Functions of several variables (2 or 3)

Lecture 7

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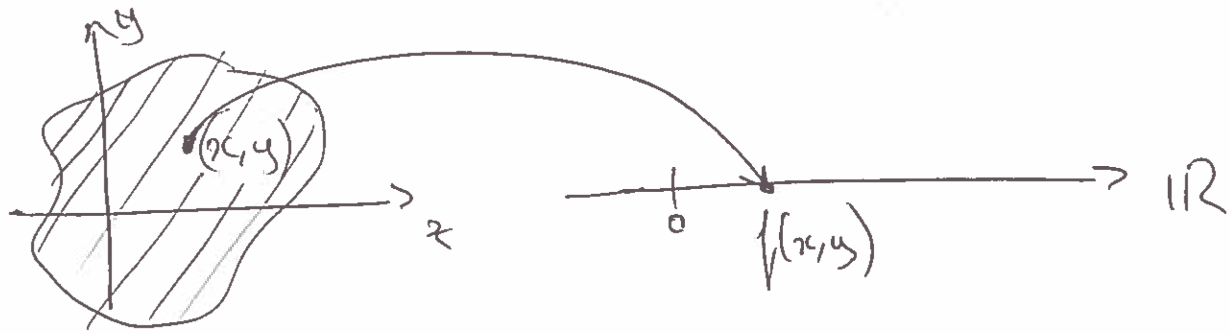
<u>Definition</u>: A function f of 2 variables with domain $D \in \mathbb{R}^2$ is a rule that assigns to each $(x,y) \in D$ a unique number $f(x,y)$

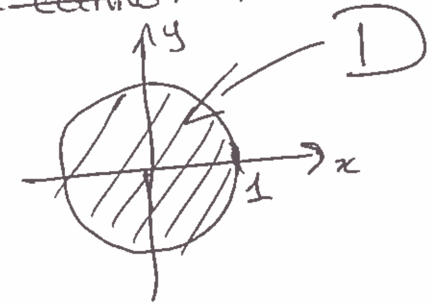


<u>examples</u> of domains:

a) $f(x,y) = \sqrt{1-x^2-y^2} \Rightarrow D = \{(x,y) : x^2+y^2 \leq 1\}$

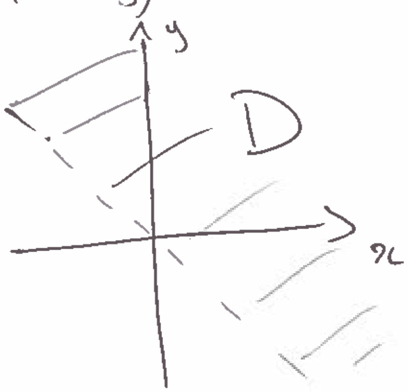
$1-x^2-y^2 \geq 0$

(~~you cannot have~~



b) $g(x,y) = \ln(x+y)$

$D = \{(x,y) : x+y > 0\}$



$\rightsquigarrow x+y=0$ (the line $y=-x$ is not included)

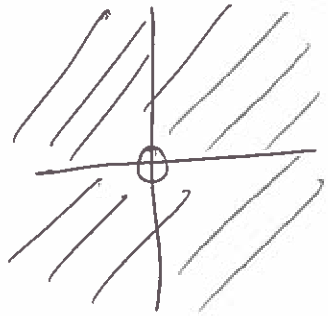
⚠ Advertise MLC

⚠ Homework 1 are returned at MLC

⚠ office hours 1-2 pm
~ come ask questions

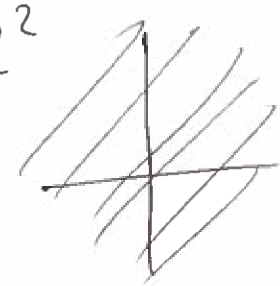
⚠ fix the error signs
the previous class ...

) $h(x,y) = \frac{1}{x^2+y^2}$ $\quad D = \{(x,y) : (x,y) \neq (0,0)\}$



everywhere except the origin.

