Lecture notes 1 (Introduction → vectors) ①

* Presentation: Can Selçuk: cselcuk@math.ubc.ca

websites: blogs.ubc.ca/cselcuk

common website: www.math.ubc.ca/~cbm/math253 → 2017/

* Office hours: monday and ~~wendes~~ wednesday

Starting the 11/09/2017 1pm to 2pm

13/09/2017 " "

→ LSK 300B room

* My office: pulp and paper PPC 323

⚠ Building is closed after 4:30pm

* Administration issue:

* Grades: 2 midterms in classe → 20% each

homework 5% (every 2 weeks)

Webwork 5% (every week) (not up yet → will be tomorrow

Final exams 50%

→ First homework will be due 15 september (next week)

→ 11 october and 15 november (wednesday)

* Textbook Apex Calculus (can be found online) for free

- <u>Motivates lecture</u>

~> show video: example of multivariable function $f(x, y, z, t)$.

~> In applied science: you need to understand well this lecture as you will need this knowledge to compute and visualize your data/solution.

~> You will learn later the computational aspects but the underlying math are multivariable functions (and partial ~~derivatives~~ differential equations - PDE)

~> you have learned so far how to deal with one variable $y = f(x)$, especially derivatives and integrals.

~> we will extend these notions to functions of two or three (or more) variables $z = f(x, y)$, $w = f(x, y,$

~> one of the biggest differences between single and multivariable calculus is

<u>Visualization and Geometry in 3D $(\mathbb{R}^3)$</u>

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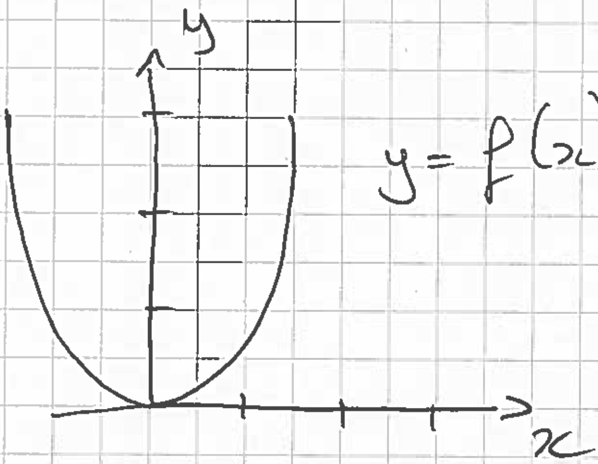
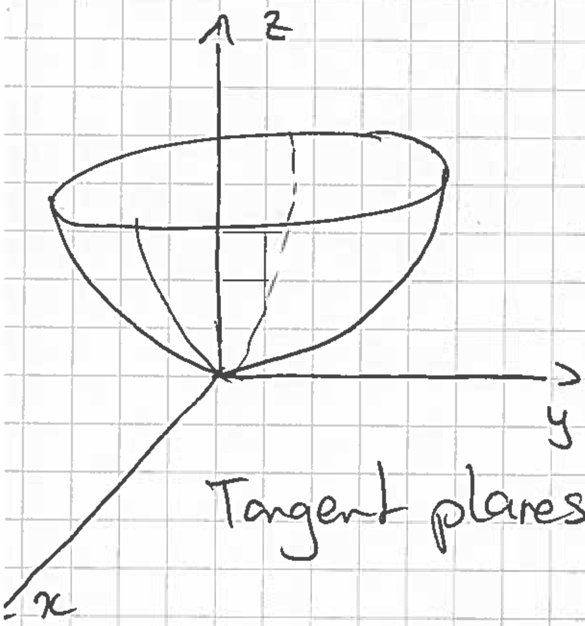
* Most of the function you have seen so far can be understood in the plan.



