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ABSTRACT

This paper investigates the effect of endogenous horizontal product differentiation on trade patterns and the gains from trade under Bertrand and Cournot oligopoly. Firms differentiate their products to mitigate competition, but only if the investment required is not too high. Investment in product differentiation takes place in a much wider range of cases and results in a greater difference between products under Bertrand than Cournot competition. In our model, trade in homogeneous products never takes place under Bertrand competition: Bertrand firms export only if they differentiate their products. Cournot firms may trade in either homogeneous or differentiated products. If there is trade, consumers tend to be better off with Bertrand than Cournot competition due to greater product differentiation and more aggressive pricing, but higher levels of investment can raise Bertrand profit above Cournot profit and also above the monopoly profit at autarky when investment costs are sufficiently low.

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1. Introduction

In the 1980s, the role of imperfect competition in international trade emerged as a central focus of attention in international economics. This development allowed international trade theory to address important empirical realities, including the extent of intra-industry trade and the associated implications for trade policy. Work on imperfect competition in international trade was channeled into two distinct streams, however, depending on whether the assumed form of imperfect competition was monopolistic competition or oligopoly.[1]

Strikingly, the role of product differentiation has been treated very differently in these two research streams. Following the pioneering work of Krugman (1979, 1980), the analysis of international trade based on monopolistic competition treats product differentiation as a fundamental determinant of trade patterns and source of gains from trade. In this literature consumer demand is typically represented by Dixit-Stiglitz preferences (Dixit and Stiglitz (1977)). Such preferences imply that product differentiation is horizontal as consumers have a taste for variety but no one variety is intrinsically superior to another.[2]

The early work on oligopoly in international trade, such as Brander (1981) and Brander and Krugman (1983), abstracts from within-industry product differentiation entirely, focusing instead on intra-industry trade involving cross-hauling of homogeneous products. A substantial literature

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[1] Neary (2010) refers to the oligopoly version as only "half a theory" of international trade due to its use of partial equilibrium analysis (or very simple general equilibrium models). More complete general equilibrium models of oligopolistic trade include Lahiri and Ono (1995) and Neary (2009). See also Section 4.1 of Etro (2014) which outlines a model structure that nests perfect competition, monopolistic competition and oligopoly in a general equilibrium framework with international trade, and Bernhofen (2001) which integrates monopolistic competition and oligopoly into a single framework.

[2] Melitz (2003) provides a highly influential analysis of trade under monopolistic competition with firm-level heterogeneity induced by productivity differences among firms. Furthermore, recent work, such as Bertoletti and Etro (2013), allows for vertical (product quality) choices in models of trade with monopolistic competition.

dealing with trade in vertically differentiated products under oligopoly did develop, including Shaked and Sutton (1984), Motta (1994), and Zhou, Spencer and Vertinsky (2002), among others. In such work different firms produce goods of different quality.

In this paper we analyze the role of endogenous *horizontal* product differentiation in trade under oligopoly. One main objective is to investigate whether endogenous horizontal product differentiation in an international oligopoly context is a potentially significant determinant of the pattern of trade and source of gains from trade. The second main objective is to compare the consequences of Bertrand and Cournot oligopoly for product differentiation decisions and the resulting trade and welfare effects.[3]

We are not the first to consider horizontal product differentiation in international oligopoly. Such differentiation is incorporated in Bernhofen (2001) but is exogenously given rather than being chosen endogenously by firms. As we show here, allowing for firms to choose differentiation investments has major consequences. The closest paper to ours is Bastos and Straume (2012), which builds on the general equilibrium Cournot oligopoly model developed by Neary (2003, 2009) and allows for endogenous horizontal product differentiation. Bastos and Straume (2012) does not analyze the Bertrand model, however, so it does not contain comparative results. Also, that paper assumes an interior solution in which Cournot firms always differentiate their products. Our formulation allows for the important possibility that Cournot firms will engage in intra-industry trade in homogenous products.

[3] As shown in Eaton and Grossman (1986), among others, the form of oligopoly has important consequences for oligopoly behavior in an international context and for the associated incentives for trade policy.

