Video 10 - Bond Valuation and Coupon Payment Formulas

The following is a supplementary transcript for tutorial videos from
https://blogs.ubc.ca/financefundamentals/

Hi everyone, today we will be learning about bonds and how to appropriately apply the bond valuation formula and the coupon payment formula to calculate the market price of a bond. By the end of this video, you will learn the terminology and notation used to describe the characteristics of a bond; how to calculate a bond's present value using the annuity formula; and how to calculate the coupon payments of different periods.

Video at 00:32
First of all, what even is a bond? Bonds are a type of debt security where the issuer owes the bondholder a predetermined sum of money at a predetermined date in the future and, for the most part, in between, must periodically pay the bondholder interest. These are known as coupon bonds, and the interest is paid at fixed intervals of time, such as monthly, semi-annually, or annually, and the interest paid each period is also predetermined, which is why bonds are often referred to as fixed income securities. The other type of bonds are known as zero coupon bonds which, like their name suggests, not make any intermittent payments of interest and only repay the amount borrowed on the predetermined date in the future. When people talk about bonds, they are generally referring to coupon paying bonds, so the bonds studied in these videos will refer to coupon paying bonds, unless you are specifically told that it is a zero coupon bond. Regardless of whether the bond pays coupons or not, you can think of bonds as a loan or an "IOU" (I owe you) from the issuer, such as a company or the government, to the bondholder, that can be traded in the public debt markets.

Video at 01:40
All bonds share certain features and before we can calculate the value of bonds, it is first important to understand the correct terminology and notation to describe the characteristics of a bond. The amount of money that the issuer borrows from the bond holder is called the principal or the face value, abbreviated as "FV". Bonds usually have a face value of $1,000, so when a company raises money by issuing bonds, this is usually done in $1,000 increments. The issuer must repay the principal on a predetermined date in the future known as the maturity date. The fixed interest payments paid by the issuer each period are called the coupons and are denoted

by the letter "C". Rather than a dollar amount, these coupons are usually expressed as a percentage rate to capture the size of the coupon relative to the principal. We call this the coupon rate. "n" is used to represent the number of periods until the maturity date. You can think of this as a number of times the issuer must pay the coupon until the bond is due. Finally, in order to express the value of the bond as a single price, we must discount all the bond's cash flows to one point in time. To account for the time value of money, we use "r", the market rate, as the discount rate for the bond's cash flows. Although the effective rate is always used to discount cash flows, rates in the world of bonds are also subject to annual percentage rates, or APR, where the discount rate is actually quoted in annual terms. In the world of bonds, the yield to maturity is the lingo used to describe the APR quotations of the effective discount rates.

Video at 03:14

If we move all the cash flows of a bond into one point in time using the discount rate "r", we can calculate the present value of the bond. This is also the value or the market price of the bond. At the market price, the issuer sells the bond to the bondholder, or the bond holder buys the bond from the issuer. You can think of this as a bondholder buying the issuer's promise to pay back the predetermined coupons and principal by the maturity date. Intuitively, it makes sense that the market price of a bond is its present value because how much the bondholder is willing to lend to the issuer today is equal to the value of the future cash flows the bondholder expects to receive from the bond.

Video at 03:53

The market rate reflects the return a bondholder could earn on comparable bonds. We will use PV to represent the present value or the market price of the bond. The price of a bond can be found by moving each cashflow individually back to today. As we do so, you may have noticed that the cash flows of a bond look similar to that of an annuity, and you would be absolutely correct! The coupon payments of a bond are an annuity, due to its fixed nature, but a bond has an added principal amount that must be paid on the maturity date, which annuities do not have. Thus, the formula to calculate the price of a bond borrows the annuity formula to calculate the present value of the coupon payments, plus a simple present value formula to discount the final principal payment.

$$price\ of\ bond = PV = \$C \times \left[\frac{1-(1+r)^{-n}}{r}\right] + \frac{FV}{(1+r)^n}$$

Video at 04:37

Let's take a look at a quick example. What is the price of a three-year $1,000 bond that pays annual coupons at 8%, and the annual discount rate for similar bonds is 10%? Feel free to pause the video and try calculating the price of this bond yourself. It may be helpful to draw a timeline of the cash flows, too.