$y = f(x)$

Concepts like slope, area under a curve have higher dimensional analogues:

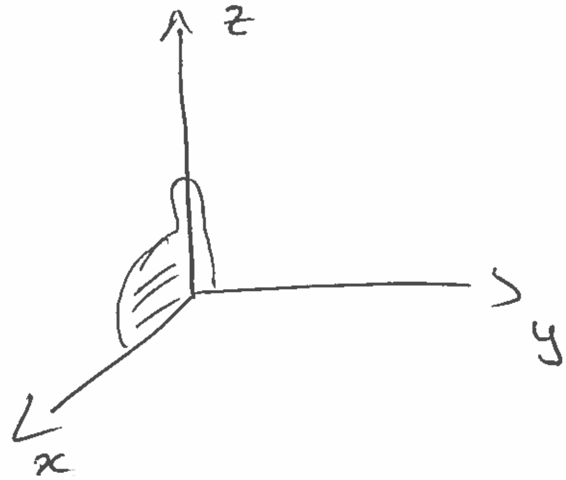


$z = f(x, y)$

Tangent planes, volume, surface area....

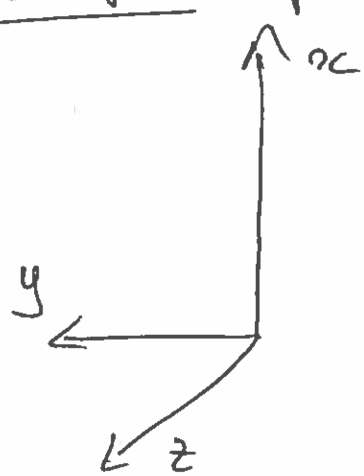
$\rightsquigarrow$ We start with the chapter 10.1 of the Apex.

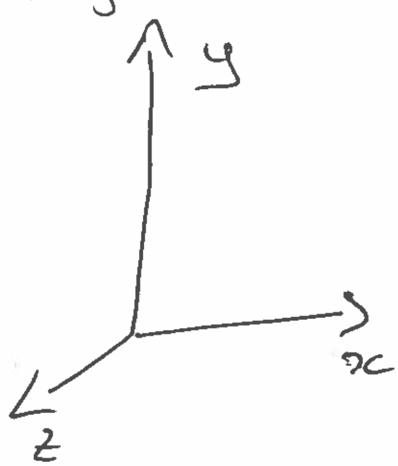
* 3-d Coordinates



We use the right-hand system: we draw the positive coordinate axes satisfying right hand rule (thumb is aligned with the z-axis, index finger with the x-axis and the middle finger with the y-axis)

<u>examples</u> of correct (right-hand) systems:

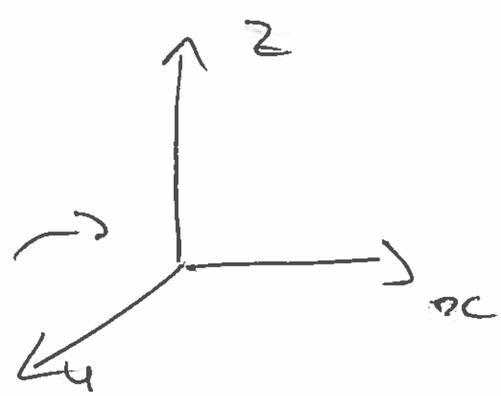




(these are obtained by rotating the right hand.)

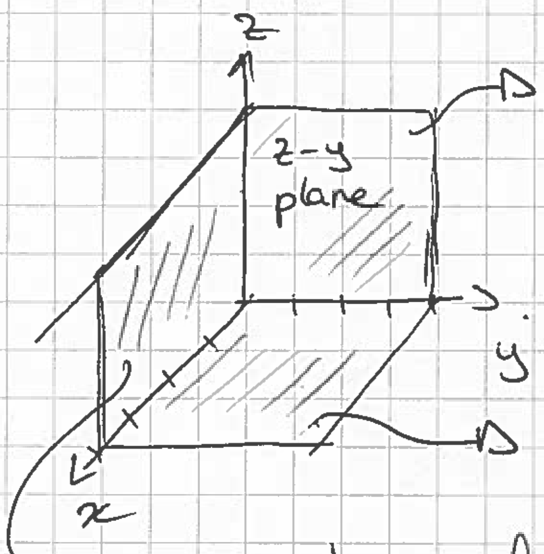
<u>example</u> of uncorrect system

mirror image



Planes :

