THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 320 2016WT2: WEEKLY QUIZZES

Full Name: _____ Exam ID: ____

Signature: _____ UBC Student #: _____

Important notes about this examination

1. You have 25 minutes to complete this quiz.
2. **Answer all questions in PEN and write CLEARLY and LEGIBLY.**
3. You are allowed to bring up to (the equivalent of) a 3-inch 3-ring binder of notes and 3 textbooks, and nothing else. Justify all you answers.
4. Use the back of the pages for your notes, or if you need extra space for the answer to any question.
5. Good luck!

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   i. speaking or communicating with other examination candidates, unless otherwise authorized;
   ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
   iii. purposely viewing the written papers of other examination candidates;
   iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Please do not write in this space:

Quiz Number:

Tutorial Section:

Greedy banks resequencing debits again :(

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Predatory banks take the debits to an account that occur over the day and reorder them to maximize the fees they can charge. For each debit that results in taking an account into overdraft (having negative balance in the account) or where the account is already in overdraft, the bank charges the customer an overdraft fee: 10% of the debited amount. For example if the sequence of debits D is \$3, \$4, \$5 and the initial account balance $B = \$8$, the optimal order (for the bank) is: \$4, \$5, \$3 with overdraft fees: $0.1 * (\$5 + \$3) = \$0.80$. Assume that the debit amounts $D = [d_1, \ldots, d_n]$ and initial balance B are in whole dollars (so \$4 is ok, but \$4.1 is not).

SUBSET SUM problem: Suppose we have an array $A = [a_1, \ldots, a_n]$ containing positive integers. For some value k, we want to know if A contains a subset of elements that sums to exactly k.

An example of an instance for SUBSET SUM: $A = [3, 7, 13, 19, 29, 37]$ and $k = 55$. This is a YES-instance, since $55 = 7 + 19 + 29$. Another example: the same A with $k = 54$, which is a NO-instance (feel free to try all 64 combinations).

Note that SUBSET SUM is NP-complete.

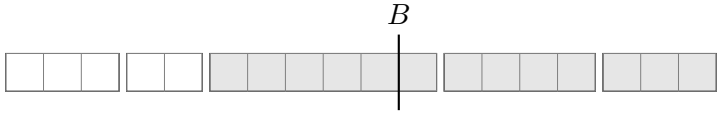
1 Greedy CEO Part 1

After hearing about the possibility of collecting the amount of fees described in the previous part, the CEO of Greedy Banks Consortium wants to find out for which collections of debits D and initial balances B (both containing only integer values) it's possible to charge this maximal fee

$$0.1 * (d_{max} + d_{sum} - (B + 1)),$$

where d_{max} is the largest and d_{sum} is the sum of all debits in D. Let's call this problem the **GREEDY CEO problem**. So the answer to an instance of this problem is "YES" if it is possible to achieve this upper-bound on fees, and the answer is "NO", otherwise.

An example of a YES-instance (with $D = 2, 3, 3, 4, 6$ and $B = 10$) is illustrated in the following diagram in which rectangles represents different debits (the number of boxes in the rectangle shows the amount of debit), shaded rectangles are debits in overdraft and the vertical line show the initial account balance:



Note that the largest debit ($6) is putting the account into overdraft and most of it (except exactly $1) is covered by the account balance.

An example of a NO-instance is the following: $D = 1, 3, 4$ and $B = 5$. Here, if we "place" $d_{max} = 4$ to right position (just $1 over the balance B), it creates two gaps, one before and one after d_{max}, of size 2, which we cannot "fill" in with debits $1 and $3 (as we are not allowed to break debits into smaller pieces).

You were tasked to write an efficient (polynomial) algorithm for the GREEDY CEO problem. You suspect that such an algorithm might not exist, so you want to prove to your boss that the problem cannot be solved by any efficient algorithm (assuming $P \neq NP$). You came up with the following reduction from the SUBSET SUM problem to the GREEDY CEO problem:

Reduction: Let $a_1, \ldots, a_n, k$ be an instance of the SUBSET SUM problem. Let $a_{max} = \max\{a_1, \ldots, a_n\}$. In order to achieve this maximum overdraft fee, we have to "pack" a subset of debits just before d_{max}. That means they would have to sum up to $B + 1 - d_{max}$. Let's construct an instance of the GREEDY CEO problem from the instance of the SUBSET SUM problem by using numbers in A as the debit amounts (so $D = A$) and setting $B = k + a_{max} - 1$.

Explain why this reduction is **incorrect**! Your explanation **must** include a simple counterexample to the correctness of the reduction.