

# Save the Last Dance for Someone Other Than Me

January 7, 2017

You're arranging a formal dance evening.  $n$  people have signed up to lead and  $n$  people to follow. There will be a series of  $k$  dances (with  $k \leq n$ ), each of which will match each leader with one follower. However, the same leader and follower will never be paired for more than one dance. (So, every person will dance with  $k$  different people across the  $k$  dances.) Your job is to create the series of  $k$  matchings for the  $k$  dances.

As with SMP, you have complete preference lists over the followers for each leader and complete preference lists over the leaders for each follower. (Where leaders and followers in this problem play similar roles to men and women—or vice versa—in SMP.)

# 1 Definitely Not the Two-Step

Someone suggests that for  $k = 2$ , we can run Gale-Shapley for the first matching and then simply have each partner in a pair from the first round move their first round partner to the end of their preference lists and run Gale-Shapley again for the second round. (I.e., pairs from the first round mutually declare each other their least favorite options for the second round.)

Work through the following counterexample to the correctness of this strategy—with  $n = 3$  and the preferences of the leaders ( $l_1, l_2, l_3$ ) on the left and the followers ( $f_1, f_2, f_3$ ) on the right—to show that it does **not** necessarily work. Specifically: (a) identify the first dance's matchings, (b) write the new preference lists for the second dance, (c) identify the second dance's matchings, and (d) briefly explain why these matchings do not meet the requirements of the problem.

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l1: 2 1 3    f1: 2 1 3
l2: 1 2 3    f2: 1 2 3
l3: 3 1 2    f3: 3 1 2
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