Video 13 - Calculating a Bond's Realized Return

The following is a supplementary transcript for tutorial videos from

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Hey everyone, thanks for tuning in. Today, we are going to be discussing "RoR", or the realized rate of return, on a bond. First, we will compare the difference between expected and actual returns; next, we will learn how to calculate the total realized return, also known as the holding period rate of return; and, lastly, we will look at how to adapt this formula to calculate the annual realized return, or "RoR". Let's get started!

Video at 00:29

We base our investment decisions on future expectations. When buying a bond, we make educated guesses about future interest rates, inflation, and that company's ability to fulfill its obligations (solvency). Since nobody can predict the future, market prices reflect expected, rather than actual, returns. But, sometimes, our actual returns turn out to be different from what we expected, kind of like getting bangs: you go in expecting to look like Zooey Deschanel, and come out looking more like Edna from The Incredibles.

Video at 00:59

Recall that we use the yield to maturity, or "YTM", to measure the expected return of a bond. YTM is an APR rate (annual percentage rate), calculated by

$$yield\ to\ maturity = YTM = r \times k$$

where "r" is the period rate, and "k" is the number of compounding periods in a year. So, what could cause our actual (realized) returns to differ from our expected returns (YTM)? Let's find out!

Video at 01:24

The yield to maturity makes three assumptions that don't always hold up in reality. First, it assumes that you hold the bond until maturity. If you don't hold the bond until maturity, changes in market interest rates may affect the market price of the bond, causing you to earn more or less when you sell the bond, compared to receiving the full face value if you hold the bond until maturity. That is, there may be an additional capital gain or loss, beyond what was baked into the investment at issuance via premiums or discounts.

Video at 01:53

Second, it assumes that you reinvest all the coupon payments you receive at that same YTM. This requires that market interest rates don't change during the period in which you hold the bond. However, this is rarely the case, as market interest rates are constantly fluctuating, allowing you to earn a higher or lower return than the original bond interest rate.

Video at 2:13

Another reason that this assumption may not hold in reality is that coupons received closer to the maturity date would have to be reinvested in shorter-term investments, which often yield lower returns. This assumption is implicit in the annuity part of the bond valuation formula. By discounting all of our coupons to their present values using the market rate of interest on the bond, we are assuming that we would be indifferent between receiving the present value of the coupon today and receiving the future value later.

Video at 02:41

The third assumption the yield to maturity formula makes is that the bond issuer doesn't default on any of their payments. That is, you receive the full amount of coupons and principal that was promised. But, these three assumptions don't always hold in reality. If you don't hold your investment until maturity, or you don't reinvest all of your coupons at the same rate, your realized return may differ from your expected return. Sometimes, we want to know our realized return in order to evaluate the actual success of an investment. So, how can we calculate it?

Video at 03:12

In order to calculate our realized return, we have to understand the two components that make up the return of a bond: capital gains (or losses), and coupon payments. We earn a capital gain on a bond when its present value (or market price) goes up. From the present value of an annuity formula, we can see that, when the market interest rate falls, the coupons are not being discounted as much, which makes the present value (or price) of the bond higher. This means that we can sell the bond at a higher price than we expected.

Video at 03:41

You can also think about this from the perspective of the investor: when the market interest rate falls, it means that similar bonds on the market are offering lower returns, which makes the bond's higher fixed coupons more attractive. As investors would rather hold bonds that pay higher coupons, the increased demand for this bond will push up the price of the bond in the market,

Video at 04:01

For instance, let's take a bond with a face value of $1,000 that pays 6% annual coupons and has 10 years until maturity. If the current market interest rate is 8%, then the market price of this bond is

$$\text{price = PV of bond} = A \times \left[\frac{1-(1+r)^{-n}}{r}\right] + \frac{FV}{(1+r)^n} = \$60 \times \left[\frac{1-(1.08)^{-10}}{0.08}\right] + \frac{\$1{,}000}{1.08^{10}} = \$865.80$$

But suppose now that the market rate goes up to 10%. The price of the bond would fall to

$$\text{price = PV of bond} = A \times \left[\frac{1-(1+r)^{-n}}{r}\right] + \frac{FV}{(1+r)^n} = \$60 \times \left[\frac{1-(1.10)^{-10}}{0.10}\right] + \frac{\$1{,}000}{1.10^{10}} = \$754.22$$

Thus, the investor's capital loss on this investment is $865.80 - $754.22 = $111.58.

Video at 04:45

We also earn a return on our bond through the coupon payments we receive. When we receive a coupon payment, we can choose whether to reinvest it and continue to earn a return on this money. The value of these reinvested coupon payments at the time that we sell the bond is part of our total realized return.

