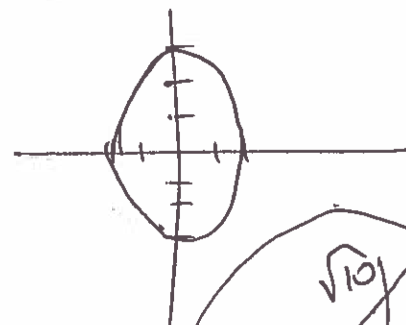
Cylinders and Quadric surfaces

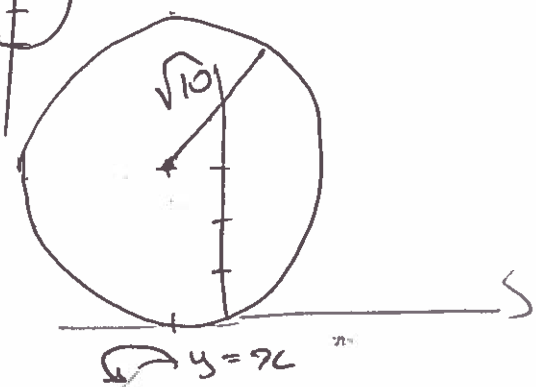
In the xy plane, a curve given by an equation involving x^2, xy, y^2, x, y or const are given by circles, ellipses, paraboles or hyperboles $\Rightarrow$ these are quadric curves.

example : $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1 \Rightarrow$ ellipse



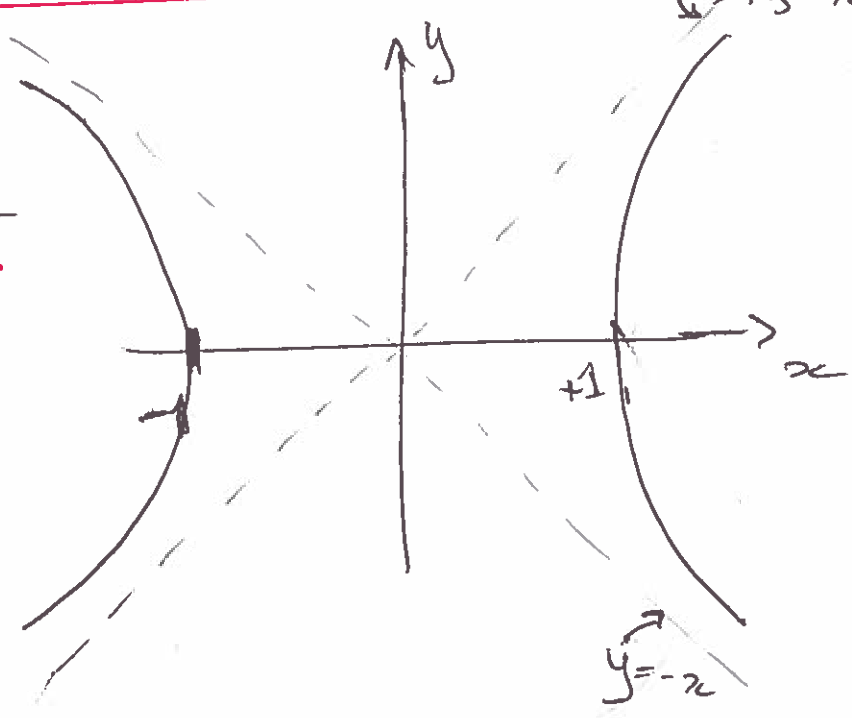
or the circle defined by $x^2+y^2+2x-6y=0$

