Video 11 - Bond Valuation: Coupon Rates and Yield to Maturity

The following is a supplementary transcript for tutorial videos from
https://blogs.ubc.ca/financefundamentals/

Hi everyone, in today's video we will be diving deeper into the coupon rate and discount rate, and yield to maturity of a bond. By the end of this video, you will learn about the difference between the coupon rate and the discount rate, and how the relationship between the two affect the price of a bond. We will also be learning more about what the yield to maturity of a bond is.

Video at 00:26

When we calculate bond prices, there are two rates we need to consider: the coupon rate and the discount rate. The coupon rate is a fixed rate that determines how much the issuer must pay to the bondholder on an annual basis. You can think of the coupon rate as the interest rate. Remember, the coupon rate is always quoted in annual terms using annual percentage rates, or APR, so you can use the coupon payment formula to calculate the dollar amount of each coupon for different coupon payment frequencies. On the other hand, the market discount rate fluctuates with the market, and is the rate of return that the bondholder will require to hold this bond and similar bonds. You can think of this as the percentage return that the bondholder could earn if he invested in another bond of similar characteristics and risk. It is important not to confuse the coupon rate and the discount rate, as it is the relationship between the coupon rate and the market discount rate that determines the present value or price of the bond.

Video at 01:21

To illustrate this relationship, let's work with an ongoing simple example of a 5-year $1,000 bond that pays annual coupons at a rate of 5%. What we will soon see is that, as we change the annual discount rate relative to the annual coupon rate, to the annual coupon rate, the price of the bond will also change. Let us first imagine that the annual discount rate is equal to the coupon rate of 5%. We can run the numbers through the bond valuation formula and find that the price of the bond is equal to $1,000.

$$price\ of\ bond = PV = \$C \times \left[\frac{1-(1+r)^{-n}}{r}\right] + \frac{FV}{(1+r)^n}$$

$$price\ of\ bond = PV = \$50 \times \left[\frac{1-(1+0.05)^{-5}}{0.05}\right] + \frac{FV}{(1+0.05)^5} = \$1,000$$

Notice that the price of the bond is equal to the principal of $1,000. When the coupon rate is equal to the market discount rate, the price of the bond is equal to the principal amount, and we say that this bond sells at par. This is because the annual cost of capital of 5%, being the annual discount rate, is equal to the amount that the bondholder receives in the form of coupon payments, so the bondholder is willing to pay exactly $1,000 today to receive $1,000 in 5 years. Oftentimes, you will see the price of the bonds being quoted as a percentage of the principal or the par value; thus, the par value of a bond is set at 100, meaning that the price of the bond is 100% of the principal.

Video at 02:33

Now, imagine that, after the bond was issued, the annual discount rate dropped from 5% to 3%. Using the bond valuation formula, we calculate that the price of this bond is equal to $1,091.59,

$$price\ of\ bond\ =\ PV\ =\ \$50 \times \left[\frac{1-(1+0.03)^{-5}}{0.03}\right] + \frac{FV}{(1+0.03)^{5}}\ =\ \$1,091.59$$

which is greater than the principal of $1,000. Why is this the case? Recall that the market discount rate is the rate of return that the bondholder requires to hold this bond and similar bonds. This means that, for other bonds on the market, the bondholder's required rate of return is 3%, but this bond is actually paying more than what the bondholder requires with the coupon rate of 5%. If the bondholder can earn a coupon rate that is greater than their required rate of return, then the bondholder would be willing to pay more for this great deal. Thus, when the coupon rate is greater than the market discount rate, the price of the bond is greater than the principal amount, and we say that this bond sells for a premium. This bond would be quoted at 109, meaning that the price of this bond is approximately 109% of the principal of $1,000.

Video at 03:36

Lastly, imagine that after the bond was issued, the annual discount rate rose from 5% to 10%. Using the bond valuation formula, we calculate that the price of this bond is equal to $810.46,

$$price\ of\ bond\ =\ PV\ =\ \$50 \times \left[\frac{1-(1+0.10)^{-5}}{0.10}\right] + \frac{FV}{(1+0.10)^{5}}\ =\ \$810.46$$

which is less than the principal of $1,000. Intuitively, we know that the bondholder requires a rate of return of 10% on similar bonds; however, this bond only has a return of 5%, because the bond is only paying coupons of 5%. If bondholders can earn a return of 10% on similar bonds, then they would only buy this bond if it sold for less than the principal. Thus when the coupon rate is less than the market discount rate, the price of the bond is less than the principal amount, and we say that this bond sells at a discount. This bond would be quoted at 81, meaning that the price of this bond is approximately 81% of the principal of $1,000.

Video at 04:32

In finance, it is always emphasized that cash flows are discounted using the effective rate, which is the rate that matches the frequency of the cash flows. However, the reality of the finance industry is that rates are always quoted in annual terms using APR. This is also true for the world of bonds. When the effective rates used to discount the coupons of a bond are annualized, this is known as the yield to maturity. The yield to maturity annualizes the effective periodic discount rates through the following formula:

$$yield\ to\ maturity\ (YTM) = r \times k$$

where “k” is the number of periods in a year. (Note: sometimes you will see “m” used instead).

