**Challenging Concept**: Understanding the restrictions on the base of an exponential function and how the base affects the location/direction of the curve.

In my experience, students can often graph exponential functions when the base is specified, but have difficulty picturing and understanding what happens to the function y = ax when abstract restrictions are applied. For example, students can graph y = 2x but cannot draw a sketch of y = ax, 0<a<1.

The first step, or perhaps pre-step, in the GEM process is that the “teacher provides background content information” (Khan, 2007). This is reiterated by the professor in the simulation case study when he states that “The time to ... [use computer simulations] ... [is when] students know what it is they’re looking at.” (Khan, 2010; p. 226). In line with this, I would first graph y=2x with the class and discuss the general shape of the graph and that an exponential function has the variable in the exponent. I would do this as a class because, even though students should be able to generate a table of values and graph it by grade 12, some will struggle and even more will stick to whole numbers for x, avoiding rational numbers and thereby miss some of the nuances of the graph. Once the students have the idea of a basic exponential function, the rest of the GEM steps would be as follows.

|  |  |  |
| --- | --- | --- |
| GEM Stage | Teacher Strategy | Technology Chosen and Why |
| Generate relationships | * Have students work collaboratively to determine the following relationships for a graph of y=ax:  1. If we want the nice exponential function curve, what are the restrictions on “a”? 2. What values of “a” give the curve starting low on the left and high on the right? Opposite? 3. What point is invariant on all basic y=ax curvers? 4. How can you manipulate the function so that it is reflected below the x-axis?  * Have the students formalize their ideas into mathematical sentences – this would be the *initial draft* of the relationships they have defined. | Graphing calculators because students can try 100 graphs in 10 minutes where graphing them by hand (especially ones with fractions, negative exponents, etc.) is time consuming.  The graphs are visual and give them immediate feedback on whether they are right or wrong – if a student hypothesizes that “a” has to be a fraction to reflect the graph across the y-axis, then they see immediately if their hypothesis works and can test it on other fractions. |
| Evaluate relationships | * Ask the students to answer or do the following:   + Does your model work? Check by comparing with other groups, collaborate.   + After the students have had time to discuss results with other groups, move to a class discussion and ask: Can we back it up with algebra? How do these reflections relate algebraically to other transformations we have done? (y = f(-x) for example).   + Often students will state that the graph is reflected across the y-axis if “a” is a fraction. I would ask them to graph y = (5/3)x as a counter example to this. | Continued investigation using the graphing calculators to check hypothesis. |
| Modify relationships | After evaluating their original mathematical sentences/relationships, I would ask them to modify their results to reflect any inconsistencies they found and then have a final class discussion on what those results were. |  |

References:

Khan, S. (2007). Model-based inquiries in chemistry. *Science Education, 91*(6), 877-905.

Khan, S. (2010). New pedagogies for teaching with computer simulations. *Journal of Science Education and Technology, 20*(3), 215-232.