Hydraulic Geometry: Empirical Investigations and Theoretical Approaches

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Abstract

Hydraulic geometry equations relate the dimensions of the wetted channel to the stream discharge the channel conveys. One approach to hydraulic geometry considers temporal changes at a single location due to variations in discharge, and is referred to as at-a-station hydraulic geometry: another approach considers the spatial changes for a common discharge (such as the bankfull flow), and is referred to as downstream hydraulic geometery. Both are typically represented using empirically fitted power functions. In the first part of this chapter, the basic concepts are reviewed, and the physical basis for hydraulic geometry is presented. A set of reference equations describing the downstream scaling of Froude-similar channels is derrived: they are exact power functions of the form $P = \kappa_1 Q^{2/5}$, $R = \kappa_2 Q^{2/5}$ and $v = \kappa_3 Q^{1/5}$, where the coefficient values are determined by the channel shape, gradient and a flow resistance parameter. A review of the literature indicates that while at-a-station relations have mostly been used to assess aquatic habitat, most of

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the research on downstream hydraulic geometry has focussed on the factors determining the coefficients and exponents of the power functions or on the physical origin of the observed relations. Empirical studies of downstream relations have demonstrated that bankfull channel width generally increases at a rate slightly higher than suggested by Froude-scaling, while channel depth typically increases at close to the rate associated with Froude-scaling, but there are notable exceptions. Other key findings include identification of gradient, grain size and riparian vegetation type as important variables influencing downstream hydraulic geometry. Progress has been made in understanding these empirical observations by developing theoretical models of hydraulic geometry: in particular, the incorporation of grain size and bank strength has advanced our understanding. Based on this review of the literature, the future research directions for at-a-station relations relate to improving their applicability to the practical problems for which they are often used, while those for downstream relations relate to improving our ability to model downstream hydraulic geometry and incorporating those relations in models of landscape evolution.

Keywords: Downstream hydraulic geometry, At-a-station hydraulic geometry, Open channel hydraulics, Rational regime theory, Stream channel

1. Introduction

Consider the following two situations: in the first, an observer moves virtually instantaneously from the headwaters to the mouth of a river system while at the same time measuring the channel dimensions and average stream velocity at regular intervals along the river; in the second, an observer remains at a single point and repeatedly measures the channel dimensions and flow velocity over a relatively long period of time (say a decade or more). The observer traveling down the river system will most likely observe that, as one travels downstream, the channel becomes wider and deeper, and the average stream velocity increases: these changes occur because the volume of water carried by the stream increases as one moves downstream. This observer will also probably note that the channel gradient declines downstream as well, a phenomenon related to the processes of sediment erosion, transport and deposition. Similarly, the observer located at a single position within the river system is likely to observe that channel width, depth and velocity all change over time as streamflow increases or decreases. The channel gradient may fluctuate, but averaged over the channel reach, it will remain very nearly constant. Hydraulic geometry relations are quantitative representations of these very basic interactions between the size of the channel and volume of water that it is conveying. As a matter of computational convenience, these relations are usually expressed using simple power functions relating channel dimensions to some fractional power of stream discharge.

In the first example above (in which channel dimensions change downstream), hydraulic geometry relations are determined by the way in which self-formed river systems are established and maintained via erosion, transport and deposition of sediment by the water flowing through the system. These relations are associated with timescales at which river systems are able to adjust to environmental and tectonic forcing (typically on the order of centuries to millennia, depending on the spatial scale of the river system). In the second example (wherein channel dimensions change over time at a single channel reach) the hydraulic geometry of the stream depends on channel geometry and the way in which flow resistance changes as different parts of the channel become submerged. For these sorts of hydraulic geometry relations to hold, the channel morphology must remain stable and the processes of sediment erosion, transport and deposition can therefore be ignored. The associated timescales are those over which the channel morphology remains static (typically years to decades). There are clearly important differences between hydraulic geometry relations describing spatial variations and those describing temporal ones, and it is important to make a clear distinction between these two approaches.

The site-specific (i.e., temporal) relations are referred to as at-a-station hydraulic geometry relations, and are typically used to assess changes in water level, the area of the channel that is inundated at a given flow, as well as the average flow velocity for a range of discharges. These kinds of relations are particularly useful to biologists assessing aquatic ecosystem dynamics in general and fish habitat in particular, which depends on both the distribution of water depths and the distribution of local stream velocities. They may also be of use to engineers designing bridges that cross a stream channel or attempting to situate infrastructure above the expected high-water level for a given design flood. If channel geometry is stable, then at-a-station relations are determined entirely by the way in which flow resistance changes with discharge.

Downstream (i.e., spatial) hydraulic geometry relations usually relate average channel geometry to a formative discharge (which is often taken to be the bankfull flow or the mean annual peak flow). For these relations, the channel geometry is not assumed to be static, it is assumed to be the result of a dynamic equilibrium established between the channel morphology and the formative discharge. These relations thus reflect the interaction between sediment transport dynamics and flow resistance at the formative flow. These relations are expressions of how a particular river system is structured and how the channel network has evolved.

Downstream hydraulic geometry relations have been applied in a variety of contexts. They have been used to estimate design channel dimensions in stream restoration projects. They have been used to predict the potential response of stream channels to changes in the formative discharge due, for example, to flow regulation and/or abstraction. Downstream hydraulic geometry relations have also been used in landscape evolution models such as LandMod (Martin, 2000) and CHILD (Tucker et al., 2001) to estimate the channel dimensions and thus the sediment transport capacity along the simulated channel network: in this context, downstream hydraulic geometry relations are effectively acting as 'black-box' models describing channel dynamics.

What follows is a review of the conceptual underpinnings for developing hydraulic geometry relations, as well as of the historical context within which the basic ideas evolved. Then, the more recent research on the topic is reviewed and synthesized. The chapter concludes with a summary of the major issues and future research areas that remain to be addressed.

2. Conceptual Basis for Hydraulic Geometry

2.1. Regime Relations for Unlined Canals

While the term hydraulic geometry was first used by Leopold and Maddock (1953), the approach originated with the study of unlined irrigation canals. The first systematic analysis was conducted on canal systems in India by Kennedy (1895) who observed that *stable* canals—those that were able to transport the imposed sediment loads with the available discharge while neither aggrading nor degrading—exhibited a relation between velocity and depth of the form:

$$v = \alpha d^{\eta} \tag{1}$$

where v is the mean flow velocity and d is the mean hydraulic depth of the canal. The coefficient α and exponent η were found to be site-specific parameters. This relation is a kind of site-specific, empirical flow resistance law.

However, this relation ignores the potential relevance of additional parameters such as channel gradient (S). Lindley (1919) identified this issue, and proposed the concept of "regime", wherein the canal geometry is adjusted to some stable configuration that, while modified locally, does not change detectably over time.

The early work was consolidated by Lacey (1930), who attempted to generalize the site-specific, empirical equations developed for canals in India by introducing a variable to account for the composition of the boundary materials that he called the silt factor, F, where $F = 8D^{1/2}$, and D is the bed material particle diameter, measured in inches. He attempted to define generally applicable relations between the adjustable quantities for canal geometry (comprising the wetted perimeter, P, the hydraulic radius, R, and the canal gradient, S)[1] and the governing conditions of discharge, Q and the silt factor, F. Blench (1969) further developed this method by defining separate silt factors describing the bed, F_b, and the bank, F_s, so as to accommodate a wider range of canal configurations. The sets of equations presented by Lacey and Blench both predict general channel scaling relations of the form:

[1]in geomorphology, the wetted width and mean hydraulic depth are often used in preference to the nearly equivalent variables P and R.

$$
\begin{aligned}
W &\propto Q^{1/2} \\
d &\propto Q^{1/3} \\
S &\propto Q^{1/6}
\end{aligned}
\tag{2}
$$

where W is the width of the water surface and d is the mean hydraulic depth for the corresponding discharge, Q. Lacey (1930) originally selected simple exponents that were consistent with the data for his analysis, since he thought that these were most likely to represent physically based, general relations.

2.2. Adapting Regime Relations to Alluvial Streams

In their pioneering work, Leopold and Maddock (1953) adapted the ideas from regime relations for canals to the description of natural stream channels. The key impediment to this is the much larger range of flows experienced in natural channels and the paucity of accurate measurements of channel dimensions for a given flow: generally, researchers have been effectively restricted to the consideration of streams near gauging sections, which are chosen for their relative stability and which are not necessarily representative of typical reach conditions. Furthermore, alluvial rivers, while frequently modified by human activity, are fundamentally self-formed systems: canals are not, since the canal gradient (if not the stable canal width and depth) is imposed by the canal design.

