Solution Key

MATH 110-001 QUIZ 2

October 6, 2017

Time: 15 minutes

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Show all your work. No calculators, no books/notes are allowed.

Name (please print): ______________________

Student number: ______________________

1. Find the following limits, if it exists

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a) $\lim_{x \to -2} \frac{e^x}{(x+2)^{10}}$

		e^x	$(x+2)^{10}$	$\frac{e^x}{(x+2)^{10}}$
1 mark: left limit	$\lim_{x\to -2^-}$	e^{-2}	(+) small	$+\infty$
1 mark: right limit	$\lim_{x\to -2^+}$	e^{-2}	(+) small	$+\infty$

$\therefore \lim_{x \to -2} \frac{e^x}{(x+2)^{10}} = \boxed{+\infty}$ 1 mark: final ans

b) $\lim_{h \to 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$

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$= \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \cdot \frac{3(3+h)}{3(3+h)}$

$= \lim_{h\to 0} \frac{3-3-h}{3h(3+h)}$

$= \lim_{h\to 0} \frac{-1}{3(3+h)}$

$= \boxed{-\frac{1}{9}}$

1 mark for process

1 mark for carrying over "$\lim_{h\to 0}$"

1 mark for final answer

2. The position of a snowboarder sliding down a slope can be described by the function

$$f(t) = -2t^2 + 3$$

find the *instantaneous* velocity of the snowboarder at t=1.

Hint: the *average* velocity of the snowboarder between $t = a$ and $t = a+h$ is $v_{avg} = \frac{f(a+h)-f(a)}{h}$

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$V_{inst} = \lim_{h\to 0} V_{avg} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ $a=1$ in this case

$= \boxed{\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}} = \lim_{h\to 0} \frac{-2(1+h)^2+3+2(1)^2-3}{h} = \lim_{h\to 0} \frac{-2-4h-2h^2+3+2-3}{h}$

1 mark for writing down this formula

$= \lim_{h\to 0} (-2h-4) = \boxed{-4}$

1 mark for final answer

1 mark for carrying over "$\lim_{h\to 0}$" for every step

Bonus: Is there a number b such that

$$\lim_{x\to -2} \frac{3x^2+bx+b+3}{x^2+x-2}$$

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exists? If so, find the value of b and the value of the limit

Numerator: $3x^2+bx+b+3$
Denominator: $x^2+x-2 = (x+2)(x-1)$
for the limit to exist, one of the solutions of the numerator needs to be -2 to cancel out the $(x+2)$ in the denominator:

Method 1:
using quadratic formula:

$$x = \frac{-b\pm\sqrt{b^2-4\cdot 3\cdot (b+3)}}{6} = -2$$

$$-b\pm\sqrt{b^2-12b-36} = -12$$

$$b-12=\pm\sqrt{b^2-12b-36}$$
$$b^2-24b+144=b^2-12b-36$$

square both sides

$$12b=180$$
$$\boxed{b=15}$$

Method 2:

Numerator: $3x^2+bx+b+3$

$x=-2$: $3(-2)^2-2b+b+3=0$

$$\boxed{b=15}$$

Now with $b=15$:

$$\lim_{x\to -2}\frac{3x^2+15x+15+3}{x^2+x-2}$$
$$=\lim_{x\to -2}\frac{3x^2+15x+18}{(x+2)(x-1)}$$
$$=\lim_{x\to -2}\frac{3(x+2)(x+3)}{(x+2)(x-1)}$$
$$=\lim_{x\to -2}\frac{3(x+3)}{(x-1)}$$
$$=\frac{3(1)}{-3}$$
$$=\boxed{-1}$$

1 mark for $b=15$

1 mark for limit $=-1$