Problem-Solving Strategies

In this section, you will learn some of the things that can help make you a better problem solver. Remember that problem solving, like playing tennis, is a skill. As in learning any skill, the more you practice, the more you will improve.

Good Problem Solvers vs. Poor Problem Solvers

A great deal of research has been done on problem solving, and several things can be said about good problem solvers and poor problem solvers.

Poor Problem Solvers

1. Poor problem solvers do not believe that they can solve problems. They seem to feel that you either know the answers or you do not.
2. Poor problem solvers are impatient. If they do not see the answer quickly, they give up.
3. Poor problem solvers are careless readers. They often misread what is written. They may begin solving the problem before they know what the problem says.
4. Poor problem solvers jump to conclusions and guess. They expect to go immediately from what is given to the answer. If they cannot solve the problem in one step, they guess or give up.
5. Poor problem solvers fail to organize their work. They are unable to retrace their thinking at the end of the problem.
6. Poor problem solvers seldom check their work.
7. Poor problem solvers have a limited idea of how to approach problems. If they cannot recall a "formula," they do not know what to do and give up.

Good Problem Solvers

1. Good problem solvers believe that they can solve just about any problem if they work at it long enough.
2. Good problem solvers are persistent. They will work a long time on a problem before giving up.
3. Good problem solvers read carefully. They often read the problem several times before beginning to solve it. They are certain that they know what has been said before they begin.
4. Good problem solvers break down complex problems into smaller steps and solve each step at a time.
5. Good problem solvers organize their work so that at any point they can back up and follow the steps they have taken. They often draw pictures to describe the problem.
6. Good problem solvers habitually check what they have done, both at the end of the problem and at various steps along the way.
7. Good problem solvers try many things when they solve problems. They sometimes draw pictures or try to visualize a concrete example. They jot down notes that translate what they have read into words or symbols that are easier to understand. If they cannot solve the problem that is given, they often try a simpler, related problem. For example, they might (a) substitute small numbers for large numbers, (b) substitute whole numbers for variables, or (c) restrict the conditions given in the problem. Good problem solvers guess and test. That is, they try something that may work and test it to see if it does work. If it does not, then they try something else.

If you are not a good problem solver, you probably recognize some of the things that you do in the list of Poor Problem Solvers. You can become a better problem solver by doing the things mentioned in the list of Good Problem Solvers. However, it is not good enough to just *know* the difference between a good problem solver and a poor problem solver. Many people who know the difference between a good tennis player and a poor one still cannot play tennis! Once you know that you are not a good problem solver, you must do something about it. That will take some effort and possibly require some help.

How to Improve Problem-Solving Skills

There are several things that you can do to improve your problem-solving skills.

1. Work with a partner. When you work together, one of you acts as the problem solver and the other acts as the listener. The problem solver reads the problem aloud and talks aloud as he or she works. The listener reads and works along with the problem solver. When something is read or worked incorrectly, the listener stops the problem solver.

(The listener should *not* tell his or her partner the correct reading or answer—the problem solver must go back and make the correction.)

2. When you see your teacher or other students work a problem that you find difficult, pay attention to how the problem is being solved. Ask yourself *How are they reasoning?* If you have trouble following their work, ask them to explain their thinking. An even better idea is to ask them to "think aloud" as they solve the problem.

3. When you need help with a difficult problem, do not ask your teacher to work the problem for you. Rather, explain to your teacher how you would go about working the problem and ask him or her to tell you if your reasoning is correct. Remember, tennis players get better by playing the game and then asking the pro to tell them what they are doing right or wrong. They do not get better just by watching the pro.

4. Make sure that you understand what a problem is all about before you try to solve it. Spend time sorting out the information that is given and the question or questions being asked. *Do not* start searching for a sample problem, rule, or formula until you know what is required.

5. When doing word problems, practice the strategies outlined in the following section.

How to Work Word Problems

Many students describe word problems as difficult, awful, and impossible. Perhaps you have felt this way. If so, you will be happy to know that you *can* learn to solve word problems. It will take practice and hard work, but you can certainly do it!

In order to solve a word problem, you must translate written words and sentences into mathematical words and sentences. This requires careful reading, patience, and logical thinking. When you work a word problem, ask yourself the questions below. You might find it helpful at first to write out the questions.

