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HOMEWORK 2: SOLUTION KEY

1. $h(t) = t^2 + \frac{1}{t}$

/5 a) V_{inst} at $t=10 = \lim_{h\to 0} \frac{h(10+h)-h(10)}{h}$ } ~~limit formula: 1 mark~~

$$= \lim_{h\to 0} \frac{(10+h)^2 + \frac{1}{10+h} - 10^2 - \frac{1}{10}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{100}+20h+h^2+\frac{1}{10+h}-\cancel{100}-\frac{1}{10}}{h} \cdot \frac{10(10+h)}{10(10+h)}$$

$$= \lim_{h\to 0} \frac{10(10+h)(20h+h^2)+\cancel{10}-\cancel{10}-h}{10h(10+h)}$$

$$= \lim_{h\to 0} \frac{10(10+h)(20+h)-1}{10(10+h)}$$

$$= \frac{10(10)(20)-1}{100}$$

- calculation correct: 2 marks
- carrying over "lim" through all steps: 1 mark

final answer: 1 mark $\{ = \boxed{19.99}$ or equivalent fractions

b) $\boxed{m/s}$ 1 mark

2. a)

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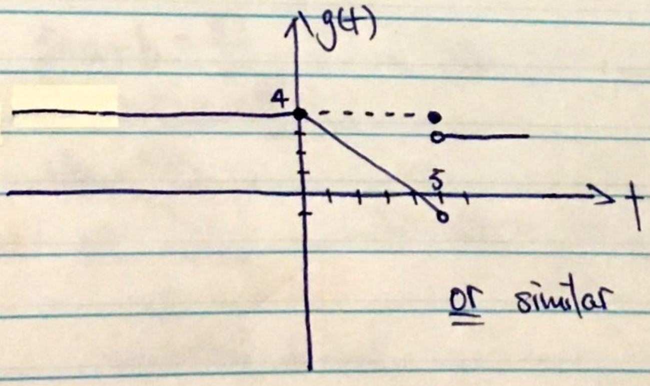

or similar

- Domain $= \mathbb{R}$: no gap in x-direction: 1 mark
- $g(t) \neq -1$ for all t: 1 mark
- $g(0) = g(5)$: 1 mark
- $\lim_{x\to 0} g(t) = \lim_{x\to 0^-} g(t) = \lim_{x\to 0^+} g(t) = 4$: 1 mark
- $\lim_{x\to 5^-} g(t) = -1$: 1 mark
- $\lim_{x\to 5^+} g(t) = 3$: 1 mark

b)

$$g(t) = \begin{cases} 4 & x \le 0 \\ 4-x & 0 < x < 5 \\ 4 & x = 5 \\ 3 & x > 5 \end{cases}$$

or similar

matches graph drawn: 2 marks

3. $$f(x)=\begin{cases} -x-4 & x<-4 \\ 2ax^2+b & -4\le x<5 \\ 11+\frac{3}{2}x & x\ge 5 \end{cases}$$

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Continuity means $\lim_{x\to a} f(x) = \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

at $x=-4$

"lim" sign: 1 mark

$$\left.\begin{aligned} \lim_{x\to -4^-} f(x) &= \lim_{x\to -4^-}(-x-4) = 0 \\ \lim_{x\to -4^+} f(x) &= \lim_{x\to -4^+}(2ax^2+b) = 32a+b \end{aligned}\right\} \begin{aligned} 32a+b&=0 \\ b&=-32a \end{aligned}$$ 1 mark

at $x=5$

"lim" sign: 1 mark

$$\left.\begin{aligned} \lim_{x\to 5^-} f(x) &= \lim_{x\to 5^-}(2ax^2+b) = 50a+b \\ \lim_{x\to 5^+} f(x) &= \lim_{x\to 5^+}\left(11+\frac{3}{2}x\right) = 11+\frac{15}{2} = \frac{37}{2} \end{aligned}\right\} 50a+b=\frac{37}{2}$$ 1 mark

$50a+b=\frac{37}{2}$ Now $b=-32a$, substitute that

$50a-32a=\frac{37}{2}$

$18a=37/2$

1 mark $\boxed{a=37/36}$ or 1.0278 equivalent decimal

$b=-32a=-32\cdot\frac{37}{36}=-8\,\frac{37}{9}=\boxed{-296/9}$ -32.889 1 mark

or equivalent decimal

4. $\sqrt[3]{t}=1-t$ for any bump or occur

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$\sqrt[3]{t}-1+t=0$ let $h(t)=\sqrt[3]{t}-1+t$

$\left.\begin{aligned} h(0)&=-1 \\ h(1)&=1 \end{aligned}\right\}$ sign change

1 mark 1 mark

$\therefore$ The function $h(t)$ is continuous, there is a sign change from $t\in[0,1]$, by IVT there is a root between $[0,1]$, thus bumping occurs → 1 mark

1 mark

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