Leopold and Maddock (1953) developed both at-a-station and downstream hydraulic geometry relations and other researchers have similarly pursued two distinct sets of hydraulic geometry. The at-a-station relations are typically constructed for flows less than or equal to the bankfull discharge. Since bed material transport will be negligible over most of this range, the channel cross section can be considered to be stable and indeed the cross sectional shape, combined with the associated flow resistance law, is the primary determinant of the form at-a-station hydraulic geometry relations, a point made very clearly by Church (1980) and Ferguson (1986).

The downstream relations are directly comparable to the earlier work on canals. While downstream relations can be constructed for almost any flow frequency, it has become common practice to use them to describe the relation between the bankfull channel geometry and the bankfull discharge, which is taken to represent the formative discharge. This is directly analogous to invoking the concept of regime articulated by Lindley (1919). Unlike the at-a-station relations, downstream hydraulic geometry is not determined by a static cross section shape, but by the way in which the boundary conditions such as bed and bank stability influence the stable channel form notionally produced by a formative flow. Downstream relations are thus strongly linked to the development of river grade, since channel stability is influenced by channel slope and characteristic grain size, both of which are important elements of river grade.

2.3. Empirical Hydraulic Geometry

Much of the research on hydraulic geometry, including the original analysis by Leopold and Maddock (1953), is based entirely on empirical analysis of the available streamflow data. For these kinds of empirical analyses, steady state continuity is the governing relation that guides our understanding of the problem. It is typically expressed as:

$$Q = Wdv \tag{3}$$

The analysis by Leopold and Maddock (1953) was based on the data available for USGS stream gauging stations (i.e., W, d, and v); they were careful to point out early in their paper that they chose the variables they did not because they were the best suited to describing the channel geometry but because the data were commonly available. Importantly, they recognized that the channel gradient, S, the resistance to flow and the sediment load supplied to the stream channel were also relevant.

Leopold and Maddock (1953) concluded that both at-a-station and downstream relations could be reasonably expressed as power functions, though they were careful to note that their analysis was based on cross sections chosen for their suitability as gauging sections, not for their representativeness of the local river conditions, and that deviations from the general power relations do occur for both at-a-station and downstream relations. They used the following set of equations to describe hydraulic geometry:

$$
\begin{aligned}
W &= aQ^b \\
d &= cQ^f \\
v &= kQ^m
\end{aligned}
\tag{4}
$$

The coefficients a, c, and k, varied between sites for at-a-station relations and between stream channel networks for downstream relations. The exponents b, f, and m also varied but to a lesser degree. Leopold and Maddock's average exponent values for at-a-station relations for 20 different river reaches are 0.26, 0.40, and 0.34, for b, f, and m, respectively. Their average exponent values for the downstream relations are 0.50, 0.40, and 0.10, for b, f and m, respectively[2]. The continuity equation was used by Leopold and Maddock to constrain the values of the coefficients and the exponents as follows:

$$
\begin{aligned}
Q &= W \cdot d \cdot v \\
&= aQ^b \cdot cQ^f \cdot kQ^m \\
&= a \cdot c \cdot kQ^{b+f+m}
\end{aligned}
\tag{5}
$$

thus:

[2]Unlike many of the researchers who have continued to investigate downstream hydraulic geometry relations, Leopold and Maddock used the mean annual discharge to define their key results, not the bankfull flow.

$$a \cdot c \cdot k = 1$$

$$b + f + m = 1 \tag{6}$$

In practice, researchers have focused their attention on the hydraulic geometry equations for width and depth, with the velocity equations being largely ignored. It is worth noting that hydraulic geometry equations for velocity do not describe any particular aspect of the channel geometry *per se*, but rather they are necessary to satisfy continuity when using an empirical approach. Thus, the empirical approach obscures another key element of hydraulic geometry, the channel gradient, S, which was explicitly included in the early work on canals.

2.4. Theoretical Hydraulic Geometry

A theoretically based governing equation that includes channel gradient can be developed using the equations for open channel flow (e.g. Parker, 1979). By combining an equation relating velocity to slope, depth and flow resistance with a continuity equation, we can define a modified but physically explicit version of (3). While this approach does not provide us with an analytical form of the hydraulic geometry equations similar to (4) and probably does not improve the accuracy with which field observations of W and d can be empirically correlated to Q, it does define the problem in a more structured way, making the key considerations (and future research questions) more transparent. A general equation for predicting stream velocity (i.e., the Chèzy equation) takes the form:

$$v = \Re\sqrt{RS} \tag{7}$$

where R is the hydraulic radius (given by the ratio A/P, where A is the cross sectional area for flow and P is the wetted perimeter) and $\Re$ is a generalized representation of some flow resistance law. In the Darcy-Weisbach formulation, $\Re = \sqrt{8g/f}$, where g is the acceleration of gravity and f is the Darcy-Weisbach friction factor, commonly estimated using the Keulegan equation (after Ferguson, 2007):

$$\sqrt{\frac{8}{f}} = \left(\frac{1}{\kappa}\right) \ln\left(\frac{11R}{k_s}\right) \tag{8}$$

where κ is the Von Karman constant (≈ 0.4) and k_s is the roughness length, which is related to the bed sediment texture when individual grains dominate the flow resistance and to the dimensions of primary bedforms like ripples and dunes when they are present. The Strickler-Manning flow resistance equation is equivalent to (after Ferguson, 2007):

$$\Re = \alpha \left(\frac{R}{D}\right)^{1/6} \tag{9}$$

where D is a characteristic grain size for the channel bed and α is a constant that depends on whether the characteristic grain size is taken to be the median (i.e., the D_{50}) or something representative of the coarser end of the bed surface distribution (e.g., the D_{84}).

Using the geometric variables applicable to the open channel flow equations, continuity can be expressed in the following way:

$$Q = PRv \tag{10}$$

Clearly, P is similar to W and R is similar to d. For a simple rectangular channel, the relation between the two sets of geometric variables $[W, d]$ and $[P, R]$ is related to the width/depth ratio, ξ, in the following way:

$$\frac{W}{P} = \frac{R}{d} = \frac{\xi}{\xi + 2} \tag{11}$$

Plotting this function reveals that the difference between $[W, d]$ and $[P, R]$ is less than 10% for relatively wide channels ($\xi \geq 17$), but approaches 35% for narrow ones ($\xi \approx 4$), suggesting that the different sets of variables are likely to produce similar hydraulic geometries for wide gravel bed streams but that the results for narrow, deep sand bed streams with cohesive banks could be quite different (Figure 1).

If we combine (7) with (9), then rearrange it to isolate the dependent geometric variables on one side of the equation and the independent variables on the other, we get a modified, theoretically based version of (3) that includes the channel gradient:

$$\frac{Q}{\Re} = PR^{3/2}S^{1/2} \tag{12}$$

This equation clearly indicates that hydraulic geometry relations depend on discharge and on some sort a relative roughness term (R/k_s or R/D), which turns up in the common flow resistance laws (e.g., (8) and (9)). Another issue is that the timescale for adjustment of the channel cross section (P and R) is typically much shorter than that required to achieve changes in the channel gradient, S. For natural stream channels over short timescales, S may be better interpreted as an independent variable rather than a dependent one (e.g. Bray, 1973), thus:

$$\frac{Q}{\Re S^{1/2}} = PR^{3/2} \tag{13}$$

Hydraulic geometry is fundamentally at least a three-variable problem. In fact, more than three independent variables are involved, since—for a unique combination of Q, $\Re$, and S values—there is a wide range of geometries that satisfy (13) (defined by $PR^{3/2} = constant$), indicating that yet more information is required.

In the case of an at-a-station relation, the missing information is the shape of the channel cross section which, once known, allows us to specify P as a function of R (and *vice versa*), and then derive a unique solution for a given set of independent variable values (see Ferguson, 1986). For downstream relations, the missing information is less easily defined, but in the author's opinion, as well as that of others (Huang and Nanson, 1998; Millar, 2005; Eaton and Church, 2007), it must be related to the factors controlling the stability of the channel bed and banks at the formative discharge. Various researchers have attempted to complete a physically based theory for downstream hydraulic geometry. These theories have been based on one of two ideas: the threshold approach and the maximum efficiency approach (Ferguson, 1986). In the threshold approach (Lane, 1957; Henderson, 1961; Li et al., 1976; Stevens, 1989), applicable to channels that do not transport the material comprising their boundary at an appreciable rate, the channel at bankfull flow is assumed to be everywhere at the threshold of motion.

