Video 2 - Interest Rates and Compounding Periods

The following is a supplementary transcript for tutorial videos from
https://blogs.ubc.ca/financefundamentals/

Hello everyone. Today, we are going to be talking about simple interest, compound interest, and compounding periods. This is an important concept, because simple interest and compound interest can yield incredibly different returns for investors. Understanding this will not only help you in finance, but also guide your future investment decisions. I will start off by explaining what simple interest is, and how it is different from compounding interest. Then, we will learn how to calculate investment returns using each method and compare the results of the two calculations. I will also explain the difference between annual compounding, semi-annual compounding, daily compounding, etc., and how to calculate them. And lastly, I will teach you what to do if you withdraw money before the end of a compounding period.

Video at 00:50
In the previous video, you learned how to calculate present and future values. You may be wondering why we use such a complicated formula when you can simply multiply the interest earned. If you have tried that already, then you will notice that you got a different result than the one we did in the last video ("Time Value of Money: Calculating Present and Future Value"). This method calculates for what is called simple interest. However, simple interest almost never applies. This is all because of a magical concept: compounding.

Video at 01:18
Compounding refers to the act of earning interest on interest. For every year of interest you have earned, in the following year, you will not only earn interest on your principal investment, but also on the interest you earned the previous year. You might be thinking: how much can I really earn from the interest on that little interest? You will be surprised. It is the best to illustrate this with a table and example.

Video at 01:41
Imagine you have decided to save for retirement at this early age. You managed to save up $100 and invested it at a 10% annual interest rate. Maybe it is too early to think about retirement, but it is never too early to start saving money, and I will show you why. If you earn

simple interest, you would receive $10 for your first year. It is the same for compound interest. For your second year, you would have still earned $10 under simple interest ($1,000*10%), but $11 under compound interest. Where did this $1 difference come from? It is because the $10 interest you earned in year 1 stayed in your account, so you effectively earned interest on your interest. Notice how 10% interest earned on $10 (interest earned in year 1) is $1, which is the $1 difference between the interest earned in year 1 ($10) vs year 2 ($11). We can keep repeating this. For the third year, simple interest would have still earned $10, but with compound interest, you actually earn $12.10. This is because the interest you earned in years 1 and 2 are now both in your account, and thus added to the principal that earns a 10% interest, which is $12.10. This process is called compound interest.

Simple Interest

year	principal	simple interest
1	$100	$10
2	$100	$10
3	$100	$10

Compounding Interest

year	principal + interest	compounding interest
1	$100	$10
2	$110	$11
3	$121	$12.10

Figure 1

Video at 02:49

You can calculate compound interest using a chart and calculating year by year like I just did, or you can use the formula:

$$ending\ balance\ after\ "n"\ years\ =\ principal\ \times (1+r)^n$$

Notice that this is the same as the future value formula that we learned in the last video ("Time Value of Money: Calculating Present and Future Value"). This will get you the end balance in your account after "n" periods at the interest rate "r". To get the interest rate earned only, you would need to use

$$interest\ earned\ after\ "n"\ years\ =\ principal\ \times [(1+r)^n - 1]$$

to subtract the principal from the future value.

Video at 03:20

The difference between simple interest and compound interest is initially small, but the snowball will soon start rolling. In year 3, as we have just calculated, the difference between interest earned is $2.10 ($12.10 compound interest - $10 simple interest), but in year 10 you will have a difference of $13.58 ($23.58 compound interest - $10 simple interest). If we add up the total earned under simple interest, you would have earned $100 of interest, $10 every year for 10 years. But under compound interest, you would have earned $159.37. That is a nearly 60% difference!

Simple Interest

Year	Simple	Total
0		100
1	10	110
2	10	120
3	10	130
4	10	140
5	10	150
6	10	160
7	10	170
8	10	180
9	10	190
10	10	200

Compounding

Year	Interest	Total
0		100
1	10	110
2	11	121
3	12.10	133.10
4	13.31	146.41
5	14.64	161.05
6	16.11	177.16
7	17.72	194.87
8	19.49	214.36
9	21.44	235.79
10	23.58	259.37

Figure 2

Video at 03:53

Let's take it to the extreme and say you left this money in the bank for 100 years, and now your grandchildren want to withdraw this money. How much do you think will be in this account? If the bank only paid simple interest, then it would be $1,100: the initial $100 and $1,000 of interest ($10 simple interest * 100 years). If the bank paid compound interest, like almost all banks do, you will have $1,378,061.23 ($FV = \$100 \times (1.10)^{100}$) in that account. I think your grandkids will definitely notice the difference.

