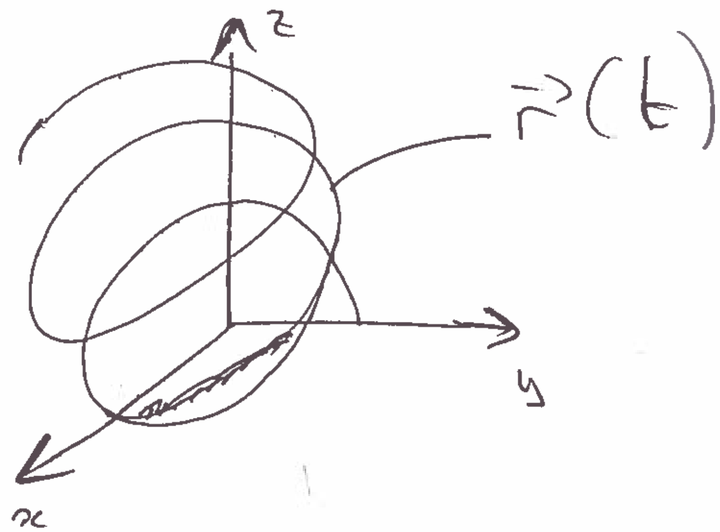
Review of curves and calculus   Lecture 5   ①

We have seen parametric equations for lines $\vec{r}(t) = \vec{r_0} + t\vec{v}$. More generally, $\vec{r}(t)$ can describe a curve in $\mathbb{R}^3$.

$$\boxed{\vec{r}(t) = \langle f(t), g(t), h(t) \rangle}$$

Example ① equation of a line
② $\vec{r}(t) = \langle \cos t, \sin t, t/4\pi \rangle$



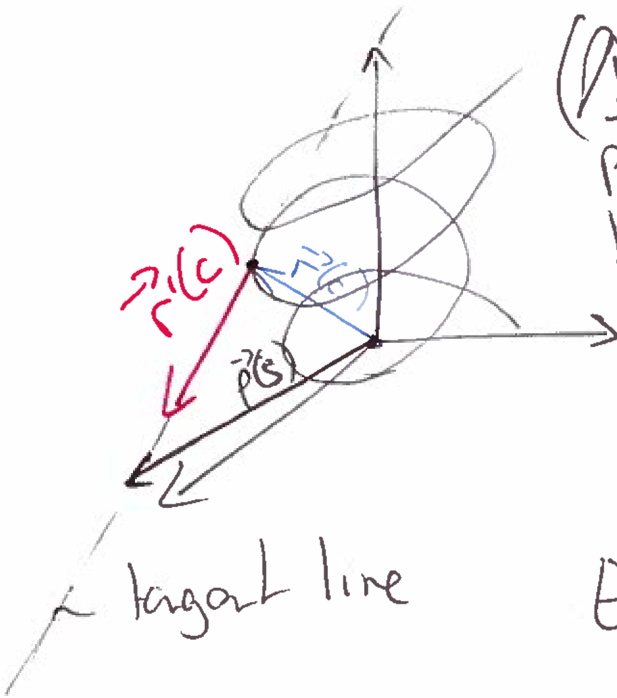
=> The derivative of a parametrized curve is

$$\boxed{\frac{d}{dt}\vec{r}(t) = \vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle}$$

One meaning is the velocity of a particle following $\vec{r}(t)$.

From this, we can get the tangent line to $\vec{r}(t)$ at the point $\vec{r}(c)$. ②

$$\Rightarrow \quad \vec{\rho}(s) = \underbrace{\vec{r}(c)}_{\text{fixed point}} + \underbrace{s}_{\text{new parameter}}\,\underbrace{\vec{r}\,'(t)}_{\text{fixed vector}}$$



(⚠ this is not present in the Apex book at definition 71)

