MATH 110-001 QUIZ 5

November 17, 2017
Time: 15 minutes

Solution Key

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Show all your work. No calculators, no books/notes are allowed.

Name (please print): _____
Student number: _____

1. Find the derivative, $f'(x)$, of the following functions. Do not simplify.

$f(x) = \sqrt[3]{1+tan(x)} = (1+\tan(x))^{1/3}$

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$f'(x) = \frac{1}{3}(1+\tan(x))^{-2/3} \cdot \sec^2(x)$

$= \frac{1}{3}(1+\tan(x))^{-2/3} \cdot \frac{1}{\cos^2(x)}$

any of these is okay!
1 pt for using chain rule
1 pt for correct final answer

$f(x) = sin^2(e^{sin^2 x})$

$f'(x) = 2\sin\left(e^{\sin^2(x)}\right) \cdot \cos\left(e^{\sin^2(x)}\right) \cdot e^{\sin^2(x)} \cdot 2\sin(x)\cdot\cos(x)$

$= \sin\left(2e^{\sin^2(x)}\right) \cdot \sin(2x) \cdot e^{\sin^2(x)}$

$= 4\sin\left(e^{\sin^2(x)}\right) \cdot \cos\left(e^{\sin^2(x)}\right) \cdot e^{\sin^2(x)} \sin(x)\cos(x)$

any of these is okay!

1 pt for using chain rule
1 pt for correct final answer

3. Find an equation of the tangent line to the curve

$$y = \frac{2}{1+e^{-x}}$$

at the point $x = 0$

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$y' = \frac{+2e^{-x}}{(1+e^{-x})^2}$ $\quad y'(0) = \frac{2}{(1+1)^2} = \frac{1}{2}$ } 2 pts

$y(0) = \frac{2}{2} = 1$

$y = mx + b$

$1 = \frac{1}{2} \cdot 0 + b \quad b = 1$ } 1 pt

$\therefore y = \frac{1}{2}x + 1$ } 1 pt

Bonus: If $F(x) = f(xf(xf(x)))$, where $f(1) = 2$, $f(2) = 3$, $f'(1) = 4$, $f'(2) = 5$, $f'(3) = 6$, find $F'(1)$.

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$F'(x) = f'(xf(xf(x))) \cdot (f(xf(x)) + x \cdot f'(xf(x))(f(x) + xf'(x)))$

$f'(xf(xf(x)))$: $=2$; $=3$; $=6$

$f(xf(xf(x)))$: $=1$ $=2$; $=3$

$x \cdot f'(xf(x))(f(x) + xf'(x))$: $x = 1$; $f'(xf(x))$: $=1$ $=2$; $=5$; $f(x) + xf'(x)$: $=2$ $=1 \cdot 4$; $=6$; $=5 \cdot 6 = 30$

$f(xf(xf(x))) + x \cdot f'(xf(x))(f(x) + xf'(x)) = 3 + 30 = 33$

$F'(x) = 6 \cdot 33 = 198$

$F'(1) = 198$ } 3 pt