Planes are determined by a linear equation :

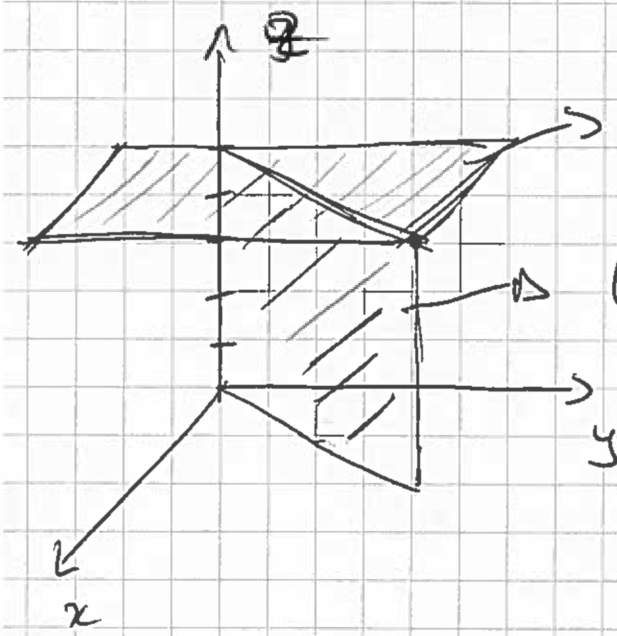


→ z-y plane eqn $x = 0$

→ x-y plane of eqn $z = 0$

→ x-z plane of eqn $y = 0$.