For channels that are assumed to transport their bed material, various approaches have been developed based on some representation of the principle of maximum efficiency proposed by Gilbert (1914), wherein a channel is assumed to adjust so that its bedload is transported most efficiently (Langbein and Leopold, 1966; Yang, 1976; Kirkby, 1977; Chang, 1979; White et al., 1982; Davies and Sutherland, 1983; Eaton et al., 2004). Many of these principles turn out to be equivalent (White et al., 1982; Davies and Sutherland, 1983), and that proposed by Eaton et al. (2004) has been successfully tested experimentally (Eaton and Church, 2004) and related to a process-form interaction that produces meandering (Eaton et al., 2006).

Equation (13) can be used to demonstrate the effect of scale alone on hydraulic geometry by applying it to channels with a range of different sizes but which are geometrically similar and Froude-similar (after Griffiths, 2003). Such channels will have the same shape, η, (given by $\eta = P/R$), relative roughness, R/D, (implying that $\Re$ will be constant), and dimensionless shear stress, τ^*, (given by $\tau^* = \tau/(\gamma_s - \gamma)D_{50}$, where τ is the average shear stress acting on the bed, γ_s is the unit weight for sediment grains, γ is the unit weight of water and D_{50} is the median bed surface size). The absolute dimensions of any channel can be related to a single length scale (L_r), provided a reference bed geometry is adopted. The reference bed geometry is given by P_o, R_o and D_o. The geometry of a channel that is L_r times larger than the reference channel is given by:

$$\begin{aligned} P &= P_o L_r \\ R &= R_o L_r \\ D &= D_o L_r \end{aligned} \tag{14}$$

If the variable definitions for P and R are substituted into (13) we get, with a slight rearrangement of terms:

$$\frac{Q}{P_o R_o^{3/2} \Re S^{1/2}} = L_r^{5/2} \tag{15}$$

By first setting the right-hand side of (15) equal to $(P/P_o)^{5/2}$ and then to $(R/R_o)^{5/2}$, we can generate hydraulic geometry equations for P and R that are exact (as opposed to approximate) power functions that have exponents of 2/5 for P and R. If we use continuity to determine the equation for v, we end up with another power function (exponent of 1/5). The complete set of equations is:

$$
\begin{aligned}
P &= \kappa_1 Q^{2/5} \qquad & \kappa_1 &= \left(\frac{\eta^{3/2}}{\Re S^{1/2}}\right)^{2/5} \\
R &= \kappa_2 Q^{2/5} \qquad & \kappa_2 &= \left(\frac{1}{\eta \Re S^{1/2}}\right)^{2/5} \\
v &= \kappa_3 Q^{1/5} \qquad & \kappa_3 &= \frac{1}{\kappa_1 \kappa_2}
\end{aligned}
\tag{16}
$$

where κ_1, κ_2 and κ_3 are coefficients that depend only on the channel shape, η, the relative roughness (which determines the flow resistance, $\Re$), and the channel gradient. Any changes in channel shape, flow resistance and/or channel slope with discharge will introduce deviations from power functions with exponents of 2/5, 2/5 and 1/5 for width depth and velocity, respectively. It is worth noting that Church (1980) derived these same results from dimensional analysis. These theoretically derived exponents therefore form a useful reference against which to compare field data (cf. Parker et al., 2007). More generally, these equations will describe the hydraulic geometry of all sets of channels for which channel shape (η), flow resistance ($\Re$) and gradient (S) are known for all discharge values.

In Figure 2, data from Andrews (1984) are compared against hydraulic geometry relations for which $W \propto Q^{2/5}$ and $d \propto Q^{2/5}$, as suggested by (16).

While there are clearly systematic deviations from the Froude-scaling trend, it does explain much of the variance in the dataset. Furthermore, the deviations appear to correlate with the characteristic riparian vegetation (which Andrews classified as either "thin" or "thick"), and thus presumably with the erosional resistance of the channel banks. The comparison between the $b = f = 2/5$ lines and the data is only approximate, since (16) relates $[P, R]$ to Q, whereas only the bankfull $[W, d]$ are reported in Andrews (1984). In addition, the coefficients κ_1 and κ_2 depend on S, which varies systematically with Q for the dataset. However, the channels in Andrews' dataset all have relatively large W/d ratios, suggesting that it is appropriate to substitute the variables $[W, d]$ for $[P, R]$, and the dependence of κ_1 and κ_2 on S is relatively weak, since S is raised to the $1/5^{th}$ power in both.

In the following sections, the recent research will be reviewed and discussed with reference to the underlying theory. The review will first address the major issues and advances for at-a-station relations and then for downstream relations. Finally, major opportunities for research will be identified.

3. Recent Research

3.1. At-a-Station Relations

Provided that the channel geometry remains stable, at-a-station hydraulic geometry simply describes how an increase is discharge is accommodated. The relations depend entirely on the channel shape and the resistance to flow (Ferguson, 1986). If both can be described mathematically, then the hydraulic geometry equations can be derived analytically: if one assumes that Manning's n remains constant at all flows, then at-a-station hydraulic geometry relations can be described using power functions in which a triangular cross section would be typified by $b = f = 0.376$ and $m = 0.25$ and a parabolic cross section would be typified by $b = 0.23$, $f = 0.46$ and $m = 0.31$ (after Ferguson, 1986).

Using the Keulegan flow resistance law in which flow resistance varies with flow depth, Ferguson (1986) showed that, while they can be defined analytically, the hydraulic geometry relations for rectangular, parabolic, and triangular cross sections cannot be exactly described by power functions. All of these derivations necessarily assume that the energy gradient at a cross section remains constant, as well, which is often not the case. Additionally, significant flood events that alter the channel morphology also alter the characteristic hydraulic-geometry. For example, Lisle (1982) showed a significant shift in the values of b and f for at-a-station relations at gauges in California that were impacted by a large flood in 1964, as well as a longer-term drift back towards to original values as the channels recovered from the disturbance.

While some workers have proposed modified curve-fitting procedures to improve the performance of at-a-station hydraulic geometries and to account for changes in channel shapes (e.g. Bates, 1990), the changes in energy gradient at a given cross section are more difficult to incorporate (cf. Carling, 1991). Given that the variations in b, f and m for individual cross sections are often as large as the variations in the mean exponent values for a range of streams (Rhodes, 1977), it is fair to conclude that the applicability of at-a-station hydraulic geometry relations is limited by the spatial variation in channel shape and the variation of the local energy gradient with discharge.

The development of both 2D and 3D computational fluid dynamics models (CFD models) has obviated the need for at-a-station hydraulic geometry equations in many circumstances. These computer models are now capable of modeling the spatial distribution of depth and velocity for a range of flows, provided sufficiently detailed information on the bed geometry and bed roughness are available. However, these models have fairly onerous data requirements, and the 2D models are restricted to streams wherein the velocity profile at any point in the stream can be mathematically described.

At-a-station relations may continue to be a suitable tool for studies in which the detailed information and the level of accuracy of CFD predictions are not required. In assessments of fish habitat, it is often desirable to calculate the percent of usable habitat, but not to determine the distribution of that habitat. Hogan and Church (1989) present a methodology for assessing instream fish habitat using at-a-station relations based on the average response to changes in flow of a number cross sections in a study reach, rather than an individual cross section. Using this approach in two different streams with complex morphologies influenced by large woody debris, they were able to detect the impacts of logging on fish habitat. Jowett (1998) also used at-a-station relations to characterize fish habitat, and found that the at-a-station based predictions were close to those from a more detailed assessment of fish habitat.

In some circumstances, such as in braided rivers, the channel geometry is not generally stable, making application of CFD models difficult. Indeed, it is often difficult to even reliably measure the discharge in braided streams. Ashmore and Sauks (2006) showed that, for flows greater than about half of the mean annual peak flow, almost all of the variation in discharge was taken up by changes in the wetted width in their study reach. Their equation for width is:

$$W = 4.722Q^{0.978} \tag{17}$$

They also demonstrated that this relation was reasonably robust, and could be used to predict the discharge from the wetted width measured from oblique aerial photographs at nearby reaches of the Sunwapta River and in subsequent years at their study reach.