d) $k(x,y) = e^{x^2+y} + \sin(y^4+3x)$ $\quad D = \mathbb{R}^2$

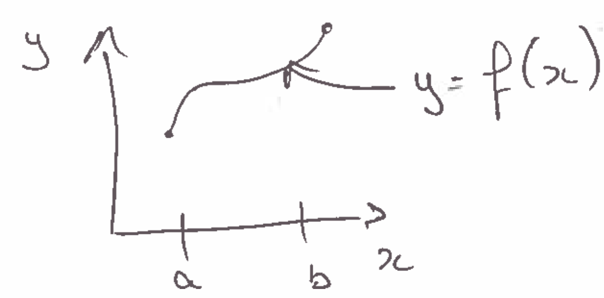


Graphs of functions

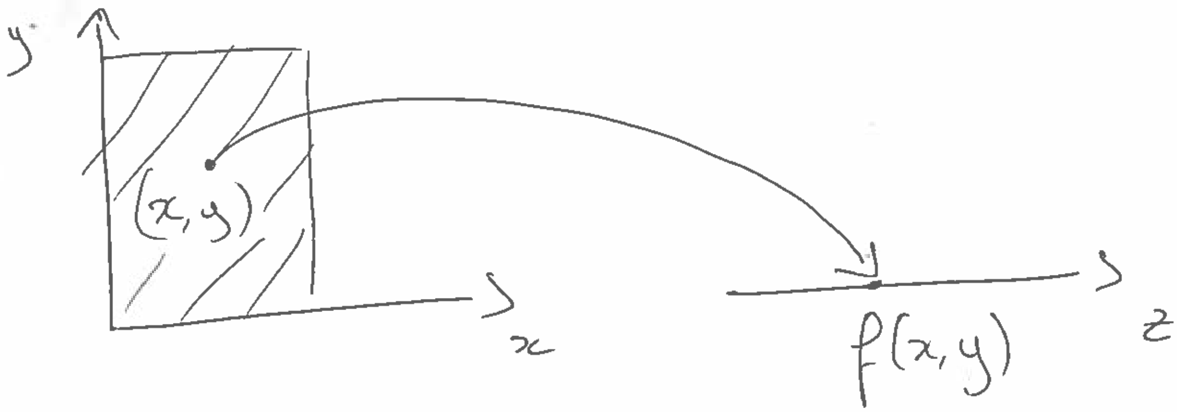
just like 1-variable case, we use graphs to understand functions.

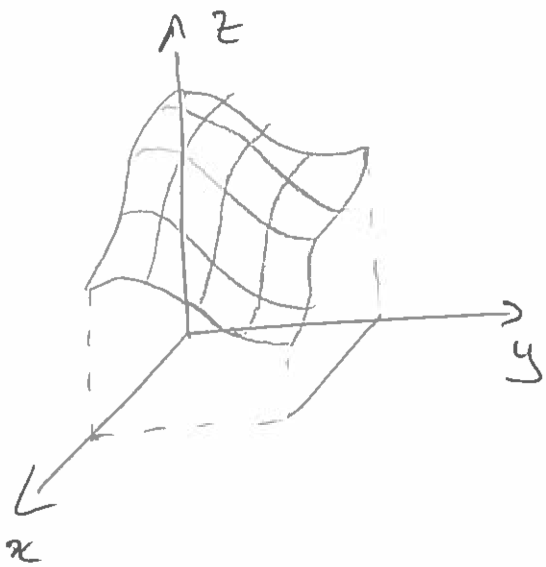
=> function of one variable:

=>



two-variables case





graph is a surface $z = f(x,y)$

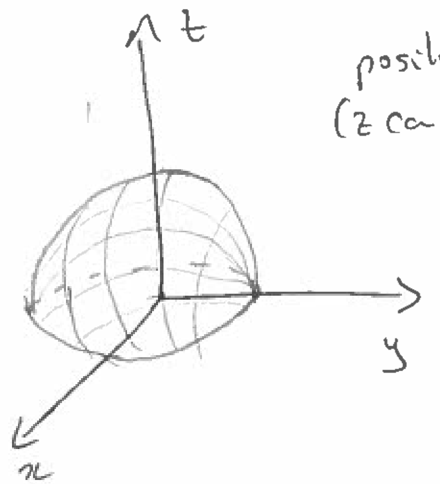
examples : $f(x,y) = \sqrt{1-x^2-y^2}$ $\quad D = \{(x,y) : x^2+y^2 \leq 1\}$

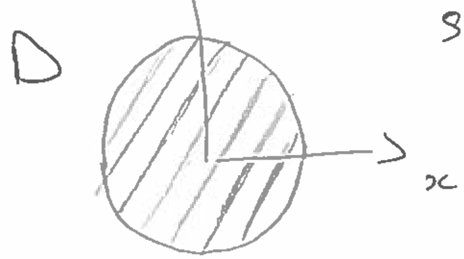
$\Rightarrow$ write $z = \sqrt{1-x^2-y^2}$ then $z^2 = 1-x^2-y^2$

positive square root (z can only be positive)

