof. Since $\pi^* = \pi'$ when $u=0$, it is sufficient to show that $\pi^* - \pi'$ is decreasing and strictly concave in u. From (4.13) and (4.20), we first obtain $\pi^* - \pi' = x^*[(c(k^0)-\alpha_0)-(c(k^*)-\alpha_0)]/3$. Since $c'(k) = -\lambda(c(k)-\alpha_0)$ (from (4.21)), it follows from (4.12), (4.14) and (4.16) that

$$c(k^0)-\alpha_0 = v/\lambda x(k^0, 0) \quad \text{and} \quad c(k^*)-\alpha_0 = v/\lambda x^*. \tag{4.22}$$

Then using (4.22), we can write $\pi^* - \pi' = (c(k^0)-\alpha_0)(x^* - x(k^0, 0))/3$. It remains to show that $dx^*/du < 0$ and $d^2x^*/du^2 < 0$. Using (4.22) and (4.13), we obtain

$$x^* = [\omega - u + \sqrt{\phi}]/6b, \tag{4.23}$$

where $\omega \equiv (a-\alpha_0)+(c^e-\alpha_0) > 0$, and $\phi \equiv (\omega - u)^2 - 24bv/\lambda$. From differentiation of (4.23), $dx^*/du = -x^*/\sqrt{\phi}$ and $d^2x^*/du^2 = -4(v/\lambda)\phi^{-3/2}$. □

Proposition 5 follows because the marginal cost of the incumbent decreases at a decreasing rate as capital investment is increased. The increase in marginal cost from the flexible use of capital in bad states (entrant's marginal cost is low) therefore tends to exceed the decrease in marginal cost in good states. On average the output of the entrant is increased by the flexible choice of capital[13] and, for this example, the effect is sufficient to make $E[\pi^* - \pi'] < 0$. However, this term can also be positive under some conditions.[14] There is a tendency for the term to be positive because changes in the entrant's output are weighted by the incumbent's output [see (4.20)] and generally a lower weight is given to the increase in the entrant's output in bad states than to the decrease in the entrant's output in good states.

Since $E[\pi^* - \pi']$ can be negative, the possibility arises that the incumbent could actually lose by adjusting its investment level in the flexible regime, relative to setting k^0 so as to minimize expected cost. Nevertheless, one might expect the benefit from actually matching investment levels to output levels [as represented by $E[\pi_k(k^* - k^0)]$] to become dominant as the shock u becomes more variable. That is, at a sufficiently high variance, the value of the flexible regime is likely to be positive and increasing in the variance of the

[13]The entrant's output is always positive if $u \geq (c^e - \alpha_0) - (a - \alpha_0)/2$.

[14]If $p = a - b(x+y)$, $C(x,k) = cx$ with $x = k$ and $C^e(y,u) = (c-u)y$, we obtain $E[\pi^*] = (a-c)^2/9b + \sigma^2/9b$ and $E[\pi'] = (a-c)^2/9b + \sigma^2/18b$, implying that $E[\pi^* - \pi'] = \sigma^2/18b > 0$. In this case $E[\pi^* - \pi^0] = \sigma^2/9b$ and $E[\pi^c - \pi^0] = (a-c)^2/72b$ so that, as in section 2, commitment is preferred if and only if $\sigma^2 < (a-c)^2/8$.

shock. In the particular case in which investment of capital has no effect on the flexibility of technology $[\gamma'(k)=0]$, it is easy to show (following on from proposition 2) that the gain from the commitment regime as represented by $\mathrm{E}[\pi^{\mathrm{c}}-\pi^0]$ is independent of the variance of the shock. The incumbent will then choose the flexible regime if the variance is sufficiently high.[15]

The comparison becomes more difficult, but perhaps more interesting, when capital investment influences the flexibility of technology. Proposition 6 reports some results for the specific case in which $\gamma(k)=\gamma_1 k$ and demand is linear. We consider the effect of a small increase in γ_1 on the choice between the two regimes starting from an initial situation where capital has no effect on the slope of marginal cost (i.e. $\gamma_1=0$).

Proposition 6. Assume $\gamma(k)=\gamma_1 k$, $\gamma^{\mathrm{e}}=0$, and linear demand, then:

(i) An increase in γ_1 always reduces investment by the incumbent: k^{c}, k^0 and k^* fall. If in addition, $c(k)$ is given by (4.21) and $\gamma_1=0$, the reduction in k^{c} exceeds the reduction in k^0.

(ii) An increase in γ_1 evaluated at $\gamma_1=0$ increases the expected profit of the incumbent at the commitment regime relative to the flexible regime.