In other circumstances, the resistance to flow may be much too complicated for the existing CFD models. Lee and Ferguson (2002) examined the change in flow resistance with stage in step-pool systems, and they found that due to very rapid changes in flow resistance with increasing discharge, velocity changed most rapidly with discharge (m varied from 0.51 to 0.84), followed by changes in depth (f varied from 0.19 to 0.36). Comiti et al. (2007) also measured flow resistance in the step-pool Rio Cordon, and they also found that changes in velocity were the primary means by which changes in discharge were accommodated ($m = 0.49$, on average), followed by changes depth ($f = 0.29$, on average). Their results also point out that, while the coefficient for the velocity relations varied over a moderate range (m varied from 0.24 to 0.63), the exponents of the width and depth relations were far more variable (b varied from 0.03 to 0.47 and f varied from 0.08 to 0.44).

For both braided and step-pool streams, at-a-station hydraulic geometry relations are likely to remain important tools for characterizing the hydraulic changes with discharge. Similarly, streams in which large woody debris produces physically and hydraulically complicated channels (e.g. Hogan and Church, 1989) are unlikely to be well described by the existing CFD models.

3.2. Downstream Relations

While Leopold and Maddock (1953) used a range of flow frequencies to investigate downstream hydraulic geometry relations, subsequent work has focused on relating the channel geometry to a formative discharge, moving beyond spatial correlation and towards causation. The concept of a formative discharge requires consideration: at some level, it is entirely artificial, since the channels of natural streams are ultimately the product of a history of flows having various magnitudes that interact with other geomorphic processes controlling the supply of sediment and organic material (primarily woody debris) to the channel. The literature on formative discharge is reviewed below.

The recent work on downstream hydraulic geometry has been classified into one of two groups: the first group comprises those papers focusing on data collection and analysis under the heading of *Empirical Hydraulic Geometry*; and the second group comprises papers focusing on developing hydraulic geometry relations from the underlying theory of open channel flow, under the heading *Theoretical Hydraulic Geometry*.

3.2.1. Defining Formative Discharge

There are two primary means of defining the formative discharge: one is based on the flow frequency or flow duration; the other is based on the morphology of the stream. The frequency-based definition of the formative discharge, referred to herein as the effective flood, is the flow that is both frequent enough and powerful enough to do the most geomorphic work (and hence be geomorphically most effective). The morphologic definition, referred to as the bankfull flood, is based on the elevation of the floodplain surface since, once the water level reaches the banktop, and additional discharge is likely to flood out onto the floodplain, only minimally increasing the water depth (and thus shear stress) in the channel itself. While the association between channel dimensions and discharge is conceptually the clearest for the bankfull flood, this definition can only be applied to streams that have a self-formed floodplain and which are neither actively aggrading nor degrading.

For systems with stable, identifiable floodplains, the bankfull flood seems to be associated with a nearly constant flow frequency. Harvey (1969) found that bankfull floods often had a return period of between 1 and 2 years, except for streams dominated by groundwater flows, in which the bankfull flow return period was greater, a conclusion also reached by Petit and Pauquet (1997) in a more recent study. In contrast, Williams (1978) considered various means of defining the bankfull flow and estimated the flood frequency of each definition: he concluded that the bankfull flow had no common identifiable flood frequency. Petit and Pauquet (1997) found that the flood frequency associated with the bankfull flood was 0.7 years for the smallest streams in their study, but rose to 1.1 to 1.5 years for streams with a drainage basin greater than 250 km^2, suggesting that the frequency of bankfull floods may be slightly scale-dependent. Castro and Jackson (2002) also observed a systematic variation in the frequency of bankfull flows: they found that the bankfull return period in humid regions of the Pacific Northwest averaged about 1.2 years, but that the return period rose to 1.4 to 1.5 years in drier regions.

Bray (1975) assessed the statistical correlation between the channel geometry of gravel-bed streams and various potential definitions of the formative discharge, including the mean annual flow, 1.5-yr event, 2.0-yr event, 5.0-yr event 10-yr event, and bankfull flood. The correlations between the observed channel geometry and the various definitions of the formative flow are all relatively strong, suggesting that the precise choice of formative discharge is not critical. Viewed another way, Bray's analysis implies that there is a finite, irreducible uncertainty associated with trying to correlate channel geometry with a single, representative discharge value. The best correlation in Bray's analysis was produced by using the 2-year flood, which is also most similar to the bankfull flood; this implies that something like the bankfull flow is an appropriate (if imperfect) representation of the formative discharge. Interestingly, Bray (1975) advocated using the 2-year flood definition instead of bankfull because it can be applied to actively incising streams where no floodplain is apparent. Andrews (1980) also concluded that the effective flood and bankfull flood were very similar, having return periods between 1.18 and 3.26 years. Andrews (1982) also showed that flows less than bankfull have little effect on the morphology of gravel-bed rivers, supporting the idea that the effective flood and bankfull flood are nearly identical.

There is some evidence that different aspects of channel morphology develop in associated with different flow frequencies (i.e., that formative discharge depends on the aspect of channel morphology being considered). Pickup and Warner (1976) argued that, for the gravel bed streams in the Cumberland Basin, UK, the channel capacity was set by a somewhat rarer bankfull flood (having a return period of between 4 and 10 years) that was capable of eroding the channel banks, but that the bed surface was adjusted to the more frequent effective flood (return period of 1.15 to 1.40 years). In contrast, Emmett and Wolman (2001) found that streams having snowmelt-dominated flow regimes often developed a highly armored surface, with the result that the effective flood was less frequent than the bankfull flood. The ratio of the effective flood to the bankfull flood in their study streams varied from 0.98 to 1.31, with the upper limit representing a doubling of the return period. Finally, various researchers have pointed out that the influence of large, rare events on channel geometry is undoubtedly important, particularly in arid environments, since extreme floods can quite quickly alter the channel geometry (e.g. Gardner, 1977; Desloges and Church, 1992; Merritt and Wohl, 2003).

In summary, then, while it is possible for different components of the channel morphology to be adjusted to flows of different frequencies, there is a general consensus that the formative discharge approximately conforms to the bankfull flood when a stable floodplain is present, or to a flood with a return period of about 1.5 to 2.0 years (at least in the relatively humid climates typical of most of the studies cited above). Furthermore, when developing downstream hydraulic geometry relations, it is preferable to either use a single, convenient, well defined estimate of the formative discharge (e.g. Bray, 1975) or to employ a range of bankfull discharge estimates (e.g. Radecki-Pawlik, 2002). Researchers should also bear in mind the fundamental limitations of the basic assumption that channel morphology can be uniquely related to formative discharge and should consider the potential role of geomorphic history in shaping the morphology of any given channel (Church, 1995): by using a single, representative discharge to explain hydraulic geometry, there is bound to be some irreducible level of uncertainty associated with the implied assumption of equilibrium between the channel geometry and the chosen representative discharge.

3.2.2. Empirical Hydraulic Geometry

Since the original work by Leopold and Maddock (1953), various studies have confirmed the general form of their downstream equations for which $b \propto 0.5$ and $f \propto 0.4$ (Mikhailov, 1970; Bray, 1973; Emmett, 1975; Charlton et al., 1978; Parker, 1979; Hey and Thorne, 1986). Figure 3A presents the downstream hydraulic geometry data from a wide range of sources, including both gravel-bed and sand-bed channels, as well as trend-lines fit to all of the data. Figure 4A presents only the gravel-bed streams and trend-lines for those data. The relations fitted to the data in Figure 3A ($b = 0.536$ and $f = 0.384$) are quite close to Leopold and Maddock's original results of $b = 0.50$ and $f = 0.40$, but the data are relatively scattered (the standard error of the estimate for W and d is about 30%). When only the gravel-bed data are included in the regression, then the scatter is somewhat reduced (the standard error drops to about 24% for width and 19% for depth), but the hydraulic geometry equations change very little. In both cases, the scatter about the common trend masks systematic deviations within several of the datasets.

For the individual datasets, the standard errors for the estimates of width and depth are lower, typically around 15% for both width and depth. The implication is that the irreducible uncertainty associated with the use of a single representative discharge to explain all of the variance in channel geometry is probably in the range of 10 to 20%, depending on the system being studied.