1. What does the problem ask me to find?
2. What information is given in the problem?
3. Do I have all the information I need?
4. If not, are there facts I should remember that I can use in the problem?
5. If I cannot remember facts I need, are there facts given that I can use to *get* the facts I need?
6. Is there some information given that I do *not* need?
7. What are the important relationships among the facts that I have?

Example 1

- **What are you asked to find?**

"What would you pay for four large oranges?"

This sentence can be translated into the following format. (See Section 3-1, pages 61-64 of the text, for a discussion of this procedure.)

?¢ = 4 large oranges

Note that although you were not asked how many *cents* you would pay for four oranges, the translation reads "?¢ = 4 large oranges." It often helps to translate statements of "how much" into appropriate units when you can. The units can be a useful clue in calculation.

- **What facts are given?**

a) one dozen large oranges costs 60¢.
b) one dozen small oranges costs 55¢.
c) I am to buy four large oranges.

Note that the word *a* has been translated as the number *one*. The problem actually says "60¢ a dozen."

- **Do you have all the information you need?**

Yes, it appears so, but you will not be sure until you see how to work the problem.

- **Is some information given that you do not need?**

Since you are asked only about *large* oranges, it would appear that you can ignore the price of the *small* oranges.

- **What are the important relationships among the facts given?**

To decide what relationships are important, look again at what you want to find:

?¢ = 4 large oranges

You need a relationship between cost and number of large oranges. The problem states that one dozen large oranges costs 60¢.

At this point, you may realize that you *do* need to remember a fact that is not given in the problem:

1 dozen = 12

With this information you can proceed by determining how many dozen (or portion of a dozen) oranges you are buying.

$$?¢ = 4 \text{ large oranges} \times \frac{1 \text{ doz oranges}}{12 \text{ oranges}} =$$

⅓ dozen large oranges

Now multiply by the cost of one dozen.

$$?¢ = \tfrac{1}{3} \text{ doz large oranges} \times \frac{60¢}{1 \text{ doz large oranges}}$$

$$= 20¢$$

Actually, you could have worked the problem in one step, like this:

$$?¢ = 4 \text{ large oranges} \times \frac{1 \text{ doz oranges}}{12 \text{ oranges}} \times \frac{60¢}{1 \text{ doz large oranges}} = 20¢$$

The following examples are similar to the one you just worked. After each example, the questions that need to be asked are repeated. Answer all of the questions pertaining to each example, then compare your answers with answer key

Answers to Example Problems

Example 2

1. ? $ = 80 L. (The actual question is how much would it cost to fill the tank? The cost is expressed in dollars, and is related to the fact that filling the tank requires 80 liters.)

2. a) It is a foreign car.
   b) Its fuel tank holds 80 liters.
   c) Gasoline costs $1.02 for each gallon.

3. Yes. A relationship is needed between liters and gallons. 1 gal = 3.78 L

4. You can ignore the fact that the car is foreign, although it helps to remind you that the fuel tank capacity is measured in liters.

5. Fact c) implies that $1.02 corresponds to 1 gal, so you can write the ratio $1.02/1 gal. The relationship between gallons and liters gives the ratio 1 gal/3.78 L.

   The problem is solved like this:

$$? \$ = 80 \text{ L} \times \frac{1 \text{ gal}}{3.78 \text{ L}} \times \frac{\$1.02}{1 \text{ gal}} = \$21.58$$

Example 3

1. ? s = 1 leap year

2. Nothing

3. Yes. 1 leap year = 366 days
   1 day = 24 h
   1 hour = 60 min
   1 minute = 60 s

You may not see immediately that you need these relationships. If you do not, then ask "How can I get from what I have (year) to what I want to find (seconds)?" A little thought suggests that a day is closer to a second than a year, so write the first relationship. Then repeat the question "How can I get from days to seconds?" An hour is closer to a second than is a day, so write the next relationship. Keep it up until you get to what you want.

4. No

5. $$? \text{s} = 1 \text{ y} \times \frac{366 \text{ days}}{1 \text{ y}} \times \frac{24 \text{ h}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ h}} \times \frac{60 \text{ s}}{1 \text{ min}}$$

$$= 31\,600\,000 \text{ s}$$

Problem Solving

The following example shows how these questions might be used to reason through a problem.

Example 1

Large oranges sell for 60¢ a dozen, while small oranges cost 55¢ a dozen. What would you pay for four large oranges?

Example 2

A foreign car has a fuel tank that holds 80 liters of gasoline. How much will it cost to fill the tank if gasoline costs $1.02 per gallon?

Example 3

How many seconds are there in a leap year?