One empirically relevant implication of our analysis is that firms engaged in Bertrand competition are much more likely to undertake product differentiation than firms engaged in Cournot competition. Bertrand firms differentiate their products in a wider range of cases than do Cournot firms and, if differentiation takes place, variety as measured by a lower substitutability of products is always greater under Bertrand competition. In our model, trade in homogeneous products never takes place under Bertrand competition. Bertrand firms will either differentiate their products or they will not export. Cournot firms, however, may trade in either homogeneous or differentiated products.

Section 2 describes the basic model structure. Section 3 deals with product differentiation and intra-industry trade in the Bertrand model, and Section 4 considers the Cournot model. Section 5 compares the two sets of results. Section 6 then examines the gains from trade and Section 7 contains concluding remarks.

2. Basic Model Structure

We consider a duopoly model in which each firm is based in a different country. The oligopoly model is similar to Brander and Spencer (2015). The innovation in this paper is to consider an international context that incorporates an export decision. Each firm has a sequence of three decisions to make: the export decision in stage 1, the product differentiation decision in stage 2 and the Cournot output or Bertrand price decision in stage 3. The two firms act simultaneously at each of these three decision stages. The stage 3 price (or quantity) decisions are made separately for each country. Thus our model is what has been referred to as a *reciprocal markets* model (Brander 1995) or a *segmented markets* model (Helpman 1987).

In the first stage each firm decides whether to export and, if it decides to export, pays some up-front fixed trade cost. We have in mind that a firm must invest in a distribution system in the export market. This cost might be very small. Possibly all that is needed is to take the time and effort to conclude an agreement with a local distributor in the export market. But the cost is strictly positive.

In the second stage each firm decides on whether and how much it wishes to invest in differentiating its product from the rival's product. One possibility is to interpret the differentiation investment as an advertising cost aimed at making the product more distinct from the other product in the eyes of consumers. For example Coke and Pepsi engage in extensive advertising campaigns to differentiate their products. Many customers cannot distinguish between the products in blind taste tests but exhibit strong loyalty to one product or the other, presumably induced by advertising or by cosmetic variations in things such as bottle design or logos. Another possibility is to interpret the investment as the cost of changing some physical characteristic of the product that differentiates it from the other product, as when car manufacturers adopt new colors and or new body shapes for cars or undertake other differentiation activities of a costly but essentially horizontal nature.

One issue concerns whether differentiation expenditures are country-specific or whether they apply across countries. For example, if firms invest in local television advertisements in each country or create local product variations in color or design then the differentiation investment is country specific. Alternatively, differentiation expenditures for basic product design might apply equally to both countries. Quite possibly both types of differentiation investments might be relevant in a given case. However, to keep things as simple as possible, we assume that differentiation expenditures are country-specific.

Finally, in the third stage, each firm decides on its price (in the Bertrand case) or its quantity (in the Cournot case) in each of the segmented markets, domestic or foreign, that it is engaged in. If there is no trade, each firm charges a monopoly price in its domestic market.

2.1 Demand

We start by considering Country 1. Firm 1, located in Country 1, produces quantity x_1 in Country 1 and firm 2, located in country 2, produces and exports quantity x_2 to Country 1. Goods x_1 and x_2 can range between being perfect substitutes (homogeneous) to being totally unrelated. The aggregate or representative utility function is the same in both countries and is taken to be

$$U = a(x_1 + x_2) - \tfrac{1}{2}(x_1^2 + x_2^2) - sx_1x_2 + M \tag{1}$$

where M is consumption of a numeraire good. This commonly used quasi-linear utility function rules out income effects of demand.[4] The parameter s represents the degree of substitutability between products x_1 and x_2.

The feasible range for s is between 0 and 1. If trade leads to sales by both firms in the same country, then s = 0 captures the extreme case in which product differentiation is sufficiently large to make the demand for each product independent, giving each firm a monopoly over its own product. If $s = 1$ these goods are perfect substitutes and are, in effect, identical or homogenous. To measure the degree of differentiation, we define a parameter $v = 1 - s$ (v for "variety") where $0 \leq v \leq 1$. However, it is convenient to use s in the specification of the demand structure, yielding the following inverse demand functions:

[4] In general income effects may be important in explaining the empirical pattern of markups across countries and over the business cycle, as in Bertoletti and Etro (2013).

$$p_1 = \partial U/\partial x_1 = a - x_1 - sx_2.$$

$$p_2 = \partial U/\partial x_2 = a - x_2 - sx_1. \tag{2}$$

If the firm in one of the countries decides not to export then only the domestically produced good in the other country is available and inverse demand is $p_i = a - x_i$, where i refers to the domestic firm. For country 1, if $x_2 = 0$, then consumers in country 1 have no choice of variety and inverse demand is $p_1 = a - x_1$.