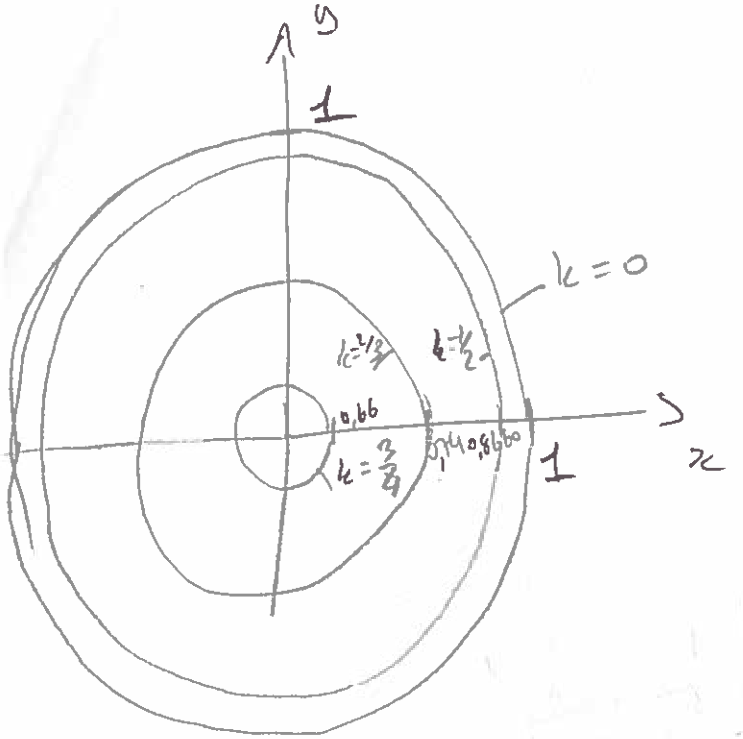
$\Rightarrow x^2+y^2+z^2 = 1$

This is a unit sphere.





we also use contour plots: we draw the contours $f(x,y)=k$ in the xy plane



$$k=\sqrt{1-x^2-y^2}$$

$$k^2=1-x^2-y^2$$

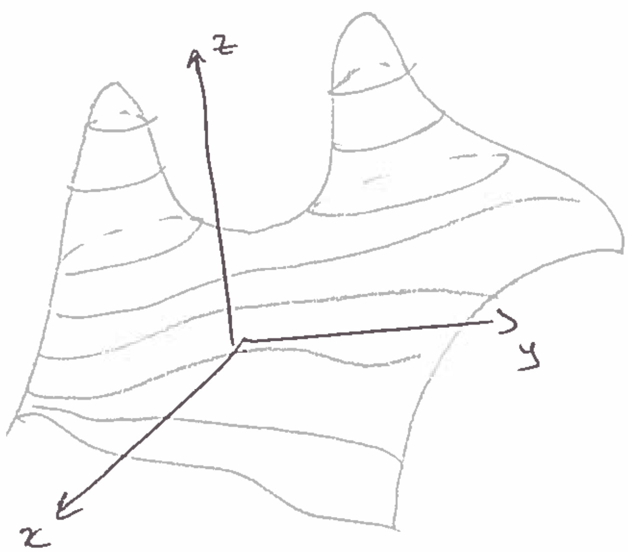
$$x^2+y^2=(1-k^2)$$

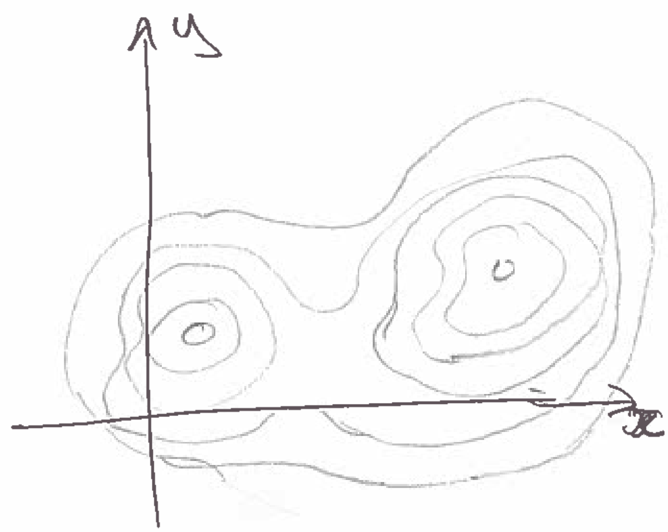
$\Rightarrow$ radius of the circle is $\sqrt{1-k^2}$

$\Rightarrow$ $k=0$ radius is 1

$\Rightarrow$ $k=\frac{1}{2}$ radius is $\sqrt{1-\frac{1}{4}}=\sqrt{3/4}$ $\approx 0,8660$

you have seen this on contour maps.





Visualizing functions of 3 variables is harder: $f(x,y,z)$, the "graph" would be in 4-space $w=f(x,y,z)$. We can draw the contour-surfaces $f(x,y,z)=k$ but it is hard to see all the surfaces

(5)

example $f(x,y,z) = z^2 + y^2 + x^2 = k$

$\Rightarrow$ these are contour surfaces of a sphere of radius $\sqrt{k}$