In some of the case studies, the exponents of both width and depth relations are systematically different from the general trend, while in others the coefficients (primarily of the width relation) are different. Coefficient variations have been related to the vegetation found on the channel banks (Charlton et al., 1978; Andrews, 1984; Hey and Thorne, 1986; Huang and Nanson, 1997) and to the material forming the channel boundary (e.g. Lane, 1955; Ferguson, 1986; Church, 1992), both of which influence the erodibility of the channel boundary and thus the shape of the channel. Variations of the exponents seem to be produced in a number of ways. Pitlick and Cress (2002) report that, for a section of the Colorado River (the data for which are presented in Figure 3), width increases downstream less quickly than depth ($b = 0.32$ and $f = 0.53$), a result attributed to a decline in sediment supply in the downstream direction. Ellis and Church (2005) and Tabata and Hickin (2003) observe that in anabranches of multiple thread channel systems (shown in Figure 3), width changes more quickly than expected as Q increases, and depth changes less quickly ($b = 0.60$, $f = 0.25$ and $b = 0.64$, $f = 0.19$, respectively). These two studies are examples of simple scaling relations, in which only one variable is changed (i.e., discharge) and the others (i.e., channel gradient, bed sediment texture and bank strength) remain constant.

Wohl (2004) identifies a limit scale for mountain streams below which regular hydraulic geometry relations are poorly developed. Once the ratio of total stream power to the characteristic grain size drops below about 10,000 W/m^2 (or equivalently kg/s^3), the fluid forces presumably become too small to shape the channel.

As is evident in Figure 3, there are systematic differences in both channel geometry and sediment transport characteristics between gravel- and sand-bed streams. At the same discharge, sand-bed channels tend to be deeper than gravel-bed channels and to have lower channel gradients (Xu, 2004): however, the intensity and duration of sediment transport tends to be much higher in sand-bed streams (Schumm, 1985; Dade, 2000; Church, 2006). The sedimentology of reach also affects how a stream will respond to environmental changes (e.g. Gaeuman et al., 2005).

In their analysis of a large number of both gravel-bed and sand-bed streams from a range of environments, Lee and Julien (2006) were able to generate a single set of equations by including both grain size and channel gradient as independent variables. The success of their approach demonstrates the importance of considering more of the relevant independent variables than merely Q. Their equations for predicting width and depth are:

$$\begin{aligned} W &= 3.004 Q^{0.426} D_{50}^{-0.002} S^{-0.153} \\ d &= 0.201 Q^{0.336} D_{50}^{-0.025} S^{-0.060} \end{aligned} \tag{18}$$

A similar analysis of gravel-bed rivers was conducted much earlier by Bray (1973). He observed that, for gravel-bed rivers in Alberta, Canada, the inclusion of channel gradient, S, as an independent variable improved the fit of the equations for d and v, but not for W. Bray (1975) also attempted a physical interpretation of his empirical results: he reported that the average dimensionless shear stress, τ^*, for his 1973 dataset was nearly constant at a value of about 0.039, or roughly 30% higher than the threshold for gravel entrainment, τ_c^* (assuming $\tau_c^* = 0.03$). Bray's equations, in SI units are:

$$
\begin{aligned}
W &= 4.05 Q^{0.515} S^{-0.035} \\
d &= 0.107 Q^{0.265} S^{-0.199} \\
v &= 3.02 Q^{0.220} S^{0.234}
\end{aligned}
\tag{19}
$$

In recently glaciated environments, the effects of the last glaciation on the character and distribution of alluvial sediment are quite pronounced, as are the effects on channel gradients (e.g. Brardinoni and Hassan, 2006). In extreme cases, the spatial complexity of post-glacial landscapes introduces enough variability that the underlying hydraulic geometry relations can be nearly obscured (Coates, 1969; Arp et al., 2007). Other geomorphic processes, such as debris flows, and the distribution of bedrock type can also influence downstream hydraulic geometry relations, resulting in deviations from the normal hydraulic geometry exponents and in step changes in scaling relations

(Montgomery and Gran, 2001). In their study of bedrock channels in a wide range of environments, Wohl and David (2008) found that the scaling typical of alluvial channels (where $b \approx 0.50$ and $f \approx 0.3$) holds, but that variability the relations was not unambiguously associated with changes in bedrock type, suggesting that other factors such as bedload supply also exert a significant influence on the form of the hydraulic geometry relations.Together, these results suggest that a single set of hydraulic geometry equations can only be expected to hold so long as the boundary conditions (primarily the valley slope and the material forming the channel bed and banks) remain relatively constant or vary in a smooth, predictable way.

Other researchers has sought to improve the predictive ability of their statistical models by combining the important variables to form non-dimensional ones. For example, Andrews (1984) included sediment size in his analysis of the hydraulic geometry of gravel-bed streams in Colorado, but rather than using D_{50} as an independent variable, he normalized W, d and Q by D_{50} of the surface, following an earlier analysis by Parker (1979). The dimensionless variables are:

$$W^* = \frac{W}{D_{50}}, \qquad d^* = \frac{d}{D_{50}}, \qquad Q^* = \frac{Q}{D_{50}^2\sqrt{(s-1)gD_{50}}} \tag{20}$$

The parameter s is the specific gravity of the sediment grains. Interestingly, while using Q^* does increase the information content of the variables used to derive the statistical model, it does not greatly improve the model fit when considering gravel-bed streams (cf. statistics for trend-lines in Figure 4A and B). Nevertheless, the dimensionless approach is quite useful for distinguishing between sand-bed and gravel-bed rivers (see Figure 3A), and relations between $[W^*, d^*]$ and Q^* for sand-bed and gravel-bed rivers taken together follow a common trend-line with exponents similar to that for Froude scaling.

The work presented by Andrews (1984) demonstrates the influence of vegetation on channel form. The most important results of the analyses presented by Andrews (1984) are: **(1)** that vegetation type has a discernable effect on the form of the hydraulic geometry relations; and **(2)** that the sets of relations for different vegetation types are associated with a constant dimensionless shear stress. His equations for thick bank vegetation ($W^* = 3.19 Q^{*\,0482}$, and $d^* = 0.491 Q^{*\,0.370}$) are associated with dimensionless shear stresses that are, on average, nearly twice the entrainment threshold for gravel ($\tau^* \approx 0.058$), while his equations for thin vegetation ($W^* = 4.94 Q^{*\,0478}$, and $d^* = 0.485 Q^{*\,0.377}$) are associated with dimensionless shear stresses close to the threshold ($\tau^* \approx 0.03$). Notice also that vegetation appears to influence the coefficients but not the exponents of the hydraulic geometry relations. These results support the idea that hydraulic geometry relations are constrained by boundary material strength and further suggest that denser riparian vegetation increases material strength such that the channel can support higher average shear stresses. Andrews' equations indicate that more densely vegetated channels are, on average, narrower than less densely vegetated ones, which is presumably the means by which bank vegetation can influence in-channel shear stresses.

Hey and Thorne (1986) further improved our understanding of the effects of bank vegetation on downstream hydraulic geometry. They grouped gravel-bed streams in the United Kingdom into four categories (types I, II, III and IV) representing increasingly dense riparian vegetation. In relations between the channel geometry and a set of independent variables including bank vegetation type, bankfull discharge (Q), sediment texture (D_{50}, D_{84} both of the surface) and the sediment load (Q_s) estimated from a sediment transport equation applied to the bankfull discharge, they found that the coefficient for the width relation decreased with increasing vegetation density, similar to the pattern reported by Andrews (1984) and that the coefficient for depth increased. In addition to the standard geometric variables they also analyzed P and R, which are arguably more appropriate variables to consider, and the R^2 values for the equations predicting P and R were both slightly better relative to the equivalent equations using W and d. The most parsimonious models for predicting width that Hey and Thorne present, wherein all coefficients and exponents are significant at the 95% confidence level, includes only vegetation type and Q (i.e., where $a = 4.33, 3.33, 2.73, 2.43$ for types I, II, III and IV, respectively). The most parsimonious equation for predicting depth includes Q and D_{50}, but does not depend on vegetation type (for types I, II, III and IV).

Figure 5A presents a plot of all data for which information on the characteristic bank vegetation is available. The trend-lines in the figure have been fit to all of the data from Charlton et al. (1978), Andrews (1984), and Hey and Thorne (1986). The channel widths for more densely vegetated channels consistently tend to fall below the regression line, and widths for more sparsely vegetated channels tend to fall above the line. Interestingly, there is little evidence for a similar systematic segregation of channel depth as a function of vegetation type. When the data are transformed into the dimensionless variables $[W^*, d^*]$ and plotted against Q^*, the general pattern remains virtually unchanged, as do the standard errors. However, while b and f are statistically different for relations between $[W, d]$ and Q, they are not for relations between $[W^*, d^*]$ and Q^*.