Eg $s = t - c$

where does this line intersects the y-z plane ? ③

~> y-z plane is defined by $x = 0$

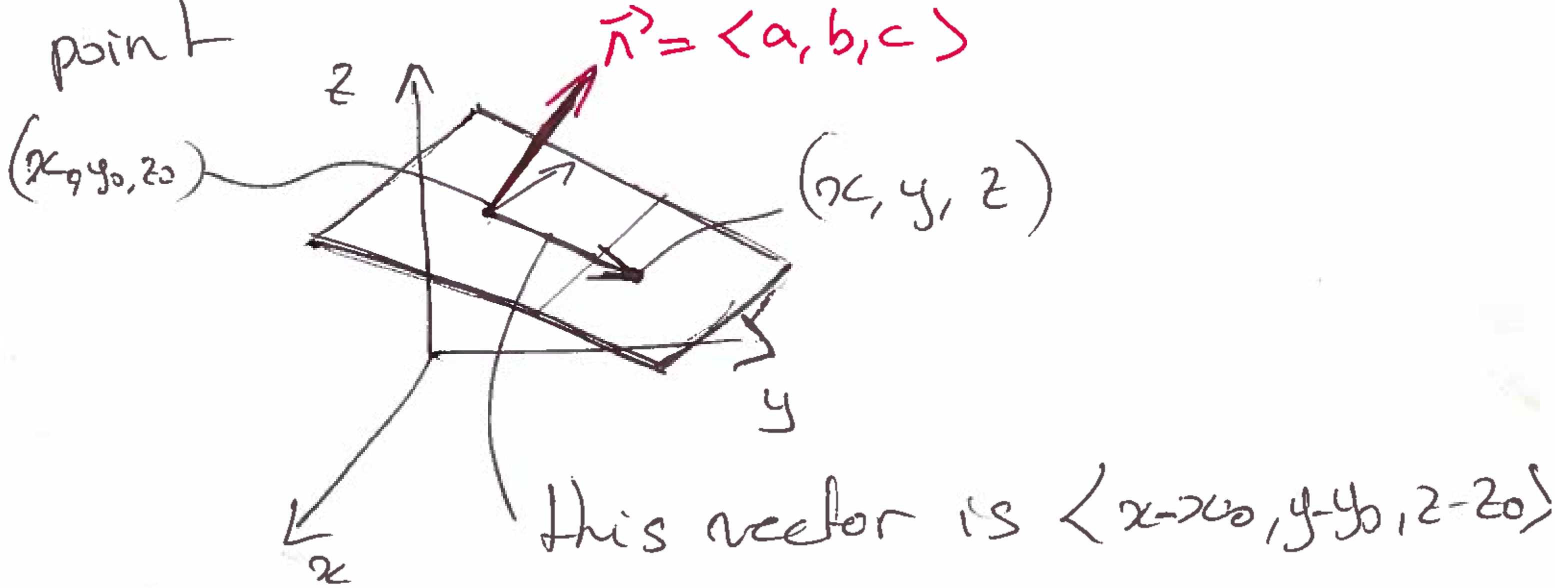
~> $\boxed{\begin{matrix} y = -8 \\ z = +9 \end{matrix}}$

⚠ This was done in a lecture 4

The point where the line intersects the y-z plane is $(0, -8, 9)$.

=> is there a similar way to determine a plane (with vectors) ?

=> A Plane is determined by a point on it (x_0, y_0, z_0) or a normal to this point

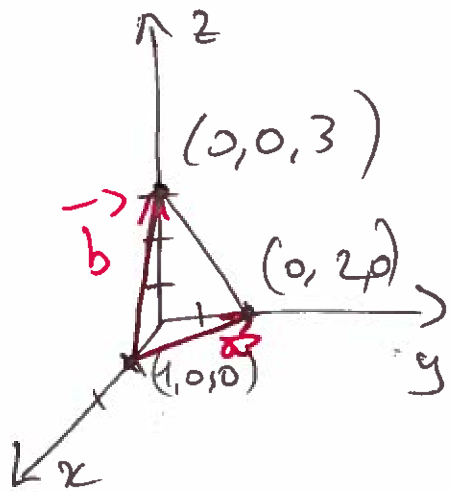


(x, y, z) is on the plane determined by $\vec{n}$ and (x_0, y_0, z_0) if and only if the vector $\langle x-x_0, y-y_0, z-z_0\rangle \cdot \langle a, b, c\rangle = 0$

$$\Longleftrightarrow \boxed{ax + yb + cz + d = 0} \qquad \vec{n} = \langle a, b, c\rangle$$

with $d = -ax_0 - by_0 - cz_0$

Example: find the equation of the plane passing through the ~~planes~~ points $(0,0,3)$, $(0,2,0)$ and $(1,0,0)$



$\vec{a} = (-1, 2, 0)$
$\vec{b} = (-1, 0, 3)$

Compute $\vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 6\vec{i} + 3\vec{j} + 2\vec{k}$