Video at 05:02

For example, if a bond with semiannual coupons has an effective semiannual rate of $r_{semiannual}$ = 5%, then the bond has a yield to maturity of 10% ($5\% \times 2$). Since it is the yield to maturity and not the effective rate that gets reported, the yield to maturity is an important concept when talking about bonds.

Video at 05:18

Let's see the yield to maturity in action. Imagine that we wanted to buy a 10-year $1,000 government bond that paid monthly coupons at a rate of 9%, and had a yield to maturity of 12%. What is the price of this bond? Right off the bat, we can expect that this bond will sell at a discount. How do we know this? Well, since both the coupon rate and the yield to maturity are expressed in APR, we can directly compare these two rates. As we learned, when the coupon rate (9%) is less than the required rate of return (12%), the bond will sell at a discount.

Video at 05:49

We can solve for the price of this bond in three easy steps: first, we need to know the dollar amount of the monthly coupons, for which we will use the coupon payment formula.

$$coupon = \frac{coupon\ rate \times FV}{k} = \frac{9\% \times \$1{,}000}{12} = \$7.50$$

Secondly, we need to find "r", the effective monthly rate of return, which we will use as the discount rate for the bond valuation formula, because recall that discount rates represent the opportunity cost of using one's money to buy this bond, and the opportunity cost of buying this bond is the rate of return or yield that the bondholder could earn on similar bonds, which is the effective periodic rate of return. Either way, the discount rate, rate of return, or yield are used interchangeably and are all represented by "r".

$$r_{monthly} = \frac{APR}{k} = \frac{12\%}{12} = 1\% \text{ per month}$$

Lastly, we will use "C" from step 1 and "r" from step 2 in the bond valuation formula to calculate the price of the bond. Following these three steps, we can see that the price of the bond is, like we suspected, selling at a discount for $825.75.

$$\text{price of bond} = PV = \$7.50 \times \left[\frac{1-(1+0.01)^{-120}}{0.01}\right] + \frac{\$1{,}000}{(1+0.01)^{120}} = \$825.75$$

(Note: the number of periods (“n”) is $12 \text{ months per year} \times 10 \text{ years} = 120 \text{ coupons in total}$)

Video at 06:45

Here, a yield to maturity of 12% means that we expect to earn an annualized rate of return of 12%, by buying a bond that is worth $1,000 in 10 years for $825.75 today. In reality, we may not know the yield to maturity of a bond. On the public debt market, we will know the face value, maturity, and the coupon rate of a bond, as well as how much it is selling for. As an investor, if we want to know our expected rate of return from this bond, in other words, the effective annual rate used to discount the coupons, then we would have to solve for "r". However, as you can see from the bond valuation formula, it is extremely complex to isolate just "r"; thus, it may require a lot of trial and error to find the value of "r" so that the present value of all the cash flows from the bond is equal to the price of the bond, $825.75.

Video at 07:36

The good news is that there is a function built into Excel that will solve for "r" for us. By using the rate function in Excel, we can input (in the following order) the number of periods (n=120), the periodic dollar amount coupon (C=$7.50), the price (PV=$825.75), and the face value (FV=$1,000), and Excel will tell us the effective "r", which, in this case, is the effective monthly rate of 1%.

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=rate(120, $7.50, -$825.75, $1000)
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From here, we know that $\text{yield to maturity } (YTM) = r \times k = 1\% \times 12 \text{ months per year} = 12\%$

Video at 08:01

Once again, it is important not to confuse the coupon rate with the yield to maturity. The coupon rate is a fixed rate that determines how much the issuer must pay to the bondholder on an annual basis. On the other hand, the yield to maturity represents the internal rate of return, or the average discount rate, on the bond that makes the sum of the present value of the bond's cash flows equal to the price of the bond, expressed in APR. However, this is only true given 3 key assumptions about the yield to maturity: first, the bondholder must hold onto the bond until the end, the maturity date. Second, the issuer must make all the promise payments, including the coupons and the principal, to the bondholder in the full amount and on time, as they are legally obligated to do so. Lastly, the bondholder must reinvest each coupon at the yield to maturity rate as soon as the coupons are received. If the above 3 assumptions are all satisfied, then the bondholder can expect to earn a rate of return that is equal to the yield to maturity of the bond.

Video at 09:00

To wrap up, what are the key takeaways from this video? Well, we learned how the relationship between the coupon rate and the discount rate can affect the price of a bond; how the yield to maturity is essentially an annualized expression of the discount rate that make the present value of all the future cash flows of a bond equal to its price; and, the 3 key assumptions behind the yield to maturity that make it the annualized rate of return that bondholders can expect to earn. Well, that is all for this video, thanks so much for watching!