Video at 04:23

Now that we have understood the importance of compounding interest, let's go over the concept of compounding periods. In the previous example, we were working with a 10% annual interest rate. This means we used annual compounding, which refers to a situation where interest is given to you at the end of the year, and this interest will start to earn interest at the beginning of the next year. We will always assume that cash flows are paid or received at the end of each period, unless expressly stated otherwise. There can be compounding for any time period.

Common compounding periods include semi-annual, monthly, daily, and continuous compounding. We will go over each of them in detail.

Video at 05:01

Semi-annual compounding is when the interest is added to our investment twice a year. Using our previous example, instead of receiving 10% interest at the end of each year, you will receive it in two chunks; that is, you will earn 5% every 6 months. We call this a 10% semi-annual rate. To calculate semi-annual compounding, you take the future value formula ($P \times (1 + r)^n$), and adjust it. There are two parts that you need to adjust in this formula. One is the time period, “n”. Here, you would double “n”, because there are two semi-annual periods in each year. In other words, you will receive interest payments twice within a year. And the other is the discount rate, “r”. Here, you would halve “r”, because we want to express the 10% interest rate on a semi-annual basis. You will receive 5% interest each semi-annual period.

Video at 05:51

Still using our example of \$100, if you want to calculate your account balance after 10 years of semi-annual compounding, you use the formula

$$FV_{10} = \$100 \times (1 + \tfrac{0.10}{2})^{2\times 10} = \$100 \times (1 + 0.05)^{20} = \$265.33$$

Note here that we now have n=20 compounding periods, instead of 10, because there are 20 six-month periods in 10 years. A common mistake students make when solving this type of problem is only adjusting one part of the formula and forgetting about the other half. Always remember to adjust both parts, because *both* the compounding period and the discount rate must reflect the interest rate's compounding frequency of 6 months. Instead of earning 10% every year, you will earn 5% every 6 months.

Video at 06:34

But wait, the payments of \$50 are smaller than the payment of \$100, so how does semi-annual compounding (\$265.33) make more money in 10 years than annual compounding (\$259.37)? The trick is to remember that increasing the compounding frequency means we are putting our money to work faster. Therefore, between getting semi-annual payments of \$50 and one annual payment of \$100, we would prefer the former because we get to put \$50 into our account 6 months in, which means this \$50 is making us interest 6 months earlier than if we had to wait a whole year to put our entire $100 into our account. The sooner we put our money into our account, the sooner our money is being put to work, and the more interest we will make.

Video at 07:15

Quarterly and monthly compounding is similar to semi-annual, except the interest will be given every quarter and month, respectively. Still, $100 over 10 years at 10% for quarterly:

$$r = \frac{10\%}{4\ quarters\ in\ a\ year} = 2.5\%\ each\ quarter$$

$$n = 4\ quarters\ per\ year \times 10\ years = 40\ quarters\ or\ periods\ in\ total$$

$$FV_{10} = \$100 \times (1 + \tfrac{0.10}{4})^{4\times10} = \$100 \times (1 + 0.025)^{40} = \$268.51$$

For monthly compounding:

$$r = \frac{10\%}{12\ months\ in\ a\ year} = 0.8333\%\ each\ month$$

$$n = 12\ months\ per\ year \times 10\ years = 120\ months\ or\ periods\ in\ total$$

$$FV_{10} = \$100 \times (1 + \tfrac{0.10}{12})^{12\times10} = \$100 \times (1 + 0.008333)^{120} = \$270.70$$

Again, notice how the number of periods (“n”) and discount rate (“r”) both match the compounding frequency (quarterly and monthly) of the interest rate.