Further evidence that gravel-bed channels with denser vegetation have narrower, deeper channels is presented by Huang and Nanson (1997) and Parker et al. (2007), while Millar (2000) and Brooks et al. (2003) demonstrate that removal of vegetation produces channel widening. Interestingly, Bergeron and Roy (1985) and Allmendinger et al. (2005) found several cases in which forested reaches were actually wider than reaches with riparian areas dominated by grasses. Bergeron and Roy (1985) attribute the differences to higher density of roots in their grass-covered study reach, while Allmendinger et al. (2005) argue that the grass is critical for trapping and stabilizing the suspended sediment deposits that produce bar growth and hence channel widening, and that grass growth under the forest canopy was effectively suppressed. Thus, while grass-covered banks in the study streams of Allmendinger et al. were eroded more quickly, their inner banks were able to advance even more quickly than those in the forested reaches. This example demonstrates that even the role of vegetation on hydraulic geometry may be far from straightforward. The potential influence of large woody debris on channel geometry undoubtedly introduces further complications (Hickin, 1984).

3.2.3. Theoretical Hydraulic Geometry

For relatively simple boundary conditions and transport dynamics, it is possible to construct a complete, physically based model of downstream hydraulic geometry. Wobus et al. (2006) present a model predicting the geometry of channels incising into bedrock that is based on the distribution of shear stress (and hence inferred erosion rate) across the channel. The eroding channels all have a nearly constant width-to-depth ratio (ξ) of about 3.5, and exhibit downstream scaling in which $b = f = 0.4$, a result that is identical with the Froude scaling argument summarized by equation (16). Their model also illustrates the influence of local channel gradient, S, on channel width ($W \propto S^{0.2}$), which is particularly significant where bedrock uplift rates are spatially variable.

Finnegan et al. (2005) also developed a theoretically based equation for the hydraulic geometry of bedrock channels, then successfully tested it. They used the equations for open channel flow to derive an equation for predicting the width of bedrock channels as a function of the channel shape (indexed by the width/depth ratio, ξ), the discharge, channel gradient, and resistance to flow (in short all of the independent variables in (13) and the information required to relate W to d). Their equation is:

$$W = \left[\xi(\xi+2)^{2/3}\right]^{3/8} Q^{3/8} S^{-3/16} n^{3/8} \tag{21}$$

Once the boundary conditions become more complex and both erosion and deposition are allowed to occur (i.e., the sediment transport is no longer supply-limited), it becomes more difficult to arrive at an explicit theory of hydraulic geometry. This point was made quite well by Parker (1979), who was able to generate a series of regime relations for gravel-bed channels that, once one of the relations was specified, could be used to theoretically derive the others. His analysis was conducted using the dimensionless variables in (20), as well as a dimensionless sediment transport rate, Q_s^*:

$$Q_s^* = \frac{Q_s}{D_{50}^2\sqrt{(s-1)gD_{50}}} \tag{22}$$

where Q_s is the volumetric sediment load associated with the discharge, Q. His regime relations can be written:

$$\begin{aligned} W^* &= 3.09 \cdot 10^6 Q_s^{*1.296} Q^{*-0.296} \\ d_c^* &= 3.56 \cdot 10^{-6} Q_s^{*-1.075} Q^{*1.075} \\ S &= 1.37 \cdot 10^4 Q_s^{*1.062} Q^{*-1.062} \end{aligned} \tag{23}$$

The variable d_c^* refers to the center depth of a channel having a flat bottom and parabolic bank profiles. These relations nicely show the dependence of channel geometry on the sediment supply to the channel and the discharge available to transport it, while his derivation shows the influence that boundary erodiblity has on the geometry. Parker tested his theory against the dataset of Bray (1973), showing that by using a given relation between W^* and Q^* to eliminate Q_s^*, the regime relations were able to reasonably reproduce the remaining hydraulic geometry equations.

However, Parker's analysis assumes that the threshold for bank erosion is the same as that for bed erosion, thus ignoring the potential role of vegetation in stabilizing the stream banks and limiting Parker's analysis to channels having no significant quantities of cohesive sediment in their overbank deposits. It also reflects only the narrowest possible channel for which the supply may be transported by the available discharge.

Others have shown that there exists a wide range of solution geometries for a given set of governing conditions (e.g. White et al., 1982), even when a bank stability analysis has been applied (Millar and Quick, 1993). While some have recently argued that a unique solution can be isolated by applying the principle of least action (Huang and Nanson, 2000) or minimum energy expenditure (Huang et al., 2004), the author and others claim that the extremal hypotheses used to close regime models are really numerical formalisms that allow a 1D model to describe a 3D reality by encapsulating feedbacks occurring in the cross-stream dimension, a dimension to which 1D regime models are obviously blind, since they employ variables that describe the average cross section dimensions (Eaton et al., 2006).

Since channels achieve stability by adjusting their form until they reach their most stable condition, for which the sediment supplied to the system can be transported by the available stream flow (analogous to the bottom of a potential energy well), it is useful to consider the problem from the point-of-view of flow resistance. When the maximization of the system-scale flow resistance is used, then the partitioning of flow resistance can be related to the mechanisms responsible for establishing channel stability (Eaton et al., 2004): reach-scale flow resistance is related to changes in channel sinuosity and channel pattern (Eaton and Church, 2004), grain-scale flow resistance is related to the development of an armor layer and surface structures (Eaton and Church, 2009), and bedform-scale flow resistance may be related to the development of bars and other bedforms with dimensions on the order of the channel width.

Millar (2005) used a numerically solved rational regime model that considers the effect of bank strength (relative to the erodibility of the bed) on channel geometry in order to generate a set of regime relations similar to the equations presented by Parker (1979). Millar's equations were statistically derived from a large number of regime model simulations, and thus are approximations of the full numerical regime model. His equations are:

$$
\begin{aligned}
W^* &= 28.1 Q^{*\,0.50} C^{*\,-1.12} \mu'^{-1.66} \\
d^* &= 0.0764 Q^{*\,0.37} C^{*\,1.16} \mu'^{1.22} \\
S &= 1.98 Q^{*\,-0.33} C^{*\,-1.86} \mu'^{-0.96}
\end{aligned}
\tag{24}
$$

where variable C^* is the dimensionless sediment concentration (similar to Parkers Q_s^*), and μ' is the relative bank strength (i.e., the ratio of the critical shear stress for entrainment of the bank normalized by the critical shear stress for the bed). Millar also observed that, while S is commonly known with a fair degree of accuracy, C^* almost never is, so Millar used S as an independent variable, thereby eliminating C^* from the equations. The resulting hydraulic geometry equations depend on the dimensionless discharge, the channel gradient, and the relative strength of the channel banks. A characteristic grain size is also implicitly included, since it is used to calculate the non-dimensional parameters (W^*, d^* and Q^*), as well as the flow resistance parameter used by the model (Darcy-Weisbach f): this represents as complete a set of independent variables as can currently be specified. The equations are:

$$
\begin{aligned}
W^* &= 16.5 Q^{*\,0.70} S^{0.60} \mu'^{-1.10} \qquad (25) \\
d^* &= 0.125 Q^{*\,0.16} S^{-0.62} \mu'^{0.64}
\end{aligned}
$$

Eaton and Church (2007) use a similar regime model, as well as another numerical regime model that analyses bank strength by specifying an effective cohesion term associated with riparian vegetation (after Eaton, 2006), to examine how well regime models are able to reproduce the observed downstream hydraulic geometry. They found that regime models in which relative bank strength (μ') was held constant were able to reproduce the hydraulic geometry of a large gravel-bed stream (Colorado River data from Pitlick and Cress, 2002), the anabranch dimensions of both a wandering gravel-bed river (Fraser River data from Ellis and Church, 2005) and a wandering sand-bed river (Columbia River data from Tabata and Hickin, 2003), as well as the geometry of distributary channels on deltas (data from Andrén, 1994; Mikhailov, 1970). For datasets including smaller gravel bed streams (i.e. Emmett, 1975; Andrews, 1984), Eaton and Church demonstrated that regime models assuming a constant effective cohesion due to vegetation for a given bank vegetation type were able to reproduce the observed hydraulic geometry, while the constant relative bank strength models were not. Eaton and Giles (2009) followed up this theme and demonstrated that the effect of vegetation on hydraulic geometry declines systematically as the scale of the system increases up to $Q^* \sim 10^6$, above which the effect of vegetation becomes negligible. A similar conclusion was reached by Eaton and Millar (2004), who found that, for gravel-bed streams in the UK (Hey and Thorne, 1986), the effect of vegetation disappeared once Q exceeded about 400 m^3/s, presumably because the rooting depth of the riparian vegetation became small in relation to the height of the channel banks.

