MATH 110-003, QUIZ 4

November 15, 2016

Time: 15 minutes

Show all your work. No calculators, no books/notes are allowed.

Name (please print): ______________________________

Student number: ______________________________

1. a) Differentiate the following function

$$y = \tan^4(z^2 - \pi)$$

b) Evaluate the derivative at $z = \sqrt{\frac{\pi}{6}}$.

$$y = \tan^4(z^2 - \pi) = (\tan \square)^4$$

a) $$y' = \underbrace{4(\tan \square)^3}_{\text{Outside: derivative of } (\square)^4} \cdot \underbrace{(1 + \tan^2 \square)}_{\text{derivative of middle : } \tan \square} \cdot \underbrace{\square'}_{\text{derivative of inside : } z^2 - \pi}$$

$$= 4\tan^3(z^2 - \pi) \cdot (1 + \tan^2(z^2 - \pi)) \cdot 2z$$

$$= 8z \tan^3(z^2 - \pi)(1 + \tan^2(z^2 - \pi))$$

b) $\rightsquigarrow$ NEXT PAGE

2. Find the point(s) where the tangent line to the graph of $h(t) = e^{5t^2+7t-13}$ is parallel to the line $y = -5$.

$$\text{slope} = 0 \xrightarrow{\text{solve}} h'(t) = 0$$

$$h(t) = e^{\square} \rightsquigarrow h'(t) = \underbrace{e^{\square}}_{\text{derivative of outside}} \cdot \underbrace{\square'}_{\text{derivative of inside}}$$

$$h'(t) = e^{5t^2+7t-13}(10t + 7) = 0$$

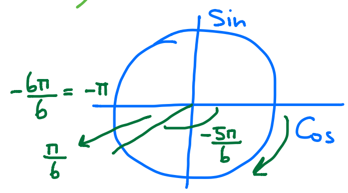
either $e^{5t^2+7t-13} = 0 \rightarrow$ NOT possible $\rightsquigarrow$ Exponential is ALWAYS positive.

or $10t + 7 = 0 \rightsquigarrow \boxed{t = \frac{-7}{10}}$

(1)

b) $y'\left(\sqrt{\frac{\pi}{6}}\right) = 4\tan^3\left(\overset{-\frac{5\pi}{6}}{\cancel{\frac{\pi}{6}-\pi}}\right)\cdot\left(1+\tan^2\left(\overset{-\frac{5\pi}{6}}{\cancel{\frac{\pi}{6}-\pi}}\right)\right)\cdot 2\sqrt{\frac{\pi}{6}}$

Locate $\underbrace{-\frac{5\pi}{6}}_{\text{Negative direction}}$ in the unit circle:



number comes from $\tan\frac{\pi}{6}$ in the memorized table $\rightsquigarrow \frac{1}{\sqrt{3}}$

Sign comes from the quadrant

$\hookrightarrow$ 3^{rd} quadrant $\rightsquigarrow$ tan is $+$ because sin is $-$ & cos is $-$

$$\Longrightarrow \tan\left(-\frac{5\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow y'\left(\sqrt{\frac{\pi}{6}}\right) = 8\sqrt{\frac{\pi}{6}}\cdot\left(\frac{1}{\sqrt{3}}\right)^3\cdot\left(1+\left(\frac{1}{\sqrt{3}}\right)^2\right)$$

$$= 8\sqrt{\frac{\pi}{6}}\cdot\frac{1}{3\sqrt{3}}\cdot\frac{4}{3}$$