$\Rightarrow (x+1)^2+(y-3)^2=(\sqrt{10})^2$



or

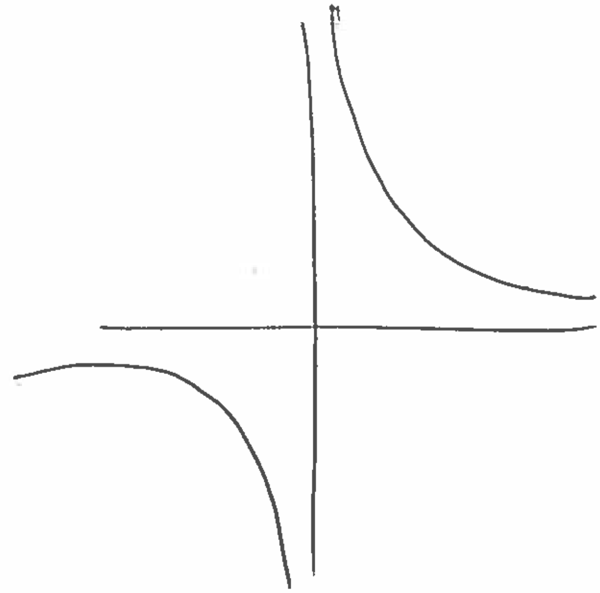
$x^2-y^2=1$



<u>note</u> $\frac{y^2}{x^2} = 1 - \frac{1}{x^2}$ and as $x^2 \to \infty$ $\frac{y^2}{x^2} \to 1$

So $y \sim \pm x$ are the two asymptotes.

or $\underline{xy = 1}$

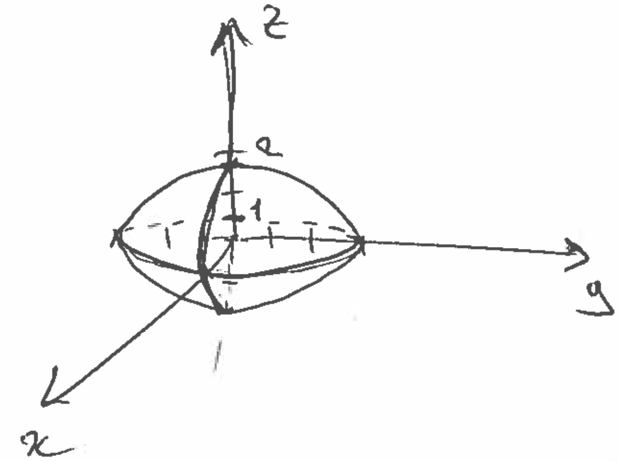


here the asymptotes are $x = 0$ and $y = 0$

=> in 3D, a <u>quadric</u> <u>surface</u> is a surface given by an equation involving $x^2, y^2, z^2, xy, yz, xz, x, y, z$, const.

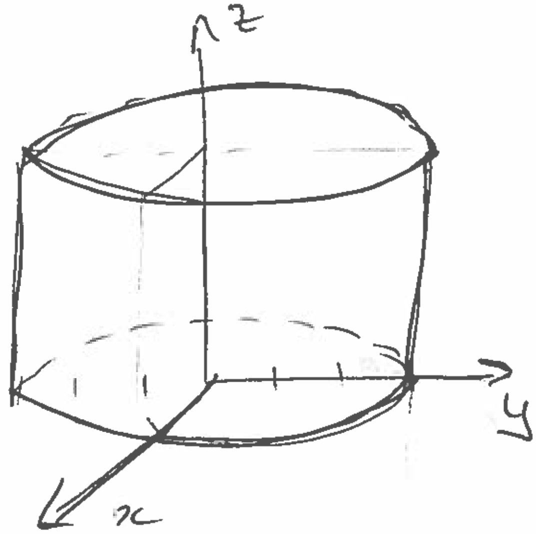
<u>example</u> : * sphere $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$

* ellipsoid $x^2+\frac{y^2}{9}+\frac{z^2}{4}=1$



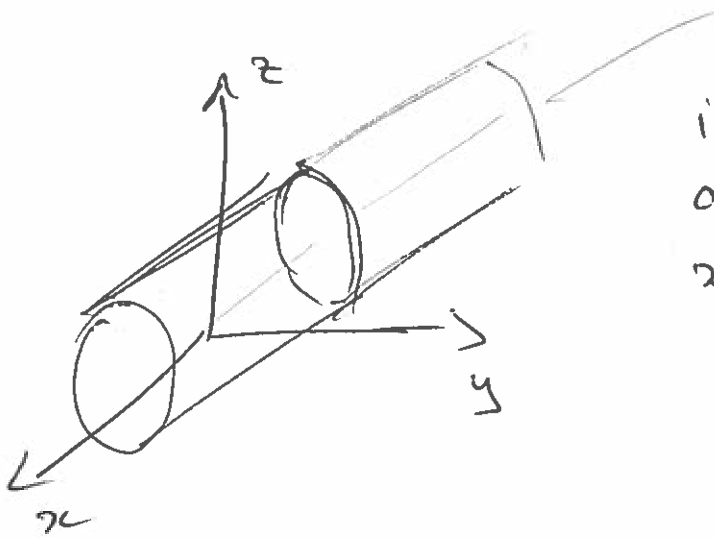
If the equation involves only 2 variables intsead of three, then the surface is a cylinder.

