Exponent Laws:

with base $x \neq 0$: bases in general have to be the <u>same</u>

(1) $x^a \cdot x^b = x^{a+b}$ Multiplying powers → Add Exponents

a) $3^7 \cdot 3^4 = 3^{11}$

b) $\left(\frac{1}{4}\right)^{0.9} \cdot \left(\frac{1}{4}\right)^{8.6} = \left(\frac{1}{4}\right)^{9.5}$

c) $z^{a-c} \cdot z^{2c} \cdot z^{a} = z^{2a+c}$

d) $3^2 \cdot 4^5 =$ CANNOT APPLY

(2) $x^a \div x^b = x^{a-b}$ Dividing powers → Subtract Exponents

a) $6^8 \div 6^5 = 6^3$

b) $\frac{5^{17}}{5^{12}} = 5^5$

c) $\frac{9^{12}}{9^{17}} = 9^{-5}$ or $\frac{1}{9^5}$

(3) $x^{-a} = \frac{1}{x^a}$ Negative Exponent → Reciprocal of Power

a) $3^{-4} = \frac{1}{3^4}$

b) $\left(\frac{1}{4}\right)^{-5} = 4^5$

b) $\left(\frac{?}{4}\right)$ = ?

c) $\left(\frac{7}{8}\right)^{-12} = \left(\frac{8}{7}\right)^{12}$

④ $(x^a)^6 = x^{a \cdot 6}$ Power of Power → Multiply Exponents

a) $(7^2)^4 = 7^{14}$

b) $(5^{20})^{\frac{1}{5}} = 5^4$

c) $\left(\left(\frac{6}{7}\right)^{0.5}\right)^3 = \left(\frac{6}{7}\right)^{1.5}$

d) $(x^a \cdot y^b)^c = x^{ac} y^{bc}$

e) $(x^a + y^b)^c =$ CANNOT APPLY

↳ Try Binomial Theorem if you really had to expand this.

→ $x \neq y$

Notation Reminder:

$-x^a$ is not the same as $(-x)^a$

eg. $-3^2 = -1 \times (3^2) = -1 \times 3 \times 3 = -9$

vs $(-3)^2 = (-3) \times (-3) = 9$

Logarithm Laws (Related to the above)

exponent power

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Recall: $3^2 = 9$ (3: base; 2: exponent, power) $\qquad \log_3 9 = 2$ (3: base; 9: power; 2: logarithm)

$$\boxed{\text{So } \log_x(x^a) = a \ \ \& \ \ x^{(\log_x a)} = a}$$

Inverses!

① $\log_x a + \log_x b = \log_x(a \cdot b)$ Add logs → Multiply powers

eg. $\log_2 256 + \log_2 4 = \log_2(1024)$
$\qquad 8 \qquad\qquad 2 \qquad\qquad 10$

Note: $\log_x y + \log_x y = \log_x(y^2) \overset{\text{ALSO}}{=} 2\log_x y$

so $O(\log_x y^2) = O(2 \cdot \log_x y) = O(\log_x y)$

② $\log_x a - \log_x b = \log_x(a \div b)$ Subtract logs → divide powers

eg. $\log_3 27 - \log_3 3 = \log_3\left(\frac{27}{3}\right) = \log_3 9$
$\qquad 3 \qquad\qquad 1 \qquad\qquad\qquad 2$

③ $\log_x(a^b) = b \cdot \log_x a$ Power of power → product with log

eg. $\log_2 64 = \log_2(8^2) = 2 \cdot \log_2 8 = 6$
$\qquad 6 \qquad\qquad\qquad\qquad 3$

Notation warning:

$\log_x a^b$ usually means $\log_x(a^b)$

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To show $(\log_x a)^b$, either use brackets

or write $\log_x^b a$. (Similar to $\sin^2 x = \sin x \cdot \sin x$)

(4) $$\log_x a = \frac{\log_y a}{\log_y x}$$ Change of base

eg. $\log_4 64 = \frac{\log_2 64}{\log_2 4} \rightarrow \frac{6}{2} = 3$

$\downarrow$

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So for variable n and positive constants a & b,

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$\log_b a$ = constant $= c$

$$= c \cdot \log_b n$$

$\rightarrow O(\log_a n) = O(c \log_b n) = O(\log_b n)$

So changing the base <u>does not affect</u> the Big-O complexity!

$\rightarrow$ Bases are usually omitted.