A similar theoretically based model relating hydraulic geometry to vegetation-related bank strength was developed by Huang and Warner (1995), and subsequently tested in the field by Huang and Nanson (1998): they concluded that vegetation can produce as much as a three-fold change in channel width and a two-fold change in depth.

A different approach to the problem was published by Griffiths (2003), who attempted to theoretically define downstream hydraulic geometry relations by invoking distorted Froude-scaling. His equations for mobile-bed channels are written as scaling functions, as follows:

$$
\begin{aligned}
P_r &= Q_r^{0.50} \\
R_r &= Q_r^{0.33} D_r^{-0.17} \\
v_r &= Q_r^{0.17} D_r^{0.17} \\
S_r &= Q_r^{-0.17} D_r^{0.83}
\end{aligned}
\tag{26}
$$

In this set of equations, all variables (i.e., Q, P, R, D, v, S) are normalized by values from a reference location, hence the subscript, r. This model is quite flexible, since it uses information from a reference location as a baseline, but allows both the sediment texture and the discharge to vary downstream. An important drawback is that it is not possible in this framework to explicitly consider (or vary) the effect of bank strength on channel geometry, thus one of the important independent variables is omitted.

4. Summary and Future Research

4.1. At-a-Station Relations

At-a-station hydraulic geometry relations were first developed by Leopold and Maddock (1953) using stream gauging rating curves at gauging cross sections: they are site-specific relations, determined by the channel geometry and resistance to flow, and they can drift over time (just as rating relations tend to do) as the channel cross section changes. In large part, research on at-a-station hydraulic geometry has remained focussed on the cross-section scale. Most of the recent work has either attempted to improve the curve-fitting procedures (e.g. Bates, 1990; Carling, 1991) or to apply at-a-station hydraulic equations to the study of aquatic habitat (e.g. Hogan and Church, 1989; Jowett, 1998).

Advances in our understanding of at-a-station hydraulic geometry can likely be made by taking advantage of recent advances in data collection methodologies (such as digital photogrammetry, LIDAR imaging, dGPS surveying, and ADCP flow measurements) and computational fluid dynamics models in order to escape the limitations imposed by restricting attention to a single cross section. For moderate to large rivers, it is now possible to collect detailed three-dimensional data on the bed topography, flow structure and sedimentology, and to numerically model the distribution of depths and velocities for a range of discharges. It should be possible to construct at-a-station relations using the mean depth and velocity averaged over a reach several channel widths in length (rather than averaged over a single cross section), and potentially to construct additional at-a-station relations that describe the distribution of depths and velocities within the reach. Such relations (which could more appropriately be described as at-a-reach hydraulic geometry relations) would provide information that is more relevant to fish habitat and instream engineering projects. By generating relations that describe reach averages, some of the between-stream variation could be reduced and it is possible that approximate general forms of the at-a-station relations could be derived for streams with similar channel characteristics (channel size, characteristic morphology, etc.). Furthermore, the temporal variability of cross section based relations could be reduced, provided the reach remained pattern-stable, since much of the temporal variability at a cross section is associated with the normal processes of adjustment and migration of a channel at equilibrium.

4.2. Downstream Relations

Downstream hydraulic geometry equations originated as watershed-by-watershed correlations between the channel dimensions and the mean annual flow (Leopold and Maddock, 1953). Relatively soon thereafter, downstream relations focussed on the correlations between geometry and a formative discharge, often taken to be the bankfull flow, as well as other important variables such as the average channel gradient (e.g. Bray, 1973; Emmett, 1975;

Charlton et al., 1978). Eventually, additional variables such as the characteristic bed sediment size and riparian vegetation type were also included in these empirical correlations (Charlton et al., 1978; Andrews, 1984; Hey and Thorne, 1986; Lee and Julien, 2006; Parker et al., 2007). Early on, the similarity of correlations between channel geometry and discharge for different river systems prompted researchers to combine data from different watersheds in order to construct relations that were purportedly representative of physiographic regions and/or stream channel types (e.g. Bray, 1973; Charlton et al., 1978; Andrews, 1984; Hey and Thorne, 1986; Lee and Julien, 2006; Parker et al., 2007), and the watershed-specific downstream hydraulic geometry relations have been nearly abandoned, with some notable recent exceptions (Andrén, 1994; Pitlick and Cress, 2002; Tabata and Hickin, 2003; Ellis and Church, 2005). The first attempts at defining a physically based theory to predict downstream hydraulic geometry emerged in the 1970's and 1980's (Li et al., 1976; Parker, 1979; Yang et al., 1981; Stevens, 1989). Currently, reasonably complete theories exist for predicting channel geometry in degrading, sediment supply-limited channels for which shear stress can be treated as a proxy for erosion rate (Finnegan et al., 2005; Wobus et al., 2006). For channels developed in alluvial deposits, there have been some advances in recent years, notably in attempting to incorporate some measure of boundary erodibility into the equation (Millar and Quick, 1993; Huang and Nanson, 1998; Millar, 2005; Eaton, 2006).

One important question that requires further research is the relation between stream channel dynamics and reach-average channel geometry. To tackle this issue, our ability to appropriately characterize the stream channel boundaries needs to be improved. This includes improving our ability to describe bank vegetation and bank sedimentology, as well as our ability to estimate the surface texture for an entire reach, rather than at a single point. It also requires the development of improved, reach-scale models that describe the way in which channel geometry, boundary erosion and sediment transport interact which, as Allmendinger et al. (2005) point out, is not always straightforward. Much of the current work has focussed exclusively on the processes of bank erosion and bank resistance, while paying little attention to the processes of sediment deposition, vegetation colonization and bank advance. This avenue of research may also improve our ability to describe the hydraulic geometry of stream in arid environments (e.g. Merritt and Wohl, 2003) and/or streams recently impacted by extreme flood events (e.g. Desloges and Church, 1992), if it allows us to reasonably model the processes of stream channel disturbance and recovery.

There is also a need to link the theory of downstream hydraulic geometry to sediment transport processes, which produce downstream changes in the supply and texture of alluvial sediment, and ultimately influence the downstream changes in channel gradient. Currently, landscape evolution models are being used to explore the issue of channel network evolution, but at present, those models force the channel network to conform to an imposed empirical hydraulic geometry relation, and thus cannot be used to understand why downstream relations have the form that they do, nor why that form is so common. By linking reach-scale models of stream channel geometry to network-scale models predicting the evolution of channel slope and sediment texture (e.g. Ferguson et al., 2001), it is likely that significant advances can be made on this front.

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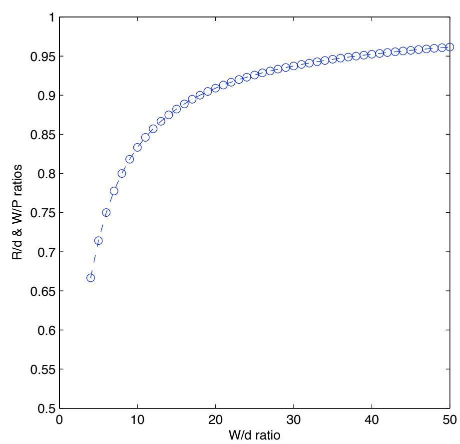

Figure 1: The ratio W/P (or equivalently R/d) is plotted against the W/d ratio, ξ

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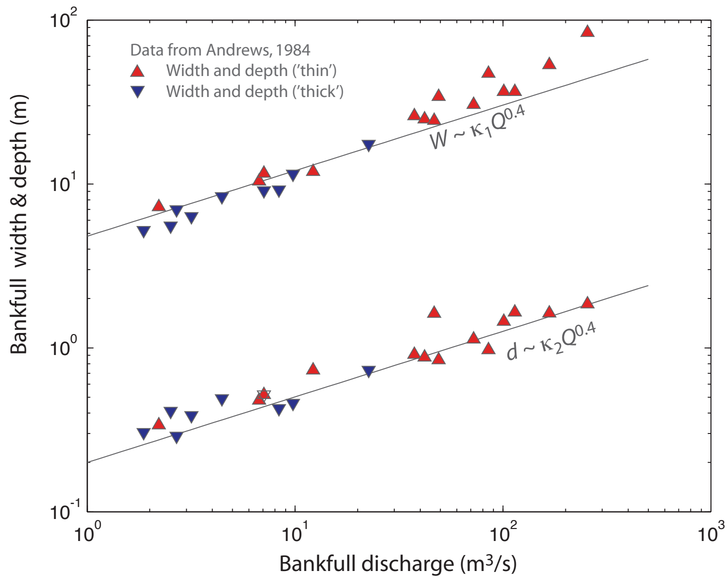

