

THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 320 2016WT2: WEEKLY QUIZZES

Full Name: _____

Exam ID: _____

Signature: _____

UBC Student #: _____

Important notes about this examination

1. You have 25 minutes to complete this quiz.
2. **Answer all questions in PEN and write CLEARLY and LEGIBLY.**
3. You are allowed to bring up to (the equivalent of) a 3-inch 3-ring binder of notes and 3 textbooks, and nothing else. Justify all you answers.
4. Use the back of the pages for your notes, or if you need extra space for the answer to any question.
5. Good luck!

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - i. speaking or communicating with other examination candidates, unless otherwise authorized;
 - ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
 - iii. purposely viewing the written papers of other examination candidates;
 - iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Please do not write in this space:

Quiz Number: _____

Tutorial Section: _____



Construction site workforce optimization

February 7, 2017

A construction company is trying to solve the following cost optimization problem. The construction will take place over the next k days. A different number of workers required each day at the construction site: n_1, \dots, n_k (the number of workers required on day i is n_i). Hiring a worker cost $h > 0$ dollars, discharging a worker costs $d > 0$ dollars. Each worker gets paid $s > 0$ dollars per day, whether it's idle or not. However, if the number of hired workers is less than the required number for that day, some of the hired workers must work overtime and they get paid double rate ($2s$ dollars) for the overtime. Each hired worker can take at most 2 shifts per day (one normal shift and one overtime shift). Hence, the number of hired workers on day i cannot drop below $n_i/2$. The construction company wants to determine the optimal number of workers for each day that would minimize the total cost of the workforce, including daily salary and the cost of hiring and discharging workers. On day 0, no workers are hired.

The *Shortest paths from a single source in a graph problem* takes as the input a directed graph $G = (V, E)$, a weight function $w : E \rightarrow \mathbf{R}^+$ that assigns to every edge a positive real number (weight) and a source vertex s , and returns a shortest path from s to every other vertex in the graph. This problem can be solved efficiently by Dijkstra algorithm.

1 Reduction to Shortest paths from a single source

Give a clear and correct reduction from the Construction site workforce cost minimization problem to the Shortest path from a single source in a graph problem.

Remark: Let n_{\max} be the maximum number of workers required over the construction period (so maximum of n_1, \dots, n_k). Note that any optimal solution will not hire more than n_{\max} workers on any day.

Gradescope #:
