Footblog

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The massive social network Footblog tracks relationships based on whether two people have "enemied" each other. ("Enemyship" is a mutual agreement, meaning that a person is not allowed to "enemy" another person unless the other person agrees to "enemy" them back. No one can "enemy" themselves.)

1 Isolationism

We investigate whether Footblog's network is a single connected component.

Footblog's founder created the first Footblog account, and that account has no "sponsor" (and cannot be assigned one). Every other account must have a single, designated "sponsor" who they have "enemied". If a sponsors b, we call b the *sponsee* of a.

There are then four major actions to consider on Footblog, some of which involve others as steps:

Joining When a new Footblog member joins, they must do so by choosing as sponsor (and "enemying") someone already in the network who agrees to be their sponsor (and their enemy). After members join, they're free to "enemy" and "unenemy" anyone except their sponsor and their sponsees.

Enemying Already described above. Remember that when one person "enemies" another, the other must agree to "enemy" that person back.

Un-Enemying Unlike making an "enemy" link, one person alone can "unenemy" another person, in which case neither "enemies" the other any more.

Change Sponsor If a person wishes to change their sponsor, they must "unenemy" their sponsor and simultaneously "enemy" a new sponsor. The new sponsor must agree to act as sponsor and enemy and **must be a new enemy** (i.e., must not already be the person's enemy). Note that while a sponsee can choose to change their sponsor, a sponsor cannot choose to change their sponsee.

You may assume these actions never happen in parallel, i.e., a defined sequence occurs of the operations: joining, enemying, un-enemying, and changing sponsors.

1. Based on these rules, sketch a brief proof that when a person changes their sponsor, their new sponsor cannot also be one of their sponsees.

2. Based on these rules, either **sketch the key points in a proof that Footblog's enemy graph forms a single connected component** or **give a small sequence of actions that creates multiple components**.

   Circle **one**: **SINGLE ONLY** **MAY BE MULTIPLE**

   Provide your proof sketch or example:

2 Centrality

Footblog has defined a notion of "centrality" for its users: a user's "centrality" is the minimum number of people they'd need to go through to get a message to the person farthest from them on the network, following "enemy" links. (The "farthest" person is exactly the one to whom there is the longest minimum-length path of enemies.)

For this problem, **assume that the Footblog network does indeed form a single connected component.**

Briefly describe an algorithm to compute the centrality of a user given a graph G represented as a number of users $n > 0$ (where the users themselves are vertices named $\{v_1, v_2, \ldots, v_n\}$, a vertex number i (where $1 \leq i \leq n$) of the user whose centrality we wish to compute, and an adjacency list A of edges (i.e., an array of linked lists, where the entries in the list $A[j]$ are the vertex numbers of the users j has "enemied"). You may use any common data structures you need. **Your algorithm must run in linear (i.e., $O(n+m)$ for n nodes and m edges) time.**

```
Centrality(n, i, A):
  // Fill in your algorithm here!
```