THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 320 2016WT2: WEEKLY QUIZZES

Full Name: _____ Exam ID: ____

Signature: _____ UBC Student #: _____

Important notes about this examination

1. You have 25 minutes to complete this quiz.
2. **Answer all questions in PEN and write CLEARLY and LEGIBLY.**
3. You are allowed to bring up to (the equivalent of) a 3-inch 3-ring binder of notes and 3 textbooks, and nothing else. Justify all you answers.
4. Use the back of the pages for your notes, or if you need extra space for the answer to any question.
5. Good luck!

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   i. speaking or communicating with other examination candidates, unless otherwise authorized;
   ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
   iii. purposely viewing the written papers of other examination candidates;
   iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Please do not write in this space:

Quiz Number:

Tutorial Section:

Parking optimization in Wonderland

February 7, 2017

A company called Wonderland offers several types of parking permits to its employees, with different durations and prices. The coop student Alice will work in Wonderland for n consecutive days. She wants to figure out the cheapest collection of parking permits that would cover all days she needs to be present at work. Alice can buy as many permits of a given type as she likes.

Let's assume there are k type of permits $1, \ldots, k$: the price of permit type t is p_t dollars and duration D_t. Alice needs to stay at work on n consecutive days $T = [1, 2, 3, \ldots, n-1, n]$. ($T$ is for "time period".) Our goal is to help Alice to compute a collection of permits to buy and when to activate them of the form (t, d), where t is the permit type and d is the starting day for that permit. Of course, this collection of permits has to cover all days in T.

1 Let's try different types

Consider the following greedy approach to the parking problem. Start with all days in T unmarked. Construct a greedy solution as follows. Let i be the first unmarked day in T. For each permit type t starting on day i, check the number of all days in T (including i) that are covered by permit (t, i) and calculate the average cost per day. Pick (t, i) with the smallest average cost per day, add it to the solution and mark all days in T covered by this permit. Repeat until all days in T are covered.

1. Does this always produce an optimal solution?

See the next page!

2. Now, assume we have more information about the parking permits offered by Wonderland: the duration of each permit type t is $D_t = 2^t$ and the average cost per day of permits is decreasing

$$p_1/D_1 > p_2/D_2 > \ldots p_k/D_k$$

Is the algorithm described above guaranteed to produce an optimal solution?

Justify your answer in each case.