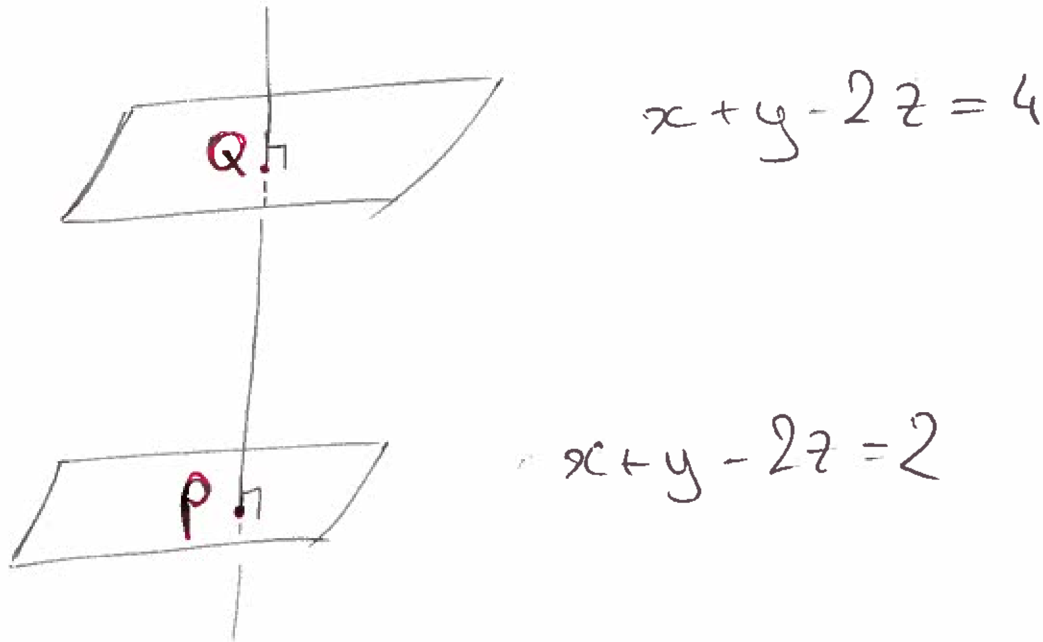
any point (x, y, z) on the plane satisfies

$$6(x - x_0) + 3(y - y_0) + 2(z - z_0) = 0$$

pick $x_0 = 0$, $y_0 = 0$, $z_0 = 3$ or $x_0 = 0$, $y_0 = 2$, $z_0 = 0$ or $x_0 = 1$, $y_0 = 0$, $z_0 = 0$

$$\Rightarrow \boxed{6x - 6 + 3y + 2z = 0}$$

Problem: Find the distance between the 2 parallel planes : $x+y-2z=2$ and $x+y-2z=4$



$\Rightarrow$ Choose any point on the plane for P. We choose $\vec{OP} = \langle 2,0,0 \rangle$. The desired distance is $|\vec{PQ}|$.

$\Rightarrow$ The line through ~~tos~~ P and Q is :

$$\boxed{\vec{r}(t) = \vec{OP} + t\vec{v}}$$

with $\vec{v} = \langle 1, 1, -2 \rangle$ (from the equations of the planes)

any point x, y, z on the line is $\vec{r}(t) = \langle 2,0,0 \rangle + t\langle 1, 1, -2 \rangle$

$$\vec{r}(t) = \langle x, y, z \rangle = \langle 2+t, t, -2t \rangle$$

$\Rightarrow$ Now find the value $t = t_q$ which corresponds to the point Q. ⑦

$$\vec{r}(t_q) = \langle x_q, y_q, z_q \rangle = \langle 2 + t_q, t_q, -2t_q \rangle \quad *$$

as the point Q is on the plane, ~~which~~ it satisfies:

$$x_q + y_q - 2z_q = 4$$

So it comes: (inject * into the equation above)

$$2 + t_q + t_q - 2(-2t_q) = 4 \Rightarrow 6t_q = 2$$

$$\Rightarrow \boxed{t_q = \frac{1}{3}}$$

$$\Rightarrow |\vec{PQ}| = |\vec{r}(t_q) - \vec{r}(t_0)| = |\vec{r}(t=0) + t_q\langle 1, 1, -2\rangle - \vec{r}(t_0)|$$

$$|\vec{PQ}| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{-2}{3}\right)^2} = \sqrt{\frac{6}{9}} = \sqrt{\frac{2}{3}}$$

$$\boxed{|\vec{PQ}| = \sqrt{\frac{2}{3}}}$$