Integration and Antidifferentiation

Students are often asked to solve an integration problem. To solve this type of problem, one of the main key is to find the antidifferential, or often called antiderivative, of the function. However students often mistake the usage of the word "integration" and "antiderivative" by mixing the definition of the other. The meaning of "integration" and "antiderivative" are actually slightly different, especially if we have the case of definite integral.

So, what is the connection between integral and antiderivative?

Let say we have a function $f(x)$, which is a curve. And suppose we are asked to find area (it can be volume too) of the curve. We can add rectangle slices to find the whole area. Using more slices mean more precise result. However, calculating such a huge numbers of slices can be tedious. Other way, we can use the integral of the function $f(x)$, let name the integral function $F(x)$. In other words, $F(x) = \int f(x)dx$. Key point is finding an integral is the reverse of finding a derivative. We need to look for an antiderivative function $F(x)$ whose derivative is $f(x)$. So, basically students are asked to find the original function. Formally, an anti-derivative of a function f is a function F such that $F = f$. This process is called anti-differentiation.

To understand better, let's use some examples:

1. Consider the derivative formula $\frac{d}{dx}x^2 = 2x$. It implies that an antiderivative of $F(x) = x^2$, or we can write it as $F(x) = \int 2xdx = x^2 + C$.
2. Consider $\frac{d}{dx}sin(x) = cos(x)$. It implies that an antiderivative of $F(x) = sin(x)$, or we can write it as $F(x) = \int cos(x)dx = sin(x) + C$.

Should be emphasized that there are many antiderivatives because if $F(x)$ is an antiderivative, then so is $F(x)+4$, $F(x)-\pi$, $F(x)+10^e$, and$F(x)+r$ for any real number r, because the integral of a constant is just 0. There is no special antiderivative for a function, they are all on equal footing. In general if we have a function, we can find an antiderivative using derivative rules backwards.

To make it clear, we can use analogy of tap and tank.

Consider formula above: $\int 2xdx = x^2 + C$.

Integration is like filling a tank from a tap. The input (before integration) is the flow rate from the tap. Integrating the flow (adding up all the little bits of water) gives us the volume of water in the tank.As the flow rate increases, the tank fills up faster and faster. With a flow rate of $2x$, the tank fills up at x^2.

We have integrated the flow to get the volume. Or imagine the reverse scenario. Imagine we don't know the flow rate. we only know the volume is increasing by x^2. We can go in reverse (using the derivative, which gives us the slope) and find that the flow rate is $2x$. So, integral and derivative are reverse to each other. And what is $+C$ means? So, we use $+C$ when we have a constant value, for example, when there is already water in the tank before we fill the tank. That's why we need to use C.

Indefinite integral versus definite integral

So far, all we have done is only the indefinite integral, which means there are no "bound" in our integral. And as we can see, the antiderivative is the integral itself in indefinite integral (only $+C$ is different). That's why students often mistake the usage of those two terms. But, if we have definite integral, which means that we have the bound from a to b for the function that we need to integrate, the usage of integral and antiderivative has a big difference. The process of finding integrals of definite integrals will be almost the same as finding integrals of indefinite integrals. The difference is we need to plug in the value of a and b to the antiderivative, so we will not have any constant (+C) anymore. The Definite integral between a and b is the indefinite integral at b minus the indefinite integral at a.

Example:

1. Consider $\int_1^2 2xdx = x^2|_1^2 = (2)^2 - (1)^2 = 4 - 1 = 3$. It implies that an antiderivative of $F(x) = \int_1^2 2xdx = 3$, which is not the same as x^2.
2. Consider $\int_0^{\frac{\pi}{2}} cos(x)dx = sin(x)|_0^{\frac{\pi}{2}} = sin(\frac{\pi}{2}) - sin(0) = 1 - 0 = 1$. It implies that an antiderivative of $F(x) = \int_0^{\frac{\pi}{2}} cos(x)dx = 1$, which is not the same as $sin(x)$.

Reminder: The function we are integrating must be continuous between a and b: no holes, jumps or vertical asymptotes.

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