$$x^2 + \frac{y^2}{9} = 1$$



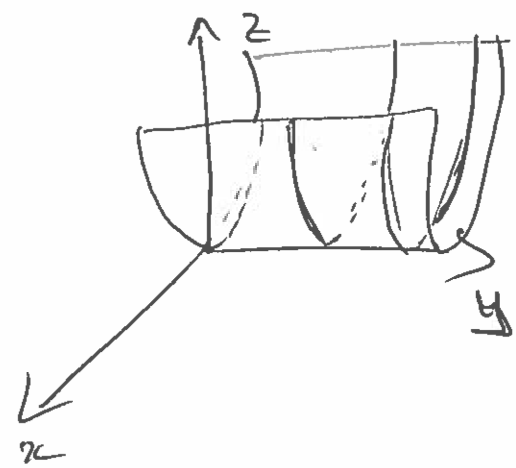
invariance along z direction

$$y^2 + z^2 = 1$$



invariance along the x direction

$$z = x^2$$



invariance along the y direction

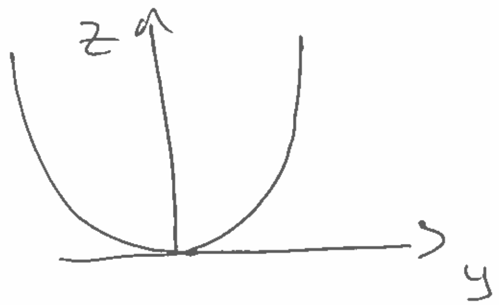
to sketch more general surfaces we use <u>trace curves</u> that we get when we cut the surface by planes parallel to coordinate planes.

<u>example</u>: use of trace curves

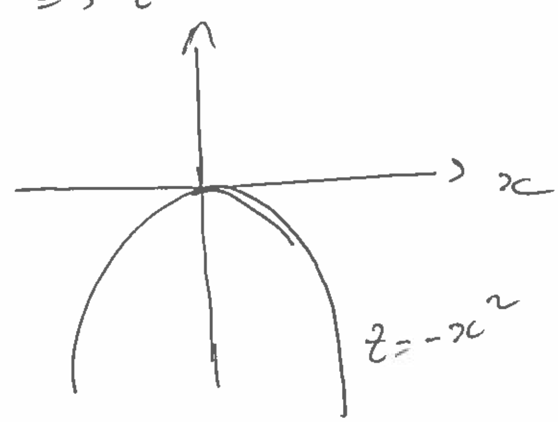
$\Rightarrow$ we want to sketch the surface

$\rightsquigarrow$ $\boxed{z = y^2 - x^2}$

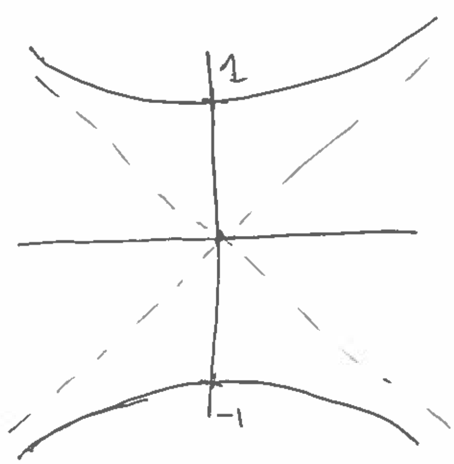
$x = 0 \Rightarrow z = y^2$



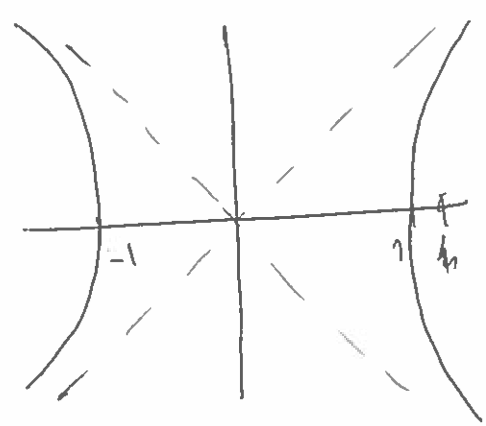
$y = 0 \Rightarrow z = -x^2$



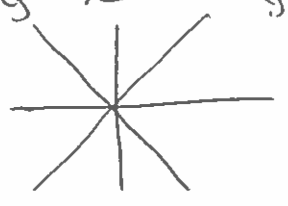
$z = 1 \Rightarrow 1 = y^2 - x^2 \Rightarrow y = \sqrt{1 + x^2}$



