

**MATH 110 Midterm 1**, October 15th, 2014**Duration: 90 minutes***This test has 7 questions on 9 pages, for a total of 50 points.*

- Read all the questions carefully before starting to work.
- All questions require a full solution; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. **None of the following are allowed:** documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First Name: \_\_\_\_\_ Last Name: \_\_\_\_\_

Student-No: \_\_\_\_\_ Section: \_\_\_\_\_

Signature: \_\_\_\_\_

Question:	1	2	3	4	5	6	7	Total
Points:	8	7	10	6	6	7	6	50
Score:								

**Student Conduct during Examinations**

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
- Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  - speaking or communicating with other examination candidates, unless otherwise authorized;
  - purposely exposing written papers to the view of other examination candidates or imaging devices;
  - purposely viewing the written papers of other examination candidates;
  - using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  - using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

**Full-Solution Problems.** In the following questions, justify your answers and **show all your work**. Unless otherwise indicated, **simplification of answers are required**.

2 marks

1. (a) Here is a table of values for two functions  $f(t)$  and  $g(t)$

$t$	-2	-1	0	1	2	3	4
$f(t)$	4	-1	2	8	1	0	5
$g(t)$	-2	-1	1	1	3	2	0

Compute the following values.

(i)  $f(g(1))$

(ii)  $g(f(0))$

(iii)  $f(f(-2))$

3 marks

- (b) Find the domain of the following function

$$h(x) = \sqrt{2x + 5} + \frac{x - 3}{x^3 - 5x^2}.$$

3 marks

- (c) Consider the parabola  $y = -x^2 - x + 6$ . Let A be the point of intersection of the parabola with the positive  $x$ -axis and let B be the point with positive  $x$ -coordinate obtained from the intersection of the parabola with the line  $y = 4$ . Find the distance between the points A and B.

7 marks
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2. Determine whether each of the following limits exists, and find their value if they exist.

(a)  $\lim_{x \rightarrow 2} \frac{1}{2}x^3 - 3.$

(b)  $\lim_{x \rightarrow -3^+} \sqrt{x + 3}.$

(c)  $\lim_{x \rightarrow 1} f(x),$  where  $f(x) = \begin{cases} 3 & \text{if } x \leq 1 \\ x + 3 & \text{if } x > 1. \end{cases}$

(d)  $\lim_{x \rightarrow 0} \frac{4x^2 - 2x}{x^2 + 3x}.$

(e) Suppose  $\lim_{x \rightarrow 0} (f(x) + g(x)) = 1$  and  $\lim_{x \rightarrow 0} (f(x) - g(x)) = 2.$  Find  $\lim_{x \rightarrow 0} f(x)g(x).$

3. A rocket is launched from a space station. The rocket's position (measured in metres) at time  $t$  (measured in seconds) is given by

$$p(t) = t^2 + t + 1, \quad t \geq 0.$$

2 marks

- (a) Find the average velocity of the rocket over the interval from  $t = 1$  to  $t = 3$  seconds. Include units in your answer.

2 marks

- (b) Let  $h \neq 0$  be a small real number. Find the slope of the secant line between the points  $(1, p(1))$  and  $(1 + h, p(1 + h))$ . Simplify your answer.

2 marks

- (c) Using limits, explain how you can calculate the exact value of the instantaneous velocity of the rocket at  $t = 1$ , and calculate this value (include units). No marks will be given if the velocity is calculated using methods that do not involve limits.

1 mark
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(d) Find the equation of the tangent line to the graph of  $p(t)$  at  $(1, p(1))$ .

2 marks
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(e) Sketch the graph of the position function  $p(t)$ . On the same sketch, draw the tangent line found in part (d).

1 mark
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(f) Now think about the function  $v(t)$  representing the rocket's velocity at time  $t$ . Which of the following options best describes  $v(t)$ ? Circle your answer.

- A. Decreasing and positive
- B. Decreasing and negative
- C. Increasing and positive
- D. Increasing and negative
- E. Constant

6 marks
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4. Let  $a, b$  be real numbers and put

$$f(x) = \begin{cases} -1 & \text{if } x \leq -1 \\ ax^3 + b & \text{if } -1 < x \leq 1 \\ \frac{x^2 - 1}{x - 1} & \text{if } x > 1. \end{cases}$$

For what values of  $a$  and  $b$  is  $f$  continuous on  $(-\infty, \infty)$ ? Make sure you show all your work and justify your claims.

**5 marks**

5. (a) Using the theorem(s) we discussed in class, carefully prove that  $x^4 - x^3 + 2x^2 - 1 = 0$  has a solution between  $x = -1$  and  $x = 1$ . Make sure you justify your claims.

**1 mark**

- (b) Explain why the equation in part (a) has at least another solution between  $x = -1$  and  $x = 1$ .

7 marks
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6. Sketch the graph of a function  $f$  which satisfies all of the following criteria:

- The domain of  $f$  is  $[-3, 1) \cup (1, 3]$
- $\lim_{x \rightarrow -1^-} f(x) = 0$
- $\lim_{x \rightarrow -1^+} f(x) = 1$
- $\lim_{x \rightarrow 1} f(x) = 3$
- The tangent line to the graph of  $f$  is horizontal at  $x = 2$
- The tangent line to the graph of  $f$  has a negative slope at  $x = -2$ .

Make sure you label the axes and clearly identify the points on the graph that are related to the above conditions. *Note:* You are NOT required to provide a formula for  $f(x)$ .



7. Let

$$f(x) = \begin{cases} x^2 - 4 & \text{if } x \leq 2 \\ |x - 5| & \text{if } x > 2. \end{cases}$$

4 marks

(a) Sketch the graph of  $f(x)$ .

2 marks

(b) Consider  $f(x)$  given above, and let

$$g(x) = \begin{cases} -1 & \text{if } x \leq 2 \\ 2 & \text{if } x > 2. \end{cases}$$

Where is  $f(x) - g(x)$  continuous? Explain why.