Homework 4 Solution Key

/30

1. $h(v(t)) = h(t) = \sqrt[3]{\dfrac{e^{t/10}}{t^2+1} + 2} = \left(\dfrac{e^{t/10}}{t^2+1} + 2\right)^{1/3}$ } 1pt for function composition

/5

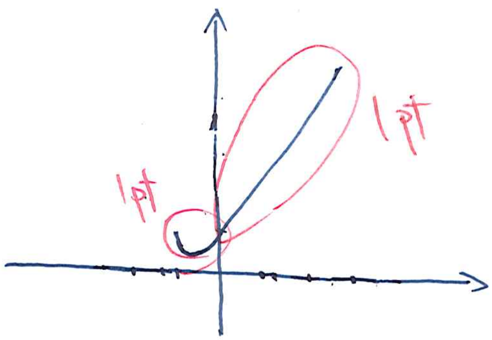
$$h'(t) = \frac{1}{3}\left(\frac{e^{t/10}}{t^2+1} + 2\right)^{-2/3} \cdot \frac{\left(\frac{1}{10}e^{t/10}(t^2+1) - 2te^{t/10}\right)}{(t^2+1)^2}$$ } 2pt for finding derivative using chain rule

$$\underbrace{h'(30) = \frac{1}{3}\left(\frac{e^3}{30^2+1} + 2\right)^{-2/3} \cdot \frac{\left(\frac{1}{10}e^3(30^2+1) - 60e^3\right)}{(30+1)^2}}_{\text{1pt for plugging in } t=30}$$

$\dfrac{\text{inch}}{\text{min}}$ 1pt for correct unit

2. i)

/9



ii) $f'(x) = \begin{cases} 2\cos(2x) & x<0 \\ 2e^{\sqrt{x+1}-1} \cdot \dfrac{1}{\sqrt{x+1}} & x \geq 0 \end{cases}$

$f'(-1) = 2\cos(-2) = \boxed{2\cos(2)}$ } 2pt

$f'(2) = \boxed{\dfrac{2e^{\sqrt{3}-1}}{\sqrt{3}}}$ } 2pt

iii) $f'(0)$ from left
$= 2$

$f'(0)$ from right
$= \dfrac{2e^{\sqrt{0+1}-1}}{\sqrt{0+1}} = \dfrac{2e^{1-1}}{1} = 2$

$\therefore f'(0) = 2$ } 1pt

$f(0)$ from left
$= 4$

$f(0)$ from right
$= 4e^{\sqrt{0+1}-1}$
$= 4e^{1-1}$
$= 4$

$\therefore f(0) = 4$ } 1pt

$y = mx + b$

$4 = 2\cdot(0) + b \qquad b = 4$

$\boxed{y = 2x + 4}$ } 1 pt

3\. a) $C'(t) = 4(\tan(t) + t^2)^{-\frac{1}{2}} \cdot (\sec^2(t) + 2t)$

12 b) $R'(v) = -\sin(e^{v^2}) \cdot e^{v^2} \cdot 2v$

c) $g(x) = \ln x + \frac{1}{2}\ln(x^2 - 1)$

$g'(x) = \frac{1}{x} + \frac{1}{2(x^2-1)} \cdot 2x = \frac{1}{x} + \frac{x}{(x^2-1)}$

} 3 pt each
- 1 pt for attempting chain rule
- 1 pt for getting chain rule correct
- 1 pt for final answer

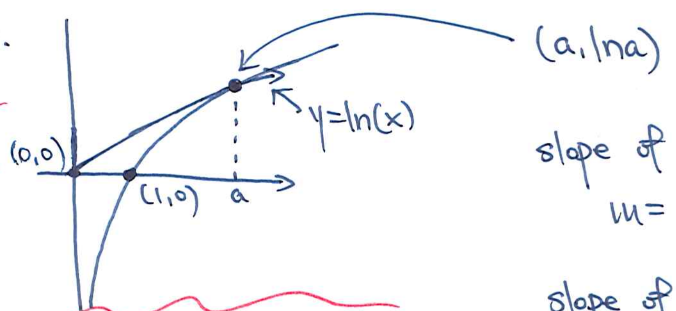
d) $T(y) = e^{\ln((\frac{1}{3})^{y^2})} = e^{y^2 \ln\frac{1}{3}}$

$T'(y) = e^{y^2\ln\frac{1}{3}} \cdot 2y\ln\frac{1}{3}$

$T'(y) = \left(\frac{1}{3}\right)^{y^2} \cdot \left(2\ln\frac{1}{3}\right) y$

} 3 pt
- 1 pt for changing base into "e"
- 1 pt for attempting chain rule
- 1 pt for final answer

4\. 4



1 pt

slope of the tangent line:

$m = \frac{\ln a - 0}{a - 0} = \frac{\ln a}{a}$

slope of $y = \ln x$ at $x = a$:

$y' = \frac{1}{x} \quad y'(a) = \frac{1}{a}$

} $\frac{\ln a}{a} = \frac{1}{a}$ } 2 pt

$\ln a = 1 \qquad \boxed{a = e}$ } 1 pt