CPSC 320 Assignment #4

February 10, 2015

Due date: Monday, 2015/02/23 at 5PM

Staple your solution behind the CPSC 320 cover page and submit in our handin box.

We guarantee that we will mark at least one part of each problem.

1. Consider the following recurrence:

$$T(n) = \begin{cases} 3T(\lfloor \frac{4n}{9} \rfloor) + 3T(\lfloor \frac{n}{9} \rfloor) + n\sqrt{n} & \text{if } n \geq 9 \\ 1 & \text{if } n \leq 8 \end{cases}$$

   (a) Draw the first two levels (the root and its children) of the recursion tree for this recurrence plus enough of the third level to understand what's going on in the tree. Label the tree by the amount of time taken by each recursive call and the total time for each level of calls. (You may ignore floors and ceilings.)

   (b) Determine the depth of the shallowest leaf in the tree. (Again, we are not concerned about floors and ceilings, and you may ignore low-order terms, e.g., reporting n rather than $n + 5$ if $n + 5$ were the correct answer.)

   (c) Determine the height of the tree (i.e., depth of the deepest leaf). (As before, ignore floors and ceilings and low-order terms.)

   (d) Based on your work, give a Θ-bound on the solution to the recurrence.

2. The newly named director of a top-secret facility wants to hire as many experts in Warp Drive as possible. Unfortunately, having been trained before the invention of Warp Drive, the director cannot tell which one(s) of the n applicants are experts in the field. Luckily, if two of the applicants spend an hour chatting, and one of them is indeed an

expert, then she will be able to tell whether or not the other applicant is also an expert. Thus, an expert can report accurately whether or not the other applicant is "Expert" or "Non-expert". The answer provided by a non-expert applicant, however, cannot be trusted. Hence the four possible outcomes of the one-hour chat are:

A says B is	B says A is	Conclusion
expert	expert	Both A and B are expert, or both are non-expert.
expert	non-expert	At least one of A and B is non-expert.
non-expert	expert	At least one of A and B is non-expert.
non-expert	non-expert	At least one of A and B is non-expert.

If more than $n/2$ applicants are non-expert, then the director cannot necessarily determine which applicants are experts using any strategy based on this kind of one-hour chats.

(a) Consider first the problem of finding a single expert among n applicants, assuming that **more than** $n/2$ of the applicants **are** experts. Give an algorithm that, given the result of $\lfloor n/2 \rfloor$ one-hour chats, reduces the problem to one of nearly half the size.

Hints: (1) In a sense, your job is to guarantee that you're throwing away at least as many non-experts as experts. How does that relate to the conditions above? (2) You will need to handle the case where n is odd, in which case, consider a statement like "If there is an applicant who has not participated in one of the one-hour chats, and *some simple condition on the number of pairs where both applicants said the other was an expert*, then discard that applicant". You'll need to decide what condition to use in the sentence in *italics* (look at some examples!).

(b) Prove that your algorithm from the previous part will retain no more than about half of the applicants **and** that more than half of the remaining applicants are experts. Be careful to keep track of each of the possible cases!

(c) Show that the experts can be identified with $\Theta(n)$ one-hour chats, assuming that more than $n/2$ of the applicants are experts. Give and solve the recurrence that descrbes the number of one-hour chats.