Firms can choose to increase the degree of differentiation between products, or equivalently, increase variety, by making differentiation investments. The combined effect of the differentiation investments of both firms determines the value of s. We make the simplifying assumption that the differentiation investment affects only the degree of differentiation (or the degree of substitutability) with no effects on other aspects of demand.

One characteristic of the utility function is that, other things equal (i.e. holding quantities x_1, x_2 , and M constant), utility is strictly decreasing in s if x_1 and x_2 are positive: $\partial U/\partial s = - x_1x_2 < 0$. As increases in s reduce product differentiation, this implies that utility is increasing in the extent of product differentiation.

The aggregate utility function given by (1) implies that consumers have an aggregate taste for variety. However, investment in differentiation has no vertical component – it does not make the product better in any absolute sense. Thus consumers might be happier if they can purchase clothing decorated by both the Nike "swoosh" and Adidas stripes rather just one variety, but neither variety is intrinsically better than the other. We acknowledge that vertical differentiation (trying to produce a better product than the rival) is important in practice and that cases of pure horizontal differentiation might be rare. However, our objective here is to abstract from vertical differentiation so as to focus on the implications of horizontal product differentiation.

2.2 Costs

There are three types of cost to consider: the cost of production, the cost of differentiation and the cost of exporting. For simplicity, we assume that output is produced at constant marginal cost, c. For production to take place, the maximum willingness to pay, a, must exceed c:

$$a > c. \tag{3}$$

The second type of cost is the investment required to differentiate the products, which might be advertising costs or product development costs. These investment amounts for the two firms are denoted k_1 and k_2. Using K to represent combined differentiation expenditures, $K = k_1 + k_2$, if both firms supply the market (one through exports), we model the effect of differentiation investments on the degree of differentiation experienced by consumers using the following convenient functional form:

$$v = 1 - s \text{ where } s = 1/e^{\beta K} = e^{-\beta K} \tag{4}$$

If neither firm invests in differentiation, then $K = 0$, $s = 1$ and variety $v = 0$, so the products are effectively identical. If at least one of the firms invests in differentiation then $v > 0$ and the products are differentiated. From (4), an increase in differentiation investment by either firm reduces the degree of substitutability and increases variety v. Variety is zero if $K = 0$ and approaches 1 as the differentiation investment approaches infinity. As previously mentioned, if there is only one firm in the market (a domestic monopoly), then consumers do not have the benefit of variety and $s \equiv 0$. With only one product, investment cannot increase demand by making products more different.

The exponential functional form is appealing for several reasons. First, it has the empirically desirable property that s must be between 0 and 1. Second, it has the plausible property that there are decreasing returns to investment in the sense that it takes larger and larger increments in investment to achieve a given increase in v. However this specific functional form is not essential. If, for example, instead of (4), we assume a power function: $s = (1 + k_1 + k_2)^{-\beta}$, which is very general, or if we use the first two terms of a Taylor series approximation (i.e. up to the quadratic term), we can obtain essentially the same results, although with more computational difficulty.

The parameter β indicates the effectiveness of investment in achieving differentiation. If $\beta = 0$ then $s = 1$ no matter how much differentiation investment is undertaken and the products remain identical. Differentiation is simply not possible if $\beta = 0$. Since negative values of β are not meaningful, we assume $\beta > 0$. Larger values of β imply a greater amount of differentiation (i.e. a lower s) for any given differentiation investment K.

There is also a fixed cost of exporting, E. This cost must be paid in the first stage, before country-specific differentiation investments are made and before output is sold. We assume that both firms face the same export cost. In much of our analysis we consider the limiting case in which exports costs are very small (approach zero) but are strictly positive.

