MATH 110 HOMEWORK 1
KEY

1. a) $f(x) = \dfrac{x^2-4}{e^{2x}\sqrt{x^2-9}}$

$x^2 - 9 > 0$

$x^2 > 9$

$\therefore x \in (-\infty,-3) \cup (3,\infty)$ or $x<-3,\ x>3$

b) $f(x) = \dfrac{e^{2x}\sqrt{x^2-9}}{x^2-4}$

$x^2-9>0 \implies x<-3,\ x>3$

$x^2-4 \neq 0 \quad x^2 \neq 4 \quad x \neq \pm 2$

$\therefore x \in (-\infty,-3) \cup (3,\infty)$ or $x<-3,\ x>3$

c) $h(t) = t^{-2/3}$

$= \dfrac{1}{t^{2/3}} = \dfrac{1}{\sqrt[3]{t^2}}$

$\therefore t \in (-\infty,0) \cup (0,\infty)$ or $t<0,\ t>0$ or $t \neq 0$

d) $h(t) = \sqrt{6+t-t^2}$

$6+t-t^2 \geq 0$

$t = \dfrac{-1 \pm 5}{-2} = -2, 3$

$\therefore t \in [-2,3]$ or $-2 \leq t \leq 3$

2. a) $f \circ g = f(g(x)) = \sqrt{\sqrt[3]{1-x}}$

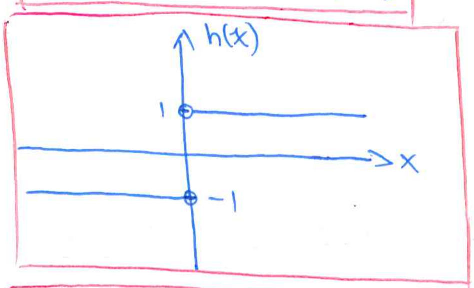
$1-x \geq 0$

$x \leq 1$ or $(-\infty,-1]$

b) $g \circ f = g(f(x)) = \sqrt[3]{1-\sqrt{x}}$

$x \geq 0$, or $[0,\infty)$

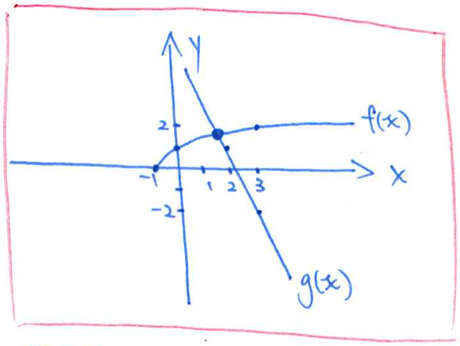
c)



d) domain: $x \neq 0$ or $(-\infty,0)\cup(0,\infty)$ or $x<0,\ x>0$

range: $y \in \{-1,1\}$ or $y=-1$ and 1

3. a)



b) $\sqrt{x+1} = -3x+7$

$x+1 = (-3x+7)^2 = 9x^2 - 42x + 49$

$9x^2 - 43x + 48 = 0$

$x = \dfrac{43 \pm 11}{18} = 3, \dfrac{16}{9}$

Clearly, the intersection is at $x = \frac{16}{9}$, not $x = 3$

$g\left(\frac{16}{9}\right) = f\left(\frac{16}{9}\right) = \frac{5}{3}$

$\boxed{\therefore \text{intersection: } \left(\frac{16}{9}, \frac{5}{3}\right)}$

c) y-intercept of $f(x) = \sqrt{x+1}$

$f(0) = 1 \quad \Rightarrow (0,1)$

intersection: $\left(\frac{16}{9}, \frac{5}{3}\right)$

$\text{slope} = m = \dfrac{5/3 - 1}{\frac{16}{9} - 0} = \dfrac{2/3}{16/9} = \dfrac{2}{3} \cdot \dfrac{9}{16} = \dfrac{3}{8}$

$\boxed{\therefore y = \frac{3}{8}x + 1}$

4. $f(x) = \dfrac{1+10x}{4-3x}$

a) $x = \dfrac{1+10y}{4-3y} \Rightarrow 4x - 3xy = 1 + 10y \Rightarrow 4x - 1 = 10y + 3xy \Rightarrow 4x - 1 = y(10+3x)$

$y = \dfrac{4x-1}{10+3x}$ $\quad \boxed{\therefore f^{-1}(x) = \dfrac{4x-1}{10+3x}}$

b) $f^{-1}(f(x)) = \dfrac{4\frac{1+10x}{4-3x} - 1}{10 + 3\frac{1+10x}{4-3x}} = \dfrac{4(1+10x) - 4 + 3x}{10(4-3x) + 3(1+10x)} = \dfrac{4 + 40x - 4 + 3x}{40 - 30x + 3 + 30x} = \dfrac{43x}{43} = \boxed{x}$

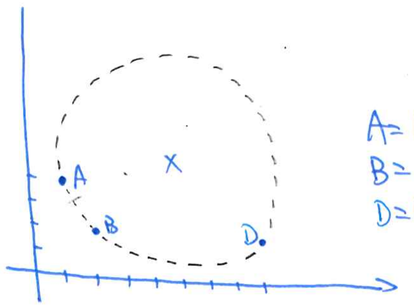
c) $f(f^{-1}(x)) = \dfrac{1 + 10\frac{4x-1}{10+3x}}{4 - 3\frac{4x-1}{10+3x}} = \dfrac{10 + 3x + 10(4x-1)}{4(10+3x) - 3(4x-1)} = \dfrac{10 + 3x + 40x - 10}{40 + 12x - 12x + 3} = \dfrac{43x}{43} = \boxed{x}$

d) $f(f^{-1}(x)) = f^{-1}(f(x))$

you have some input, x; apply some function $f(x)$;
then inverse that function $f^{-1}(f(x))$; you arrive back at the input, x

} anything relevant and makes sense

Bonus:



A = (1,4)
B = (2,2)
D = (8,2)

Let centre be (x,y), distance from C to any of the three points is the same

$$\text{dist}(AC) = \text{dist}(BC) = \text{dist}(DC)$$

$$\sqrt{(x-1)^2+(y-4)^2} = \sqrt{(x-2)^2+(y-2)^2} = \sqrt{(x-8)^2+(y-2)^2}$$

$$(x-1)^2+(y-4)^2 = (x-2)^2+(y-2)^2 = (x-8)^2+(y-2)^2$$

$$\underbrace{x^2-2x+1+y^2-8y+16}_{1} = \underbrace{x^2-4x+4+y^2-4y+4}_{2} = \underbrace{x^2-16x+64+y^2-4y+4}_{3}$$

1 and 2:

$$-2x-8y+17 = -4x-4y+8 \Rightarrow 2x+9 = 4y \Rightarrow y = \frac{1}{2}x+\frac{9}{4}$$

1 and 3

$$-2x-8y+17 = -16x-4y+68 \Rightarrow 14x-51 = 4y \Rightarrow y = \frac{7}{2}x-\frac{51}{4}$$

Intersection:

$$\frac{1}{2}x+\frac{9}{4} = \frac{7}{2}x-\frac{51}{4} \Rightarrow 2x+9 = 14x-51 \Rightarrow 12x = 60 \quad x=5$$

$\boxed{\therefore (5, 19/4) \text{ this is the centre}}$

Radius: $\sqrt{(5-1)^2+(19/4-4)^2}$

$$= \sqrt{16+9/16}$$

$$= \sqrt{265/16}$$

$$\boxed{r = \frac{\sqrt{265}}{4} \text{ or } 4.0697}$$