Proof. (i) From (4.16), (4.12) and (4.15), we obtain $\partial(\pi_{\mathrm{k}})/\partial\gamma_1=-(x^*)^2/2<0$, $\partial(-\mathrm{E}[C_{\mathrm{k}}]/\partial\gamma_1)=-\mathrm{E}[(x^0)^2]/2<0$ and $\partial\mathrm{E}[\pi]/\partial\gamma_1=-\mathrm{E}[(x^{\mathrm{c}})^2](1/2+b^2/D)<0$. This implies that k^*, k^0 and k^{c} are all decreasing in γ_1. Using $c''(k)=-\lambda c'(k)$ [from (4.21)], and using (4.16), $x_{\mathrm{k}}=-2c'(k)/3b$, (4.13) and $\gamma_1=0$, we obtain $\mathrm{d}k^*/\mathrm{d}\gamma_1=-[\partial(\pi_{\mathrm{k}})/\partial\gamma_1]/\pi_{\mathrm{kk}}=3b(x^*)^2/2\lambda c'(k^*)(\omega-u)<0$ where $\omega\equiv(a-\alpha_0)+(c^{\mathrm{e}}-\alpha_0)>0$. Similarly, from (4.12) and (4.15), $\mathrm{d}k^0/\mathrm{d}\gamma_1=3b\mathrm{E}[(x^0)^2]/2\lambda c'(k)\omega<0$ and $\mathrm{d}k^{\mathrm{c}}/\mathrm{d}\gamma_1=15b\,\mathrm{E}[(x^{\mathrm{c}})^2]/8\lambda c'(k^{\mathrm{c}})\omega<0$. Using $v=4x(k^{\mathrm{c}},0)c'(k^{\mathrm{c}})/3=x(k^0,0)c'(k^0)$, it follows that $\mathrm{d}k^{\mathrm{c}}/\mathrm{d}\gamma_1<\mathrm{d}k^0/\mathrm{d}\gamma_1$.

(ii) From $\pi=b(x)^2-vk$ using (4.15) at $\gamma_1=0$ and $x_{\mathrm{k}}=-2c'(k)/3b$, we obtain $\mathrm{d}\pi^{\mathrm{c}}/\mathrm{d}\gamma_1=(-4x^{\mathrm{c}}c'(k^{\mathrm{c}})/3-v)(\mathrm{d}k^{\mathrm{c}}/\mathrm{d}\gamma_1)=(4u/9b)c'(k^{\mathrm{c}})(\mathrm{d}k^{\mathrm{c}}/\mathrm{d}\gamma_1)$, which has expected value zero. Similarly, $\mathrm{d}\pi^*/\mathrm{d}\gamma_1=(-4x^*c'(k)/3-v)(\mathrm{d}k^*/\mathrm{d}\gamma_1)$ reduces to (using (4.16) at $\gamma_1=0$) $\mathrm{d}\pi^*/\mathrm{d}\gamma_1=-x^*c'(k^*)(\mathrm{d}k^*/\mathrm{d}\gamma_1)/3<0$. Also, $\mathrm{d}\pi^0/\mathrm{d}\gamma_1=-c'(k^0)(x(k^0,0)-4u/3b)(\mathrm{d}k^0/\mathrm{d}\gamma_1)$, which has a negative expected value. It follows that $\mathrm{d}\mathrm{E}[\pi^{\mathrm{c}}-\pi^0]/\mathrm{d}\gamma_1>0$ and $\mathrm{E}[\mathrm{d}(\pi^{\mathrm{c}}-\pi^*)/\mathrm{d}\gamma_1]>0$. □

Proposition 6 shows first that an increase in the flexibility-reducing effect of capital investment causes the quantity of committed capital to fall. Surprisingly, however, the attractiveness of the commitment regime improves relative to the flexible regime. As shown in the proof, an increase in γ_1 actually has no effect on expected profit at the committed equilibrium. The

[15]Note that variance can be increased up to a point while keeping $\underline{u}$ and $\bar{u}$ constant. If we imagine increasing the support of u, then we would also have to consider 'shutdown' possibilities, as in subsection 4.5.

lowered investment does reduce profit in good states, but this is exactly offset by the improvement in profit in bad states. The result then follows because profit is reduced in the flexible regime. Conversely, this implies that an increase in the flexibility-enhancing effect of capital investment does not necessarily make capital commitment more attractive. Waiting to respond with a capital investment that is tailored to the equilibrium level of output may become even more advantageous.

4.5. Entry deterrence

If there are some states of the world in which the entrant will choose not to produce, then entry deterrence considerations will affect the incumbent's choice between commitment and flexibility. In a world of certainty our model gives rise to three possibilities. First, unconstrained cost-minimizing monopoly behaviour by the incumbent might be sufficient to deter entry. Second, the incumbent may find it necessary and desirable to commit excess capital so as to deter entry. Third, the incumbent may find it more profitable to allow entry than to deter the entrant, probably carrying excess capacity so as to manipulate the output equilibrium. The first case is often called 'blockaded entry', the second we refer to as 'active entry prevention', and the third as 'manipulated entry'. Under uncertainty these three cases exist, but a fourth case, 'probabilistic entry deterrence' (examined in section 2 for the case of output commitment) is also a possibility. We outline below how each case emerges under uncertainty in our model.

Case 1. Blockaded entry. The conditions for blockaded entry are extreme but simple. If, in the best state of the world for the entrant ($u=\bar{u}$), the corresponding output equilibrium of the flexible Cournot regime yields negative profits to the entrant, then the entrant would not enter under any circumstances (i.e., for any other realization of u or for the commitment regime). In this case the incumbent can act as an unconstrained monopolist, and will set its capital at the cost-minimizing level after the resolution of uncertainty.