Figure 2: The data from Andrews (1984) are compared against the Froude-scaling criterion, as derived in (16)

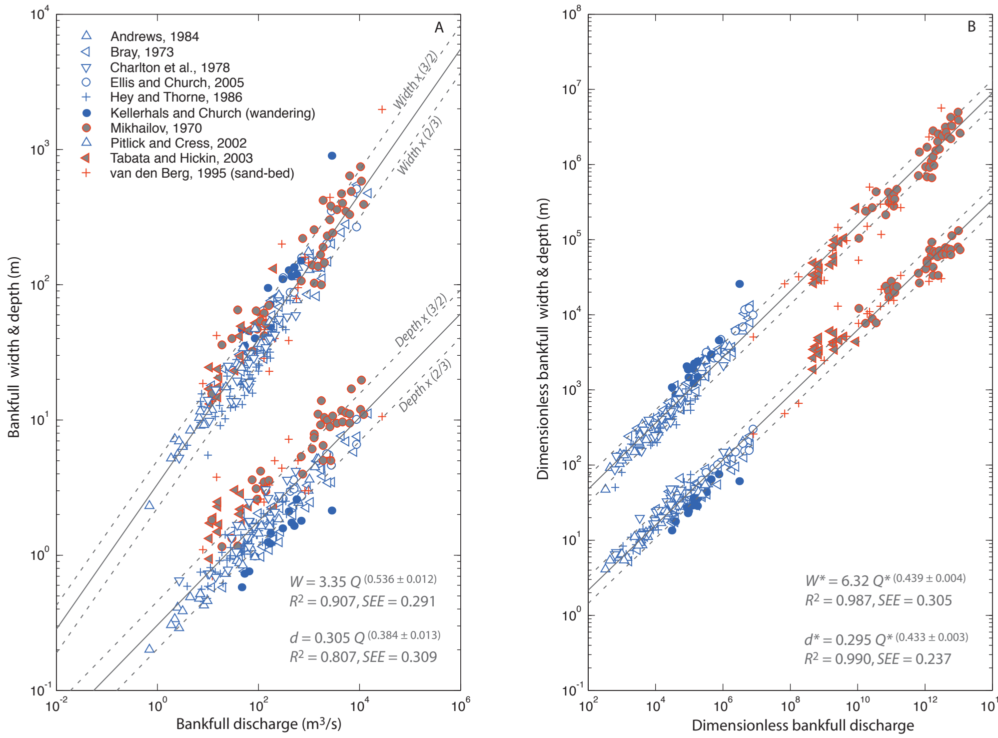

Figure 3: Downstream hydraulic geometry data from various sources plotted against bankfull discharge (Panel A) and then against dimensionless discharge (Panel B)

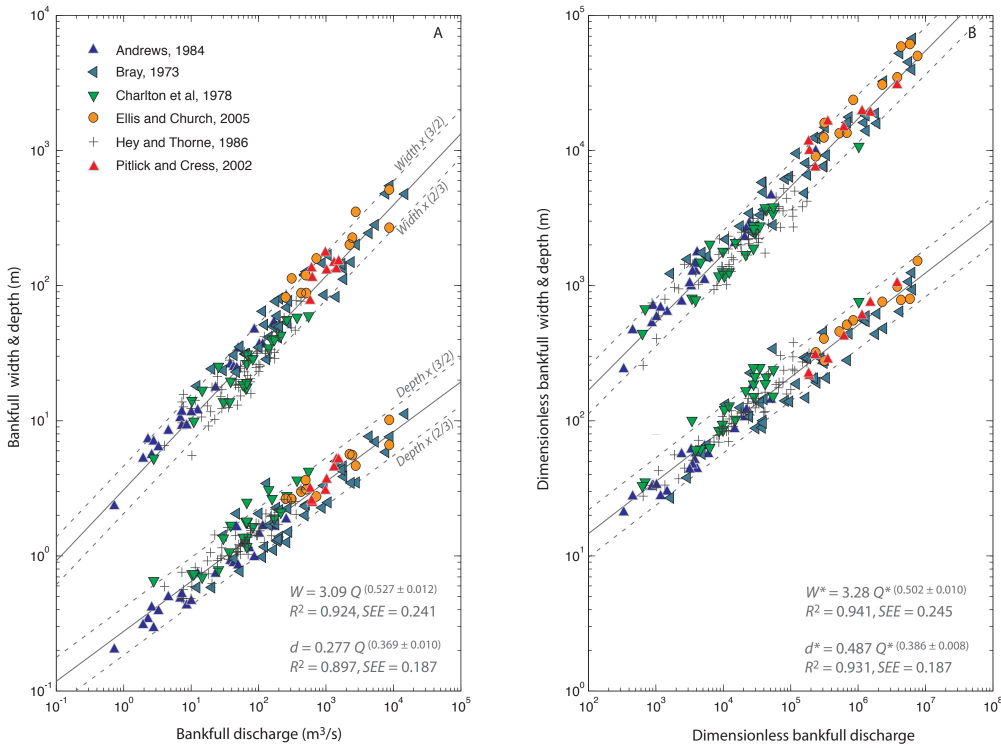

Figure 4: Downstream hydraulic geometry data for gravel bed streams plotted against bankfull discharge (Panel A) and then against dimensionless discharge (Panel B), as defined in (20)

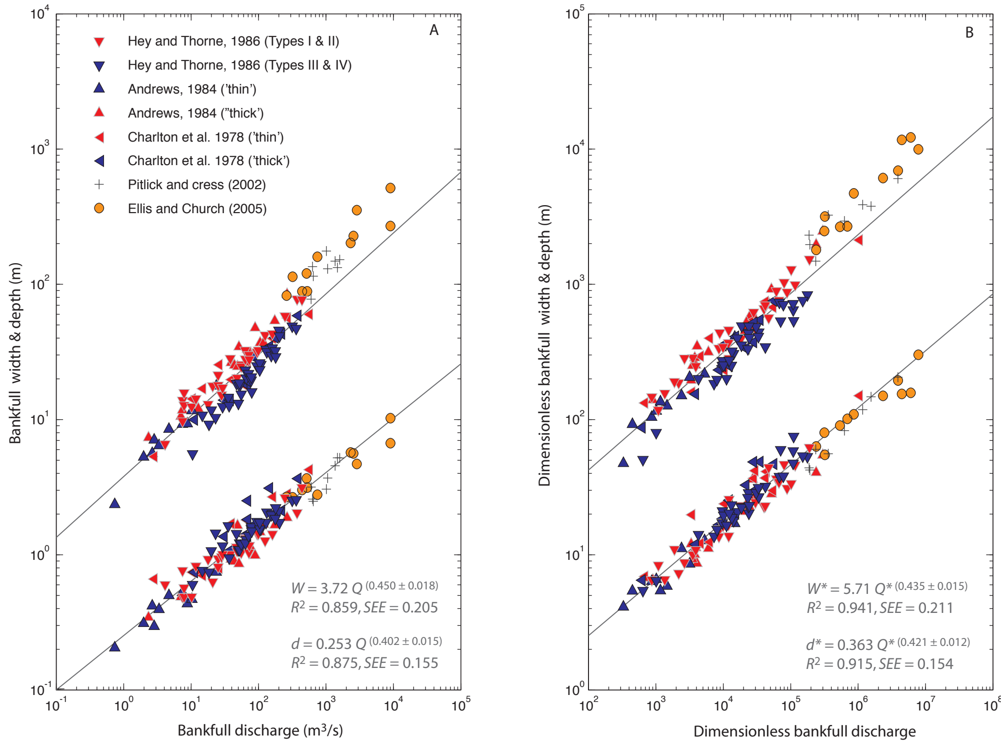

Figure 5: Data from Charlton et al. (1978), Andrews (1984), Hey and Thorne (1986), are plotted against bankfull discharge (Panel A) and dimensionless discharge (Panel B), along common trend lines for width and depth fit to all of the data (statistics of which are shown on the figure). Additional data from two large gravel bed river systems (Ellis and Church, 2005; Pitlick and Cress, 2002) are shown for reference, but were not used to determine the common trend line.