The equilibrium concept used is the subgame perfect Nash equilibrium (SPNE). Thus we start by focusing on the final stage, when firms simultaneously choose prices (Bertrand) or quantities (Cournot). We solve for the final stage equilibrium conditional on s, then consider the second stage decision of how much to invest in differentiation, followed by the first stage decision of whether to pay the fixed cost E required in order to export.

3. Bertrand Competition

3.1 *Final Stage - Pricing Decisions*

If both firms have decided to export, then stage 3 is a Bertrand duopoly in each country. Each firm sets its price for each country to maximize profit, treating the other firm's price in that country as outside its control (exogenous) and treating stage 2 differentiation investments (k_1 and k_2) and therefore s as predetermined. If a firm has decided not to export to a given country then the domestic firm in that country acts as a monopolist.

Due to constant marginal costs of production, what happens in one country does not affect cost in the other country. As a result, the markets in the two countries are independent at stage 3 and it is sufficient to analyze just one market. For a given market, final stage (variable) profit for firm i, denoted V_i , is

$$V_i \equiv (p_i - c)x_i \tag{5}$$

where $i = 1,2$. The cost of differentiation, k_i, and the export cost, E ,are not included in (5) since these costs are sunk by the time firms reach the final stage.

We first examine the case in which the two firms compete due to the decision to export by one of the firms. It is convenient to convert the inverse demand functions given by (2) to direct form. Provided goods are differentiated ($s < 1$) these demand functions are:

$$x_1 = [(a - p_1) - (a - p_2)s]/(1 - s^2);$$

$$x_2 = [(a - p_2) - (a - p_1)s]/(1 - s^2). \tag{6}$$

For homogenous products ($s = 1$), consumers will buy from only one firm if that firm charges a strictly lower price and firms will share the market if they charge the same price.

If $s < 1$, then maximizing variable profit (5) using (6), Bertrand equilibrium prices and quantities are as follows:

$$p = p^B(s) = (a\text{-}c)(1\text{-}s)/(2\text{-}s) + c \tag{7}$$

$$x = x^B(s) = (a\text{-}c)/[(2\text{-}s)(1+s)] \tag{8}$$

$$V = V^B(s) = (1\text{-}s^2)(x^B(s))^2 \tag{9}$$

If $s = 1$ (homogeneous products), then (7) reduces to $p = c$ and (8) to $x = (a\text{-}c)/2$ and variable profit is zero, which is the standard Bertrand solution with homogeneous products. As might be expected, increases in differentiation (higher v or lower s) cause prices and variable profit to rise. Interestingly, however, output is a quadratic function of differentiation that reaches its minimum at $v = ½$ implying that the lowest level of output is achieved at an intermediate level of differentiation. (See Brander and Spencer, 2015.)

3.2 *Second Stage – Investments in Product Differentiation*

In stage 2 the decision of whether to pay the export fee is predetermined and each firm chooses its differentiation investment anticipating the final stage equilibrium that will emerge from any set of second stage decisions. For a market in which there is Bertrand competition due to a decision to export in stage 1, the second stage profit for firm i for i = 1,2 can be written as:

$$\pi_i = V^B(s) - k_i = (1\text{-}s^2)(x^B(s))^2 - k_i \tag{10}$$

where s and $v = 1\text{-}s$ depend on k_1 and k_2. Export cost E is a sunk cost at this stage and is therefore not included in (10).

In setting k_i, each firm i for $i = 1, 2$ correctly anticipates the effect of k_i on its profit as in (10), but takes the investment of the other firm as fixed. Simultaneous decisions regarding differentiation investments k_1 and k_2 jointly determine $K = k_1 + k_2$, which in turn determines the degree of differentiation, $v = 1\text{-}s$ where $s = e^{-\beta K}$ (from (4)). As previously noted, β measures the effectiveness of differentiation expenditures in achieving product differentiation. The partial effect of each firm's investment on s, taking the investment of the other firm as given, reduces substitutability (and increases the degree of differentiation).

$$\partial s/\partial k_1 = \partial s/\partial k_2 = \mathrm{d}s/\mathrm{d}K = -\beta e^{-\beta K} = -\beta s < 0 \tag{11}$$

From (10) using (11), the first order condition for an interior solution ($k_i > 0$) to firm i's profit maximization problem is

$$\partial \pi_i/\partial k_i = (\mathrm{d}V^B/\mathrm{d}s)(\partial s/\partial k_i) - 1 = 2\beta s(x^B(s))^2(1 - s + s^2)/(2\text{-}s) - 1 = 0 \tag{12}$$

A corner solution in which the firm spends nothing on differentiation arises if $\partial \pi_i/\partial k_i \leq 0$ at $k_i = 0$ in which case the firm chooses not to differentiate.

