

# THX 1138

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A (conveniently-sized) set of  $2n$  actors are applying for  $n$  roles in a play. For each role, an actor is needed as well as an *understudy*, someone who plays the role if the main actor has problems. Each actor has a preference list over the roles. Each one would rather have any of the roles than any of the understudy positions; however, if they have to take an understudy position, they have the same preference order for these as for the roles. The casting director is in charge of hiring the actors. The casting director has a list of preferences over the actors for each role (and has the same list of preferences for each understudy position as for the corresponding role). We call this problem THX (the THeatre eXtension to the Stable Marriage Problem).

We want to find a solution to THX that is *stable*.

## 1 Small Example

Here is an example with  $n = 2$ .

- $a_1$  prefers  $r_1$  to  $r_2$  (and therefore  $u_1$  to  $u_2$ )
- $a_2$  prefers  $r_1$  to  $r_2$
- $a_3$  prefers  $r_2$  to  $r_1$
- $a_4$  prefers  $r_1$  to  $r_2$
- For  $r_1$ , the casting director's preference order is:  $a_1, a_3, a_2, a_4$
- For  $r_2$ , the casting director's preference order is:  $a_1, a_4, a_3, a_2$

Give a stable matching of actors to roles and understudy positions for this problem.