Case 2. Active entry prevention. This case arises if, in the flexible Cournot output equilibrium associated with the entrant's best state of the world, $\bar{u}$, the entrant earns non-negative profits, but would always choose not to produce at some level of committed capital It is also required that the incumbent earn higher expected profits through entry deterrence than by allowing entry. Active entry prevention of this type is a possibility in this setting in contrast with the model considered in section 2. In section 2, the assumption of identical costs and a common shock to demand made it unprofitable for a committed leader to deter entry with certainty.

Case 3. Manipulated entry. Both blockaded entry and active entry prevention ensure that entry will never take place. The other extreme is one in which entry cannot be prevented by rational action. This occurs if the commitment regime together with the worst realization of u (for the entrant) imply an output equilibrium with non-negative profits for the entrant. The incumbent may choose to pre-commit in this case, and will use excess capital if it does, or it may opt for the flexible Cournot regime. This is the case analyzed in sections 4.1–4.4.

Case 4. Probabilistic entry deterrence. This is a case which arises only under uncertainty. It includes all situations not covered by cases 1, 2, and 3. The condition for entry deterrence is that the entrant's profit be negative. The boundary of this condition (where $\pi^e=0$) defines a trade-off between a critical value of u, denoted $u^p(k)$, and the committed level of capital, k: The higher the level of incumbent's capital, the more favourable must be the state of the world at which the entrant will just earn zero profits if it enters. Thus an increase in the level of committed capital causes 'probabilistic entry deterrence'. We denote the probability that the entrant will choose not to produce by $q(k)\equiv\int_{\underline{u}}^{u^p(k)} f(u)\,\mathrm{d}u$ where $f(u)$ is the density function of u.

If there is no entry, the incumbent sets the monopoly level of output for the committed level of k, denoted $x^m(k)$. The stage 1 expected profit of a committed incumbent can then be written as

$$\mathrm{E}[\pi^c]=q(k)\pi(x^m(k),0,k)+\int_{u^p(k)}^{\bar{u}}\pi(x^c(k,u),y^c(k,u),k). \qquad (4.24)$$

As shown by (4.24), the expected profit of the incumbent depends on three basic elements: Its profit should it be an ex post monopolist, its profit supposing that the entrant enters and the probability of these two events. Commitment of capital has an impact on the expected profit of the incumbent through its effect on the first two elements. With respect to the last element, we find that the marginal effect of an increase in k on u^p (increasing the probability of shutdown) actually has no effect on expected profit.[16] This follows because at the borderline value of $u=u^p$, the incumbent earns the same profit whether or not entry is deterred. Thus, as was the case with commitment of output in section 2, the effect of capital investment in causing probabilistic entry deterrence does not provide a motive for increased commitment of capital. Moreover, when the possibility of shutdown is introduced by widening the support of u, the increase in variance tends to favour the flexible regime.

[16]Differentiating (2.24) with respect to u^p using $\mathrm{d}q/\mathrm{d}u^p=f(u^p)$, $x(k,u^p)=x^m(k)$ and $y(k,u^p)=0$, we obtain $\partial\mathrm{E}[\pi^c]/\partial u^p=\pi(x^m(k),0,k)f(u^p)-\pi(x^c(k,u^p),y^c(k,u^p),k)=0$.

5. Concluding remarks

The lost 'option value' associated with commitment strategies is a potentially important part of economic decision-making. Specifically, there may be a trade-off between commitment and flexibility that would lead firms (or other economic agents) to forego the opportunity to undertake strategic pre-commitment. Since the sequence of moves is an important part of market structure and an important determinant of market performance, it follows that structure and performance are influenced by the characteristics of this trade-off between flexibility and commitment. We show that a potential Stackelberg leader might choose to give up the advantages of Stackelberg leadership if uncertainty is significant, preferring to wait until uncertainty is resolved, even if this means acting on an equal footing with its rival. If both firms are allowed the opportunity to pre-commit output, we find that firms might be trapped in a low-level equilibrium in which both act before the resolution of uncertainty, even though this lowers industry profits. It follows immediately that this outcome is also bad for consumers, so the outcome might be characterized as a 'rat race' in which firms are induced to act too quickly from the social point of view, and from the industry point of view.

We also characterize the trade-off between flexibility and pre-commitment for the case of capital pre-commitment by an incumbent confronted with the possibility of entry by a firm with uncertain costs. The cost of pre-commitments has the cost that arises from having a capital stock that is the wrong size for the output the firm will produce as a result of the realization of the entrant's cost uncertainty. This means that the incumbent may choose to act as a flexible Cournot firm if the variance of cost is high. We also consider investment that change the slope (as well as the level) of marginal cost, and obtain the surprising result that an increase in the flexibility-enhancing effect of investment, while increasing the desired amount of capital that would be invested if the incumbent chose to pre-commit, may actually make the overall strategy of commitment less attractive relative to the alternative of delaying investment and taking part in a simultaneous game with the entrant. Finally, we categorize the various entry deterrence possibilities that arise in the presence of uncertainty.

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