MATH 110-001 HOMEWORK 3
Due date: Friday, November 10, 2017

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your answers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwritting please!). Your answers must be stapled, with your name and student number at the top of each page.

1. Compute the following derivatives. Remember to use the appropriate differentiation rules

a) $f(x) = x^{87}(5x-1)\sqrt{x}$ *b)* $g(x) = \dfrac{6x^{10}+x^6}{\sqrt{x}}$

c) $h(t) = \dfrac{t^{50}(2t-4)}{t+3}$ *d)* $A(y) = (20\sqrt[3]{y})cos(y)$

2. Let ω be a positive, non-zero constant and let the function $I(t)$, describing the current through my failed electrical experiment, be defined as:

$$I(t) = \frac{cos(t)+2sin(t)+3}{1+\frac{t^2}{\omega}}$$

Find the equation of the tangent line to $I(t)$ at $t = \frac{3\pi}{2}$ (Your answer should depend on ω).

3. Find values for b and c such that the function

$$W(u) = \begin{cases} bu+c & u \leq \frac{\pi}{2} \\ cos^2(u)sin(u) & u > \frac{\pi}{2} \end{cases}$$

is differentiable at $\frac{\pi}{2}$. Remember that differentiable functions cannot have jumps, corners, cusps, or vertical tangent lines.

4. Recall that given a differentiable function, say $f(x)$, we can compute its derivative, which we denote $f'(x)$. The derivative $f'(x)$ is itself a function and if it is also differentiable we could take its derivative. We call this new function the *second derivative* of f and denote it by $f''(x)$. In other words $f''(x) = (f'(x))'$.

   (a) Show that the function $f(x) = sin(x)$ satisfies $f''(x) = -f(x)$.
   (b) Find another NON-ZERO function satisfying the equation in (a) (**Hint:** Maybe the first function has a similar cousin).
   (c) Find infinitely many functions satisfying (a) (**Hint:** You already have two functions. Try combining them by adding the two or some other combination).
   (d) Find a function satisfying (a) with the property that $f(0) = -1$ and $f(\pi/2) = 3$
   (**Hint:** You will need to use both of the functions from part (a) and (b).).