CPSC 320 2019S2: Tutorial Quiz 4, Individual Portion

July 26, 2019

CS ID

Please enter your 4 or 5 digit CSID in this box:

Tiles and Tribulations

You're given a grid of size $n \times n$, where $n = 2^k$ for some $k \geq 1$, with one cell missing. Your job is to cover the board with *L-tiles*. An L-tile is three squares that form an L shape (i.e., a 2×2 square with one square missing).

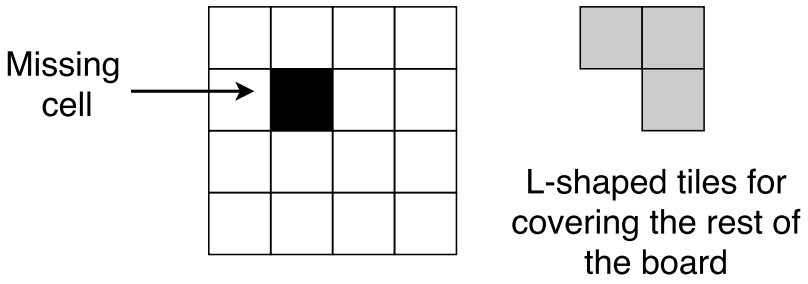

Figure 1: A 4×4 board, with a missing cell at position $(2, 3)$ (using 1-based indexing from bottom left).

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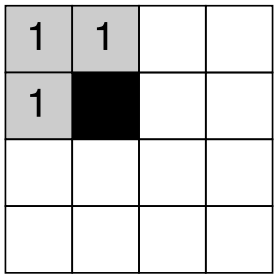
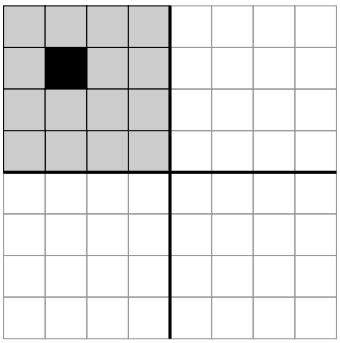
The L-tiles:

- Must cover every white square of the board.
- Must NOT cover the single black square on the board.
- Are not allowed to overlap.

1. Draw the correct arrangement of L-tiles for the 4×4 board below. We've already indicated the position of one tile; draw on the grid below where to place the other four tiles.



2. Consider the $2^3 \times 2^3$ figure below, where the top left quadrant (which contains the missing cell) is the $2^2 \times 2^2$ board that you covered with L-tiles in the question above.



Suppose you knew you could cover **any** $2^2 \times 2^2$ board with one cell missing. If we assume this is true, describe where you should place the next L-tile in the $2^3 \times 2^3$ board above to **guarantee** that you could then find a covering for the whole board, and explain why this will work.

3. Can we solve this problem for boards of size $2k \times 2k$ for all $k \geq 1$? (Note that this is a **different problem** than the one we've been addressing, which is for boards of size $2^k \times 2^k$.) If yes, sketch a proof that we can always solve the problem. If no, give and explain an example or situation where the problem is impossible to solve.