example

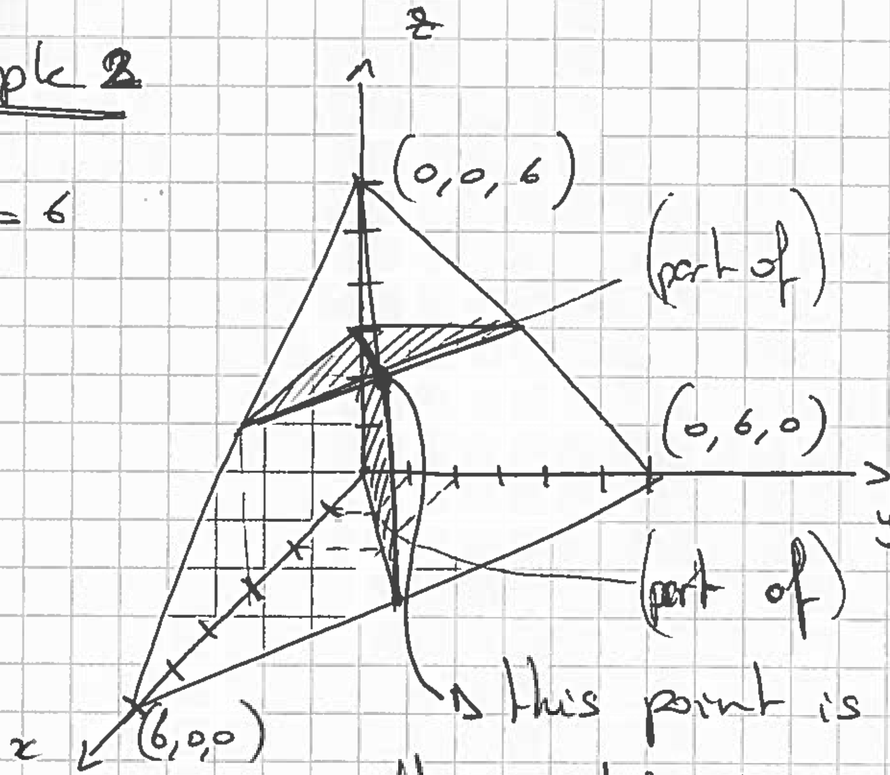


→ (part of) $z = 4$ plane

→ (part of) $x = y$ plane

* example 2

$x+y+z=6$



▷ this point is solution to the equations

$x+y+z=6 \quad z=3 \quad y=x$

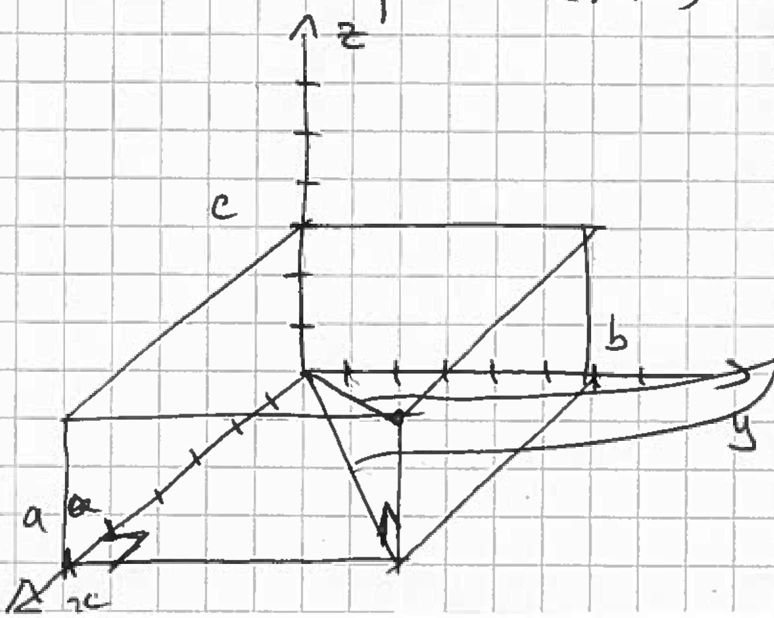
↳ $x+x+3=6$

↳ $2x=3 \Rightarrow x=3/2$

this point has coordinates $(3/2, 3/2, 3)$

* Distances

distance D from $(0,0,0)$ to (a,b,c)



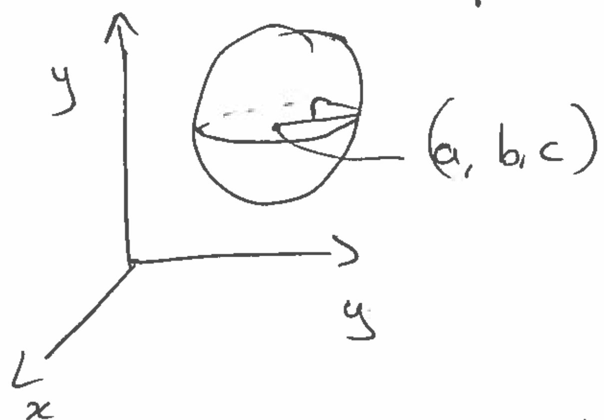
▷ length is $\sqrt{a^2+b^2}$ (Pythagorus)

▷ length is $\sqrt{\sqrt{a^2+b^2}^2+c^2}$ (Pytagorus again)

$D=\sqrt{a^2+b^2+c^2}$

* equation of a sphere

If we fix a point (a, b, c) and a distance r (radius) and let the other point very (x, y, z), we get the equation of a sphere of radius r centered at point (a, b, c)



for all points on the surface of the sphere:

$$r = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$$

a solid sphere is such that (r is fixed)

$$\underbrace{\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}} \leq r$$

distance with respect to the centre of the sphere. It has to be less than r for the interior points and equal to r for the points <u>on</u> the sphere's surface.

<u>example</u>: what is the sphere determined by $x^2+y^2+z^2 = 8x - 4y$?

$\leadsto$ solution: $x^2 - 8x + 16 + y^2 + 4x + 2 + z^2 = 16+4$

$\Rightarrow (x-4)^2 + (y+2)^2 + z^2 = (\sqrt{20})^2$

$\Rightarrow$ sphere centred at point $(4, -2, 0)$

of radius $\sqrt{20}$