Video at 04:59

Let's dissect the numbers in the problem. Right away, we know that the issuer must pay the bondholder FV=$1,000 three years from now. Until then, there will be three annual coupon payments (n=3), with the first coupon in one period from today, and the last coupon on the maturity date. An 8% annual coupon on a $1,000 bond means that the issuer must pay the bondholder C=$80 each year. If we bring back all these cash flows to a present value at a discount rate of r=10%, we see that the price of this bond is equal to

$$price\ of\ bond\ =\ PV\ = \$80 \times \left[\frac{1-(1+0.10)^{-3}}{0.10}\right] + \frac{\$1{,}000}{(1+0.10)^{3}}\ =\ \$950.26$$

In other words, at this market price, the bondholder will lend $950.26 to the issuer today in exchange for a promise that the issuer will pay three annual coupons of $80 and a principal of $1,000.

Video at 05:47

Be careful not to confuse the coupon rate and the market rate. The 8% coupon rate is a fixed rate that determines how much interest the issuer must pay, while the 10% market discount rate fluctuates with the market and is the rate of return that the bondholder will require to hold on this bond and similar bonds. The relationship between the coupon rate and the market discount rate is an important concept in finance that determines the present value of the bond.

Video at 06:12

Recall that the coupons can be paid at any fixed intervals of time, so we could have a bond that pays coupons monthly, semi-annually, annually, etc. However, the coupon rate is always expressed annually, or in APR. Imagine that our three-year $1,000 bond with an annual market discount rate of 10% now pays semiannual coupons of 8%. What does this mean? Firstly, if the coupons are paid semi-annually, this means that the issuer must make two payments a year for the three years, for a total of n=6 coupon payments to the bondholder. Since the coupon rate is expressed annually, the issuer will still pay $80 of coupons to the bondholder per year, but instead of $80 at the end of every year, like with an annual bond, the coupons will be broken into semi annual installments. Thus, each coupon is actually C=$40.

Video at 07:05

Secondly, it is crucial to discount cash flows using the effective rate that matches with the frequency of the cash flows. Here since we have semi-annual coupons, we must discount these coupons with the semi-annual effective rate ($r_{semiannual}$). Because discount rates are expressed in APR, a quoted annual rate of 10% is divided by the number of coupon payments per year, which is m=2, to arrive at the effective semiannual rate of $r_{semiannual}$ =5%. We can now plug the new values for "n", "C", and "r" into the bond valuation formula to find that the price of this bond that pays semiannual coupons is

$$price\ of\ bond\ =\ PV\ = \$40 \times \left[\frac{1-(1+0.05)^{-6}}{0.05}\right] + \frac{\$1,000}{(1+0.05)^{6}}\ =\ \$949.24$$

Video at 07:42

It is crucial to remember that the dollar amount coupons can occur at different frequencies, be it monthly, semiannual, annual, etc., but the coupon rate is always expressed annually. Just like we had done, we can easily calculate the dollar amount coupons by using the following coupon payment formula:

$$coupon\ =\ \frac{coupon\ rate \times FV}{m}$$

where "m" is the number of periods per year that the issuer will pay the bondholder. In the first example, we didn't see "m", because m=1, when the coupons are paid annually, However, in the example we just worked with, we saw that m= 2, because the coupons were paid twice per year or semi-annually. You can find "m" for any fixed period that pays coupons.

Video at 08:22

In conclusion, we have learned about what a bond is and how to use the correct terminology and notation to describe the characteristics of a bond, how to calculate the present value or price of a bond using the bond valuation formula, and how to calculate the dollar amount coupon payments for coupon rates of different frequencies using the coupon payment formula. That is all for this video, thank you so much for watching!