Because s is a nonlinear function of k_i the first order conditions do not readily yield closed form solutions for k_1 and k_2. However, the important properties of the solution can be determined. In particular, we can identify the threshold level of differentiation effectiveness β below which product differentiation would not occur.

Proposition 1: Suppose there are two firms in a market due to a decision by a foreign firm to export to that market in stage 1. In anticipation of Bertrand competition at stage 3, both firms will choose to differentiate their products at stage 2 if and only if $\beta > 2/(a\text{-}c)^2$. If $\beta \leq 2/(a\text{-}c)^2$ then no differentiation investment takes place and products are homogeneous at stage 3.

Proof: No differentiation ($s = 1$) takes place if and only $k_1 = k_2 = 0$, which occurs if and only if $\partial \pi_i/\partial k_i \leq 0$ at $k_i = 0$, in which case $s = 1$ (i.e. at $v = 0$). Substituting $s = 1$ into (12) and using $x = (a\text{-}c)/2$ from (8) shows that $\partial \pi_i/\partial k_i = \beta(a\text{-}c)^2/2 - 1$, which is less than or equal to zero if and only if $\beta \leq 2/(a\text{-}c)^2$.***

It follows from Proposition 1 that if $\beta \leq 2/(a\text{-}c)^2$ and both firms are in the market then the solution would be the homogeneous product Bertrand outcome in which price equals marginal cost. Second stage profit given by (10) is zero in this case as there are no differentiation costs.

The Nash equilibrium differentiation levels are not the levels that would maximize joint profits. There is an "under-investment" in differentiation due to a positive externality associated with differentiation expenditures. If firm 1 undertakes additional differentiation expenditures, this provides benefits to both firms. In our structure, it provides equal benefits to both firms. But firm 1 cares only about its own profit and hence will invest too little to maximize industry profit.

3.3 *First Stage – Export Decisions*

In the first stage each firm must make the decision of whether to pay export cost E to export goods to the foreign market. This is a relatively simple decision as each firm will pay export costs only if its second stage profit given by (10) is greater than or equal to export cost E. However, if firms do not differentiate their products then second stage profit given by (10) is zero and therefore cannot cover the strictly positive export cost. It follows that neither firm would pay the export cost in this case and no exports would occur.

Proposition 2: Exports will never occur under homogeneous product Bertrand oligopoly and will therefore never occur if $\beta \leq 2/(a\text{-}c)^2$.

If $\beta > 2/(a\text{-}c)^2$ exports may occur, depending on the size of E. To see the role of β as clearly as possible it is useful to consider the limiting case in which E approaches zero (but remains strictly positive).

Proposition 3: In the limiting case in which export costs approach zero, products will be differentiated and intra-industry trade will occur if and only if $\beta > 2/(a\text{-}c)^2$.

The logical foundation of Proposition 3 is that if $\beta > 2/(a\text{-}c)^2$, then each firm's variable profit under Bertrand competition at stage 3 exceeds its cost of product differentiation at stage 2. At stage 1, each firm will correctly anticipate the subsequent product differentiation and profit and will choose to pay a sufficiently small export cost. The same reasoning applies to both countries so we will observe intra-industry trade in this case. As we later show in section 6, profits might or might not be lower under intra-industry trade with Bertrand competition than if both firms act as monopolists in their home markets, but each firm faces an incentive to export to the other market and the Nash equilibrium therefore implies such exports.

