Exhausted of Marriage

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We modify SMP with the very reasonable change that not every woman need list every man in her preferences. She prefers to be unmarried to marrying unlisted men. Note that she clearly prefers any man on her preference list to any man not on her preference list. Men can similarly truncate their lists of women.

Here is the Gale-Shapley algorithm:

1: **procedure** STABLE-MARRIAGE(M, W)
2:     initialize all men in M and women in W to unengaged
3:     **while** an unengaged man with at least one woman on his preference list remains **do**
4:         choose such a man $m \in M$
5:         propose to the next woman $w \in W$ on his preference list
6:         **if** w is unengaged **then**
7:             engage m to w
8:         **else if** w prefers m to her fiancé m' **then**
9:             break engagement of m' to w
10:            engage m to w
11:        **end if**
12:        cross w off m's preference list
13:    **end while**
14:    report the set of engaged pairs as the final matching
15: **end procedure**

With one small change, we can apply this algorithm and ensure that the (not necessarily perfect) matching produced never marries a person to someone they left off of their preference list.

1. Make the small change necessary **to the algorithm above**.

2. Briefly sketch the key elements of a proof that the algorithm terminates.

3. We need a new definition of instability now that some people may end up unmarried. Here is one new type of instability that we call an *elopement instability*: m_i and w_j are both unmarried but list each other on their preference lists (in which case they have incentive to break the imposed matching and marry each other).

   Describe another new type of instability involving an unmarried woman. (Note: an analogous instability exists involving an unmarried man.)

4. Briefly sketch the key elements of a proof that your modified G-S algorithm cannot generate an elopement instability.

1   Even More Exhausted

Briefly sketch the key elements of a proof that your modified G-S algorithm cannot generate any of the other three types of instability (the classic SMP instability, the instability you defined above, and the analogous instability with the roles of men and women swapped).