Video at 05:00

Thus, the total return we earn on this bond, over the period we own it, is the amount we earn from the sale of the bond, plus the value (at the time of sale) of the coupon payments we received and reinvested, divided by the price that we initially bought the bond for:

$$\text{total holding period return} = \frac{\textit{sale price + future value of reinvested coupons}}{\textit{initial purchase price}}$$

This tells us our return over the entire investment; we call this our holding period return. Let's try it out!

Video at 05:21

Suppose I buy a $1,000 bond from my friend's microbrewery called The Yeasty Boys. The bond pays a $50 coupon at the end of each year. The market rate is 5%. We can reinvest our coupons at a rate of 3%. If I sell the bond after 3 years for $1,100, what is my holding period return?

Video at 05:41

We know that we initially paid $1,000 for the investment and sold it for $1,100. But, what is the value of our reinvested coupon payments at the time of sale? We can use a future value annuity formula to calculate the value of these coupons. If we reinvest our $50 coupons every year for three years at 3% interest, the total value of this coupon investment at the end of three years will be $154.55

future value of reinvested coupons (FVRC) = $A \times \left[\frac{(1+r)^n - 1}{r}\right] = \$50 \times \left[\frac{(1.03)^3 - 1}{0.03}\right]$ = $154.55

Pause to see if you can recalculate this future value using the basic time value formula or the annuity future value formula.

Video at 06:14

Thus, our gross holding period return is the $1,100 capital gain plus the $154.55 in coupon payments, divided by $1,000, or 125.5%

total (gross) holding period return = $\frac{sale\ price + FVRC}{initial\ purchase\ price} = \frac{\$1{,}100 + \$154.55}{\$1{,}000}$ = 125.5%

Note that this formula tells us our gross returns.

Video at 06:31

Gross returns include 100% for our initial value of our investment. Anything over 100% means we earned a positive return on our investment. Anything under 100% means we lost money on our investment. And a gross return of exactly 100% means that, without factoring in the time value of money, we exactly broke even; we didn't lose money, but we also didn't make money.

Video at 06:52

To determine our net return on our investment, which is the additional amount that we earn on our capital, we simply subtract 100% from our gross returns (i.e. net return = 125.5% - 100%). Another way to directly calculate the net return is to simply deduct the original amount paid (or invested) in the numerator

net holding period return = $\frac{sale\ price + FVRC - initial\ purchase\ price}{initial\ purchase\ price}$

In this case, the net holding period return is $\frac{\$1{,}100 + \$154.55 - \$1{,}000}{\$1{,}000}$. This net return tells us by how much our investment has grown. In this case, our investment grew by 25.5% So now we know how to calculate the total realized return of our investment, known as our holding period return.

Video at 07:28

But when we deal with interest rates, we usually want to express our returns on an annual basis, so we can better compare them between our investments. In finance, we commonly express rates in annual terms so that we are comparing apples to apples. Usually, investors will hold on to a bond for a few years, so the total realized return on the bond reflects the return over the entire holding period. We have already learned how to convert monthly or quarterly returns into annual returns, but, how do you convert, say, a six-year return into an annual return?

Video at 07:57

To express our realized return as an effective annual rate, we simply raise our gross holding period return to the power of 1/n (where "n" is the number of years we hold the bond for) and then subtract 1

$$r_{annual} = (r_{gross\ holding\ period})^{1/n} - 1$$

By n^{th} -rooting the gross period return, we are working backwards to figure out what the annual return, if compounded each year, would be.

Video at 08:18

For example, average real estate prices in the Dunbar community of Vancouver back in 2009 were $1.2million. In 2014, they were $2.4million. That is a 200% gross return ($\frac{\$2.4}{\$1.2}$), or a 100% net return ($\frac{\$2.4 - \$1.2}{\$1.2}$). But what was the return on an annualized basis? Investors would have earned an annual realized return of 14.87%

$$r_{annual} = (\tfrac{\$2.4}{\$1.2})^{1/5} - 1 = 14.87\%$$

Notice that multiplying 1 plus this annual return (14.87%) "n" times, or raising it to the power of "n" would get us to the total holding period return we originally stated

$$\text{total (gross) total period holding return} = (1 + r_{annual})^{n} - 1 = (1 + 0.1487)^{5} - 1 = 200\%$$

We refer to this annualized rate as the realized rate of return, or "RoR". It is also commonly referred to as the compound annual growth rate, the geometric mean, or the time-weighted return.