4. Cournot Competition

4.1 *Final stage – Quantity Decisions*

We now consider the Cournot version of the model, in which firms simultaneously choose outputs in the final stage taking the output of the other firm as given. As in the Bertrand case, the markets in the two countries can be treated independently so we consider one country at a time. Maximizing variable profit (5) using the inverse demand functions (2), it can be shown that the Cournot equilibrium outputs, prices and variable profits are as follows:

$$x = x^C(s) = (a\text{-}c)/(2 + s) \tag{13}$$

$$p = p^C(s) = (a\text{-}c)/(2+s) + \mathrm{c} \tag{14}$$

$$V = V^C(s) = (x^C(s))^2 \tag{15}$$

Increases in differentiation cause outputs to rise, prices to rise, and variable profits to rise. The result that output is monotonically increasing in the extent of differentiation contrasts significantly with the Bertrand model, where output reaches its minimum at v = 1- s = ½. If v =

0, then products are homogenous and output and price are at the standard homogenous product Cournot levels: $x = (a\text{-}c)/3$ and $p = (a + 2c)/3$.

4.2 *Second Stage – Investments in Differentiation*

In the second stage each firm i seeks to maximize its profit, $\pi_i = V^C(s) - k_i$ where $s = e^{-\beta K}$ (from (4)), by choosing its differentiation investment k_i, taking as exogenous the differentiation investment of the other firm. The associated first order condition for an interior solution for firm i is given by

$$\partial\pi_i/\partial k_i = (\mathrm{d}V^C/\mathrm{d}s)(\partial s/\partial k_i) - 1 = 2\beta s(x^C(s))^2/(2+s) - 1 = 0 \qquad (16)$$

As in the Bertrand case, a corner solution with $k_i = 0$ is possible. This corner solution occurs if $\mathrm{d}\pi_i/\mathrm{d}k_i \leq 0$ at $k_i = 0$. Proposition 4 identifies the critical value of β needed for differentiation.

Proposition 4: Suppose there are two firms in a market due to a decision by a foreign firm to export to that market in stage 1. In anticipation of Cournot competition at stage 3, both firms will choose to differentiate their products at stage 2 if and only if $\beta > 13.5/(a\text{-}c)^2$. If $\beta \leq 13.5/(a\text{-}c)^2$ then no investment takes place and products are homogeneous at stage 3.

Proof: No differentiation ($s = 1$) takes place if and only if and only if $\partial\pi_i/\partial k_i \leq 0$ at $k_i = 0$ and $s = 1$ (i.e. at $v = 0$) for i = 1,2. Substituting $s = 1$ into (16) and using $x = (a\text{-}c)/3$ from (13) shows that $\partial\pi_i/\partial k_i \leq 0$ if and only if $\beta \leq 13.5/(a\text{-}c)^2$. ***

Proposition 4 provides a marked contrast with Proposition 1. The no-differentiation range for the Cournot model is much greater than for the Bertrand model. Thus Bertrand firms are much more likely to differentiate their products than Cournot firms.

4.3 *First Stage – Export Decisions*

In the first stage each firm decides whether to pay the fixed cost of exporting, E. Once again it is instructive to consider the limiting case in which the export cost, *E,* while strictly positive, approaches zero. Given that $a > c$, exports will always occur in the Cournot model, even when products are identical.

Proposition 5: For the limiting case in which *E* approaches zero, exports and hence intra-industry trade will always occur in the Cournot model. If $\beta \leq 13.5/(a\text{-}c)^2$, the products will be homogeneous. If $\beta > 13.5/(a\text{-}c)^2$, products will be differentiated.

5. Comparison of the Pattern of Trade and Product Differentiation

The propositions of sections 3 and 4 examine product differentiation and exporting patterns for Bertrand and Cournot oligopoly for different ranges of β. Combining these results, Table 1 provides a comparison between Bertrand and Cournot outcomes for the case in which export costs are very small.

TABLE 1: Pattern of Product Differentiation and Trade

Values of β	Differentiation Under Bertrand	Differentiation Under Cournot	Trade Under Bertrand	Trade Under Cournot
$0 \leq \beta \leq 2/(a\text{-}c)^2$	No	No	No	Yes
$2/(a\text{-}c)^2 < \beta \leq 13.5/(\text{a-c})^2$	Yes	No	Yes	Yes
$\beta > 13.5/(\text{a-c})^2$	Yes	Yes	Yes	Yes

If product differentiation is very costly ($0 < \beta < 2/(a\text{-}c)^2$), then neither Cournot firms nor Bertrand firms will differentiate their products. Bertrand firms will not export in this case as they anticipate zero variable profit in the final stage and would therefore suffer a loss equal to export cost*, E*. As a result, each firm will produce as a monopolist in its home market. However, Cournot firms may engage in intra-industry trade in identical products and will certainly do so if export costs are very small.