Video at 07:52

For daily compounding, the interest rate will be compounded daily. This is very common in credit card balances, which is why, even though the interest rate the bank tells you is not very high, you might find your balance racking up pretty fast. Back to our old example of $100 over 10 years at 10%, this time:

$$r = \frac{10\%}{365\ days\ in\ a\ year} = 0.0274\%\ each\ quarter$$

$$n = 365\ days\ per\ year \times 10\ years = 3,650\ days\ or\ periods\ in\ total$$

$$FV_{10} = \$100 \times (1 + \tfrac{0.10}{365})^{365\times10} = \$100 \times (1 + 0.000274)^{3,650} = \$271.79$$

Video at 08:21

Lastly, we will talk about continuous compounding. This means that the interest we earn is added to our investment continuously. Theoretically, it will be compounding not only every second, but every millisecond, and even infinitesimally shorter than that. This type of compounding has its own formula, which is

$$FV_t = P \times e^{r\times t}$$

where “P” is the principal, “r” is the interest rate, and “t” is the number of periods. "e" is the magical number in math that equals to approximately 2.718. After continuously compounding for 10 years, the balance in your account will be

$$FV_{10} = \$100 \times e^{0.10 \times 10} = \$271.83$$

Video at 08:55

Let's compare the results from the different periods of compounding after 10 years. For simple interest, you have $200, annual compounding: $259.37, semi-annual compounding: $265.33, quarterly compounding: $268.51, monthly compounding: $270.70, daily compounding: $271.79, and continuous compounding: $271.83.

Compounding period	Balance
Simple	$200
Annual	$259.37
Semi-annual	$265.33
Quarterly	$268.51
Monthly	$270.70
Daily	$271.79
Continuous	$271.83

Figure 3

From this, you will have noticed that, as the compounding periods get shorter and shorter, you see a smaller and smaller difference. Also, the higher the compounding frequency, the higher the balance. This makes intuitive sense, because the more frequently the interest is compounding, the more our interest earns interest. Therefore, if you have a savings account, you want the interest to compound more frequently so that your earned interest can generate more interest more quickly. However, with credit cards and loans, you would rather that the compounding frequency is low, so that you do not owe as much in interest.

Video at 09:59

Lastly, let's end this video with a tip on solving for problems, where the money is withdrawn on a date that is not the end of a compounding period. For example, if the account compounds annually, the compounding period is usually January 1 to December 31. The end of a compounding period can be set as an arbitrary date, but we can assume the end of the year for this example. If you withdraw the money on, let's say, May 31, what will happen? Will you just forfeit the interest earned on the first 5 months of the year? No. But do you get to earn interest on the whole year? Of course not. You will earn the prorated interest. To calculate this, let's take our own example again, $100 at 10% annual interest, but this time, it is 10 years plus 5 months.

Video at 10:44

You start off by calculating the money normally earned on the 10 whole periods, which is

$FV_{10} = \$100 \times (1.10)^{10} = \$259.37$, as we have calculated before.

Then, you calculate the interest that will be earned by another whole year (year 11), which is the balance at the end of year 10 ($259.37) times a 10% annual interest rate (how much interest you would make if you kept your investment for *all* of year 11), and then times that by the fraction of the compounding period to prorate. Here, we have that we kept the money in our account for 5 whole months out of the entire year, which is $\frac{5}{12}$. You should get

$$prorated\ interest = ending\ balance \times interest\ rate \times fraction\ of\ compounding\ period$$

$$prorated\ interest = \$259.37 \times 0.10\ annual\ rate \times \frac{5\ months}{12\ months\ per\ year} = \$10.81$$

This is the interest rate you have earned in the 5 months. Add this to $259.37, and you get $270.18, and this is the balance in your account after 10 years and 5 months. Of course, you will not earn interest on the interest you earned over the 5 months until it is compounded again at the beginning of the next period.

Video at 11:33

So today we went over the difference between simple and compound interest, and why it is important. And then we looked at the different compounding periods, and the difference in the values when employing different compounding periods. Finally, we learned how to deal with questions that don't end on a whole compounding period. I hope this video was useful for not only studying finance, but also to help guide your future investment and credit card balance decisions in the future.