

THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 320 2016WT2: WEEKLY QUIZZES

Full Name: _____

Exam ID: _____

Signature: _____

UBC Student #: _____

Important notes about this examination

1. You have 25 minutes to complete this quiz.
2. **Answer all questions in PEN and write CLEARLY and LEGIBLY.**
3. You are allowed to bring up to (the equivalent of) a 3-inch 3-ring binder of notes and 3 textbooks, and nothing else. Justify all you answers.
4. Use the back of the pages for your notes, or if you need extra space for the answer to any question.
5. Good luck!

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - i. speaking or communicating with other examination candidates, unless otherwise authorized;
 - ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
 - iii. purposely viewing the written papers of other examination candidates;
 - iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Please do not write in this space:

Quiz Number: _____

Tutorial Section: _____



3-SAT Variations

March 28, 2017

Recall the 3-SAT problem. Given a collection of 3-literal clauses, we want to decide whether there is a satisfiable assignment (of *true/false* values to variables) that satisfies each clause. This problem is NP-complete even if we assume that clauses cannot contain the same variable multiple times. For instance, clauses $(x_1 \vee x_1 \vee x_1)$ or $(x_1 \vee \bar{x}_1 \vee x_2)$ are not allowed.

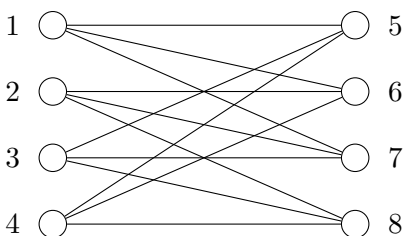
The number of occurrence of a variable x_i in the instance of 3-SAT is the number of times literals x_i or \bar{x}_i appears in the instance. For example, in instance

$$(x_1 \vee \bar{x}_2 \vee x_6) \wedge (\bar{x}_6 \vee x_3 \vee x_4) \wedge (x_2 \vee \bar{x}_3 \vee \bar{x}_5) \wedge (\bar{x}_2 \vee x_3 \vee x_5)$$

there is 1 occurrence of x_1 , 3 occurrences of x_2 , 3 occurrences of x_3 , 1 occurrence of x_4 , 2 occurrences of x_5 and 2 occurrences of x_6 .

A *bipartite graph* is a undirected graph $G = (V, E)$ in which vertices can be partitioned into two sets V_1, V_2 (so $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \{\}$) such that every edge $(u, v) \in E$ has one end point in V_1 and the other end point in V_2 . A bipartite graph is often denoted as $G = (V_1, V_2, E)$.

A graph is called *regular*, if every vertex has the same degree. For example, the following graph is a regular bipartite graph:



Note that the sizes of the two partitions (the left and right set of vertices) in a regular bipartite graph are the same.

BIPARTITE MATCHING problem: Given a bipartite graph $G = (V_1, V_2, E)$. Find a maximal matching $E' \subset E$. (Recall: no two edges in a matching share a vertex.)

The BIPARTITE MATCHING problem can be solved in time $O(|V||E|)$ by Ford-Fulkerson algorithm.

In addition, we have the following theorem:

Theorem 1 (Hall's Theorem). *A regular bipartite graph has a matching with exactly $|V_1| = |V_2|$ edges (so it involves all vertices).*

1 Darn 2-clauses!

Consider a special version of 3-SAT that requires that each variable has exactly 3 occurrences, but allows each clause to have either 2 or 3 literals. Let's call this problem 2,3-SAT3. Here is an example of an instance of 2,3-SAT3 problem:

$$(x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_4) \wedge (x_1 \vee \overline{x_4}) \wedge (x_2 \vee \overline{x_3}) \wedge (x_3 \vee x_4)$$

Note that each variable has exactly 3 occurrences.

We want to show that 2,3-SAT3 is NP-hard¹. Follow the following steps to reduce 3-SAT to 2,3-SAT3. For simplicity, you may assume that in each instance of 3-SAT, every variable has at least 2 occurrences.

- (a) Given variables y_1, y_2, \dots, y_k , $k \geq 2$, design a set of 2-literal clauses such that there are exactly two assignments for these variables that satisfy the set of your designed clauses: (i) $y_1 = y_2 = \dots = y_k = \text{false}$, and (ii) $y_1 = y_2 = \dots = y_k = \text{true}$. Each variable should have exactly 2 occurrences in these clauses.

- (b) Now consider an instance S of 3-SAT in which variable x_i has $k \geq 2$ occurrences. Using part (a) construct a new instance S' by replacing all occurrences of x_i with new variables y_1, \dots, y_k . Add necessary clauses to make the original instance S and the new instance S' equivalent (one is satisfiable if and only if the other is satisfiable). Argue why each of these new variables has exactly 3 occurrences in S' .

¹This is somewhat surprising, since both 2-SAT and 3-SAT3 are in P .

Gradescope #:

- (c) Describe a reduction from 3-SAT to 2,3-SAT3 as an algorithm.
You don't need to prove correctness of your reduction!