If product differentiation is very easy ($\beta > 13.5/(a\text{-}c)^2$), then from the third line of Table 1, both Cournot firms and Bertrand firms will differentiate their products and engage in intra-industry trade. In the intermediate case, Bertrand firms will differentiate their products, Cournot firms will produce identical products and both will engage in intra-industry trade.

6. The Gains from Trade

We now consider the gains from trade as measured by consumer surplus, profit and total surplus. We compare the Bertrand trading equilibrium and the Cournot trading equilibrium with the autarky outcome that would arise if some exogenous restriction prevented trade. For the parameter values that imply no trade under Bertrand competition, the outcome is the same as autarky.

Under autarky, each firm operates in just one country and has no incentive to differentiate its product from its rival's product. Using the superscript M to indicate monopoly outcomes the standard monopoly solutions apply:

$$p^M = (a + c)/2;\ x^M = (a - c)/2;\ V^M = (x^M)^2. \tag{17}$$

The determination of closed form solutions for the gains from trade in our model is not feasible given the nonlinearity of the product differentiation function. However, the pattern of gains can be readily inferred using simulation methods with particular parameter values. The general pattern is very robust and we present one illustrative example here. Specifically, we let $a = 14$ and $c = 2$, and we allow export costs to be very small (but still positive). It follows that the critical value of β that determines whether Bertrand firms would differentiate their products is $\beta^B = 0.014$ (rounded to three decimal places). If $\beta < 0.014$ then differentiation is too costly and Bertrand firms would not undertake differentiation expenditures. The critical value that determines whether Cournot firms would differentiate their products is approximately an order of magnitude larger at $\beta^C = 0.094$.

If differentiation occurs, the gains from trade depend on the value of β. We consider two specific values, $\beta = 0.05$ from the intermediate range in which Bertrand firms differentiate their products and Cournot firms do not, and $\beta = 1$ from the range in which both Bertrand and Cournot firms differentiate their products. We also include the case in which there is no product differentiation under either model ($\beta < 0.014$). The results are shown in Table 2 for the Bertrand model.

For each value of β, Table 2 sets out the total investment in product differentiation, *K*, the variety that is achieved as measured by *v*, the change in profit relative to autarky, the change in consumer surplus relative to autarky, and the change in total utility (which equals the sum of the change in profit and consumer surplus) relative to autarky.[5]

TABLE 2: Gains from Trade with Bertrand Competition

Values of β	Differentiation Investment, K	Differentiation, *v*	Profit Gains	Consumer Surplus Gains	Gains from Trade
$0 \leq \beta < 0.014$	0	0	0	0	0
$\beta = 0.05$	9.4	0.37	-10.3	28.9	18.6
$\beta = 0.1$	10.0	0.63	4.0	21.5	25.5

Table 2 shows that for low values of β (high differentiation costs) no differentiation occurs and therefore no trade occurs. The outcome is the same as under autarky. If differentiation costs fall somewhat (i.e. if β rises moderately) firms are drawn into competition with each other.

[5] As utility is quasi-linear with numeraire good M, one unit of utility is exactly equal to the value of 1 dollar spent on the numeraire good. Therefore surplus measures and utility coincide.

Bertrand firms differentiate their products to mitigate this competition, but profits are lower than under autarky. Consumer surplus rises substantially. Consumer surplus under autarky is not shown in the table, but it is 18, so trade more than doubles consumer surplus. Even after subtracting the loss of profit, the total gains from trade are substantial.

It is interesting that in the high β case, the Bertrand firms are actually better off in the trading equilibrium than when acting as monopolists in their home countries. Product differentiation is then sufficient to make prices high enough that profits at the cross-hauling equilibrium exceed autarky profits despite the effects of competition. Consumers still gain because they value variety.

Table 3 shows the corresponding results for the Cournot model.

