

# Solution Key

## MATH 110-001 QUIZ 4

November 03, 2017

Time: 15 minutes

Show all your work. No calculators, no books/notes are allowed.

Name (please print): \_\_\_\_\_

Student number: \_\_\_\_\_

1. Find the derivative,  $f'(x)$ , of the following functions

$$f(x) = x^{\frac{5}{2}}e^x$$

$$f'(x) = \frac{5}{2}x^{\frac{3}{2}}e^x + x^{\frac{5}{2}}e^x$$

$$= \frac{5}{2}x^{\frac{3}{2}}e^x + x^{\frac{5}{2}}e^x$$

$$= e^x \left( \frac{5}{2}x^{\frac{3}{2}} + x^{\frac{5}{2}} \right)$$

$$= x^{\frac{3}{2}}e^x \left( \frac{5}{2} + x \right)$$

any of these three is correct

1 pt for using product rule  
1 pt for final answer

$$f(x) = \frac{x^2}{4 + 3e^x}$$

$$f'(x) = \frac{2x(4+3e^x) - x^2(3e^x)}{(4+3e^x)^2}$$

$$= \frac{8x + 6xe^x - 3e^x x^2}{(4+3e^x)^2}$$

any of these two is correct

1 pt for using quotient rule  
1 pt for final answer

3. Find an equation of the tangent line to the curve

$$y = \frac{e^x}{1+x^2}$$

at the point  $x = 1$

$$y' = \frac{e^x(1+x^2) - e^x(2x)}{(1+x^2)^2}$$

$$y'(1) = \frac{e(2) - e(2)}{2^2} = 0$$

$$y(1) = \frac{e}{2}$$

$$y = mx + b \quad \text{or} \quad y - y_1 = m(x - x_1)$$

$$\frac{e}{2} = 0 \cdot 1 + b \quad b = \frac{e}{2}$$

$$\boxed{\therefore y = \frac{e}{2}}$$

1 pt for using quotient rule  
and attempt to plug in  $x=1$   
(ok if small computational mistake)

1 pt for attempt to find equation  
of tangent line

1 pt for final answer  
(ok if consistent with error carry-over)

Bonus: Suppose that  $f(2) = -3$ ,  $g(2) = 4$ ,  $f'(2) = -2$ , and  $g'(2) = 7$ , and

$$h(x) = \frac{g(x)}{1+f(x)}$$

Find  $h'(2)$ .

$$h'(x) = \frac{g'(x)(1+f(x)) - g(x)(f'(x))}{(1+f(x))^2}$$

$$h'(2) = \frac{g'(2)(1+f(2)) - g(2)f'(2)}{(1+f(2))^2}$$

$$= \frac{7(1+(-3)) - 4(-2)}{(1-3)^2}$$

$$= \frac{-14+8}{4}$$

$$= \frac{-6}{4} = \boxed{-\frac{3}{2}}$$

1 pt for using quotient rule  
(no computational error allowed)

1 pt for final answer