CPSC 320 Notes: The Futility of Laying Pipe, Part 1

UBC recently replaced its aging steam heating system with a new hot water system. A set of locations needs water delivered and there's another set of intermediate points through which we can deliver water. Some of these points can be connected—at varying costs—by laying new pipe, others cannot. You'd like to figure out the cheapest way to connect every delivery location to water.

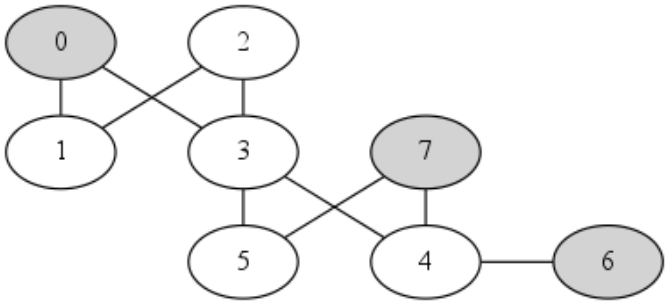
1 Steiner. . . Something-or-Others

Let's abstract and formalize this problem. We'll call it the Steiner Problem (SP).

An instance of SP is an undirected graph $G = (V, E)$ and a subset $S \subseteq V$ of the vertices to which we must deliver water. A solution to the instance is a subset $E' \subseteq E$ of the edges which connects all vertices in S (and perhaps some in V). The **best** solution is the one with the fewest edges. (We could make this into an "input file format", e.g., by reading a number n indicating $|V|$ followed by n lines, where line i lists the vertex numbers of all vertices connected to i, etc.)

(Although we've ignored the costs, we could easily have included them by making the edges weighted.)

1. Here's a SP instance, where shaded nodes are in S. Indicate a solution to this problem.



2. Build three trivial SP instances with their solutions.

3. Build two small but non-trivial SP instances with solutions.

4. Give at least two problems similar to this one that have we solved before.

5. Now, think about what a solution to SP looks like. Is it a path? A cycle? A tree? A graph?

6. Once you've figured that out, give a **very** similar problem we've solved before in **polynomial time**. Can we just use the solution we used for that problem? If you wanted to try using the solution to that problem, how would you modify it when reporting a solution to the SP instance?

7. Describe how to turn SP into a decision problem (one where the answer is `YES` or `NO`). (Remember how we did this for, e.g., independent set (IS). The original version of IS was "given a graph, find the largest independent set". The decision version was "given a graph and a number k, is there an independent set at least as large as k?". Do the same sort of transformation to SP.)

8. Prove that the decision problem is in NP. Remember: it's in NP if it's "efficiently certifiable". The "certificate" is usually what we'd think of as the solution to the non-decision problem.