MATH 110-001 HOMEWORK 4
Due date: Friday, November 24, 2017

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your answers must be clear and neatly written, as well as legible (no tiny drawings or micro-handwritting please!). Your answers must be stapled, with your name and student number at the top of each page.

1. It's Jerry's birthday and he wants an inflatable bouncy castle at his party. The volume of the bouncy castle after inflating it for t minutes is given by the function:

$$V(t) = \frac{e^{\frac{t}{10}}}{t^2 + 1}$$

The height of the bouncy castle in inches is given by the function:

$$h(V) = \sqrt[3]{V} + 2$$

The party is about to start and Jerry is worried about how long the structure is going to take to fully inflate. Calculate the instantaneous rate of change of the height with respect to time at $t = 30$ minutes.

2. Consider the following function:

$$f(x) = \begin{cases} sin(2x) + 4 & x < 0 \\ 4e^{\sqrt{x+1}-1} & x \geq 0 \end{cases}$$

   i) Sketch the graph of $f(x)$ from $x = -2$ to $x = 3$ (a table of values would greatly help here)
  ii) Find the derivative of $f(x)$ at $x = -1$ and $x = 2$.
 iii) Find the equation of the tangent line at $x = 0$ (It exists...)

3. Compute the following derivatives. Remember to use the appropriate differentiation rules

     a) $C(t) = 8\sqrt{tan(t) + t^2}$ *b)* $R(v) = cos(e^{v^2})$

     c) $g(x) = ln(x\sqrt{x^2 - 1})$ *d)* $T(y) = (1/3)^{y^2}$

4. Find the value of a so that the tangent line to $y = ln(x)$ at $x = a$ is a line through the origin. Sketch the resulting situation.