Video at 09:04

Now, try this problem on your own. Pause the video and calculate the realized rate of return of this bond:

Let's say you buy a 6-year 5% coupon bond with a face value of $1,000 from a company called Millenia, *which specializes in avocado toast and adult colouring books. The bond makes semiannual coupon payments and has a YTM of 8%. You hold the bond until maturity, and you reinvest the coupons in a pyramid scheme that promises to make you your own boss and to pay a return of 6% per year, compounded semiannually.*

(1) What is the RoR of this bond?

(2) Is this lower or higher than the YTM? Why?

Video at 09:13

Let's solve it together. In order to calculate the realized rate of return on this bond, we must know: (a) our capital gains, (b) the value of our reinvested coupon payments, and (c) what we originally paid for the bond. When we hold a bond to maturity, we can think of it as though we are "selling" the bond at the end of its life in exchange for the principal payment. So we can include this $1,000 payment in our holding period return as if it were the selling price. If we sold the bond before maturity, we would, instead, use the price at which we sold the bond.

Video at 09:43

In order to determine how much we have earned, we need to calculate what we originally paid for the bond. We can calculate the issue price using our bond annuity formula

price = PV of bond = $A \times \left[\frac{1-(1+r)^{-n}}{r}\right] + \frac{FV}{(1+r)^n} = \$25 \times \left[\frac{1-(1.04)^{-12}}{0.04}\right] + \frac{\$1{,}000}{1.04^{12}}$ = $859.22

- Remember that coupon rates are expressed in APR. We earn 0.05*$1,000 = $50 in a year, so if we're paid semiannual coupons, we receive $25 every six months (semiannual).
- 6 years of coupons * 2 coupons per year = 12 coupons in total (n=12).
- The YTM = r * k.

  Therefore, $r_{periodic} = \frac{YTM}{k} = \frac{8\%}{2}$ = 4%, so our appropriate discount rate is our effective semiannual rate ($r_{semiannual}$) of 4%.

The price we paid to purchase this bond is $859.22.

Video at 10:38

Second, let's calculate the value of the bond coupon payments at the time of sale if we reinvested them at 6%.

$$\text{FVRC} = A \times \left[\frac{(1+r)^n - 1}{r}\right] = \$25 \times \left[\frac{(1.03)^{12} - 1}{0.03}\right] = \$354.80$$

- The $25 comes from the $25 semiannual coupons that we receive and reinvest.
- The 3% is our effective semiannual rate ($r_{semiannual}$) that we are reinvesting these coupons at, since our semiannual rate is $r_{semiannual} = \frac{YTM}{k} = \frac{6\%}{2}$ = 3%.
- We receive a total of 12 coupons (n=12).

Applying the future value annuity formula gives us $354.80.

Video at 11:15

Thus, our total gross holding period return is $1,000 (the principal we receive at the end), plus $354.80 (the value of our reinvested coupons), divided by $859.22 (initial purchase price), which is 158%

$$\text{total (gross) holding period return} = \frac{sale\ price + FVRC}{initial\ purchase\ price} = \frac{\$1{,}000 + \$354.80}{\$859.22} = 158\%$$

This also means that we earned a net return of 58% (158% - 100%) over the six years.

Video at 11:37

But, in order to compare our realized rate of return to our expected YTM, we must put them on the same playing field by converting our six-year rate of return into an annual rate of return. Let's annualize this return by raising it (gross six-year return = 158%) to the power of 1/6th and subtracting 1. In other words, one year is $1/6^{th}$ of our six-year holding period. This gives us the annual realized rate of return (or RoR) of 7.4%

$$r_{annual} = (r_{gross\ holding\ period})^{1/n} - 1 = (158\%)^{1/6} - 1 = 7.4\%$$

This is lower than the yield to maturity of 8%, because we reinvested the coupons at a lower interest rate of only 6%.

Video at 12:09

Let's recap the differences between the yield to maturity (YTM) and rate of return (RoR) Yield to maturity is quoted as an APR (or stated rate), while RoR is quoted as an EAR (or effective rate). YTM assumes that the investor holds the investment until maturity and reinvests the coupons at the same rate as the bond; RoR is based on the realized return on the bond, which may differ when these assumptions do not hold. Lastly, we must solve for yield to maturity by taking r * k, where "r" is found by applying the present value of annuity using a spreadsheet or financial calculator, while rate of return can be calculated by hand from our capital gains and applying a future value annuity formula to our coupons.

Video at 12:50

In this video, we learned that realized returns can differ from expected returns when we don't hold the bond until maturity or reinvest the coupons at the bond interest rate. We can calculate the total realized return over the holding period of a bond, or we can express this rate in annual terms as a "RoR" (realized rate of return) Thanks for watching, and we'll see you next time!