$z = -1 \Rightarrow y = \sqrt{1 - x^2}$



$z = 0 \Rightarrow y^2 - x^2 = 0 = (y - x)(y + x)$



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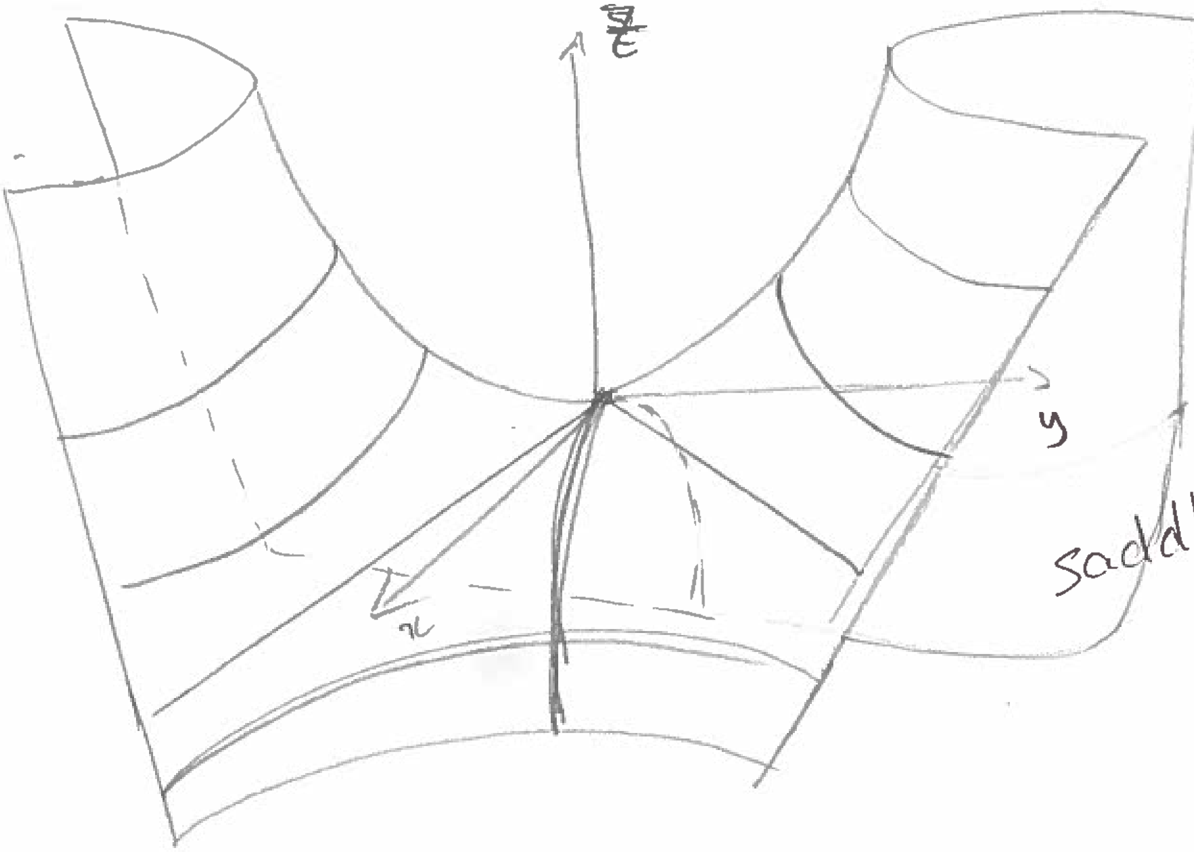

horizontal traces => hyperboles
vertical traces => paraboles

=> cutting by horizontal planes give us contour curves