TABLE 3: Gains from Trade with Cournot Competition

Values of β	Differentiation Investment, K	Differentiation, *v*	Profit Gains	Consumer Surplus Gains	Gains from Trade
$0 \leq \beta < 0.014$	0	0	-4.0	14.0	10.0
$\beta = 0.05$	0	0	-4.0	14.0	10.0
$\beta = 0.1$	4.6	0.37	1.0	15.9	16.9

In the Cournot case, for low and intermediate levels of β, no differentiation occurs so the equilibrium outcome involving intra-industry trade with cross-hauling of identical products is unchanged. In each case the firms lose by being drawn into competition with each other but consumers experience significant gains. Comparing the results of Table 3 with Table 2, for low values of β consumers are better off with Cournot competition, but for intermediate values of β consumers are better off in a Bertrand world as they get more differentiation (which is good) and relatively aggressive competition (which is also good for consumers). For high values of β, Cournot firms do just slightly better by differentiating their products and exporting than they would as monopolists under autarky.

In this paper, as in Brander and Spencer (2015), allowing for endogenous product differentiation may reverse the standard result that prices are lower under Bertrand oligopoly than under Cournot oligopoly. For a given common level of differentiation, the standard ranking applies, but the difference between differentiation levels under Bertrand competition and Cournot competition can be sufficiently great that Bertrand competition generates higher prices than Cournot competition.

7. Concluding Remarks

The primary objective of this paper is to investigate whether endogenous horizontal product differentiation has significant implications for trade in oligopolistic industries. We find that such product differentiation can have important consequences. In the Bertrand case, trade is ruled out unless products are differentiated. The reason is that firms must pay positive (albeit possibly very small) up-front fixed costs if they wish to export. If Bertrand firms anticipate producing homogeneous products they also anticipate earning zero variable profits from exporting and will therefore not pay the required fixed export costs. However, if Bertrand firms endogenously choose to differentiate their products, they will trade as long as trade costs are sufficiently small. Whether differentiation occurs depends on the effectiveness of differentiation expenditures (or, equivalently, on the cost of differentiation).

Cournot firms will export and engage in intra-industry trade even if they produce homogenous products as long as export costs are not too high. Although Cournot firms may also differentiate their products, we find that there is a large range of differentiation effectiveness over which Bertrand firms would differentiate their products but Cournot firms would not. In a closed economy version of this model Brander and Spencer (2015) offer similar reasoning to explain why homogenous product Bertrand competition is rarely observed, while homogenous product Cournot oligopoly is thought to be empirically relevant.[6]

The ability to engage in differentiation has a major effect on the gains from trade. With Bertrand oligopoly, there is no trade without differentiation. But, an increase in the effectiveness of differentiation (or a reduction in differentiation cost) above a threshold level induces trade and generates large gains from trade. With Cournot competition, there are gains from trade regardless of the cost of differentiation provided trade costs are small. If trade takes place, consumers tend to be better off with Bertrand than Cournot competition due to greater product differentiation and more aggressive pricing, but higher levels of investment can raise Bertrand profit above Cournot profit and also above the monopoly profit at autarky when investment costs are sufficiently low. Consumer surplus can nevertheless be higher under Bertrand competition because of the benefits from variety.

The analysis in this paper demonstrates that product differentiation plays a central role in determining the pattern of trade and the gains from trade under oligopoly, just as it does under monopolistic competition. However, the specific relationships between product differentiation and the pattern and gains from trade depend very much on the type of oligopolistic rivalry under consideration. Certainly the Cournot model and the Bertrand model yield substantially different effects. We would also expect that endogenous horizontal product differentiation has important implications for trade policy and multinational locations decisions, although those issues are not analyzed here.

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[6] See, for example, Slade (1995) and Carvajal et al. (2013).

The models developed in this paper deal with a simultaneous move strategic environment in which firms are symmetric. A variety of interesting issues would arise in a sequential move context. For example, many real examples would seem to correspond to a Stackelberg model in which a market leader has established its product characteristics and a follower makes later differentiation decisions. The possibilities of entry and entry deterrence would also seem to be potentially very interesting.[7] Other forms of asymmetry would also be potentially relevant, including variable trade costs, such as a per unit transport cost or an import tariff. However, while all of these extensions would be of interest, we believe that the basic insights developed in this paper would be robust.

[7] See Etro (2014) for a valuable review of free entry in oligopoly models as developed in the literature on endogenous market structure.

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