Video 25 - Risk and Return: Mean-Variance Criterion and Diversification

The following is a supplementary transcript for tutorial videos from

https://blogs.ubc.ca/financefundamentals/

Hi everyone. In today's video, we are going to talk all about risk: why we hate it, how we can measure it, and how to lower it through diversification. By the end of this video, you will learn the fundamental trade-off between risk and return, how to measure the standard deviation or variance of returns, the difference between systematic and unsystematic risk, and how to lower unsystematic risk through diversification.

Video at 00:27

We buy stocks, bonds, or other securities because we expect to earn a return on those assets. Returns can come in the form of payments like dividends or coupons, or as an increase in the asset's market price, which is known as a capital gain. But these investments carry an element of risk; we may not make as large of a return as we expect. As anyone who invested in Blackberry can attest, we may actually end up losing money on our investment. So, our return isn't 100% certain: that is, there is risk.

Video at 00:55

But see, investors don't like risk. Most people are risk averse, meaning that they will prefer an asset with a more predictable return over one with greater variability. So, to incentivize investors to take on riskier investments, these investments must earn an extra return. That is, investors can earn a risk premium in excess of what they would earn on a risk-free investment. In general, the higher the risk, the greater the return you can expect to earn on an investment. Sadly, you cannot have both low risk and high returns; there is a trade-off between risk and return. Investors can, say, decide to pick a level of risk they are comfortable with and then aim to maximize the return at that level, or choose their desired level of return and then aim to minimize the risk while meeting this target.

Video at 01:43

Some assets have greater variation in their returns, making them more unpredictable, and therefore more risky. Stocks have historically been the most unpredictable, and therefore risky, class of assets, while bonds have shown more consistent returns and are considered a less

risky investment, because the fixed coupon rates are predictable and the bond issuers are legally obligated to pay dividends. But notice that, overall, the average return of the stock market is much higher than that of the bond market. Those willing to take on the added risk of stock investments are rewarded with higher returns on average across time. This isn't just a coincidence: the market will ensure that riskier assets pay higher returns. You can imagine that if two assets that paid the same return had two different risks, then no one would ever invest in the riskier of the two assets, causing the price to fall, which will increase the return on investment. We will discuss this more in our next video on the capital market line.

Video at 02:35

So now that we know what risk is, how can we measure it? We can measure the risk of an investment by calculating the variance of its returns -- that is, how much the actual return tends to vary from the average or expected return. We use the historical returns (R_i) of an asset to calculate its variance for each possible return. We will subtract the expected, or average, return ($E(R)$) and square this value. Effectively, this gives greater weight to values that deviate more from the expected value. Then, we multiply this by the probability of getting this return (p_i) based on how many times it has occurred in the past. We do this for every possible return ($i = 1 \ldots n$), and then add these terms together ($\sum_{i=1}^{n}$). We can also express this formula like this:

$$variance\ of\ returns\ =\ Var(R)\ =\sigma^2\ =\ \sum_{i=1}^{n} p_i \left[R_i\ -\ E(R_i)\right]^2\ =\ E(R^2)\ -\ E(R)^2$$

where R_i is the return of the asset, p_i is the probability of that return, and $E(R)$ is the average return of the asset, also known as the expected return (expected value).

We can take the square root of the variance to get our standard deviation. The lower the standard deviation or variance of an investment's historical returns, the less risky its returns are expected to be in the future. Investors prefer assets with a low standard deviation on their returns.

$$standard\ deviation\ of\ returns\ = stdev(R)\ =\sigma\ =\ \sqrt{Var(R)}$$

Video at 03:32

We know that total risk is the variance of the returns, but total risk can be further divided into systemic (or market) risk, and unsystematic (or idiosyncratic) risk. Unsystematic risk refers to

the risks that affect an individual asset or the specific firm rather than the entire market or a large number of securities. Some examples of unsystematic risks include labor strikes, lawsuits, or poor company management, because these events affect only *one* company. If we were to own every security in the market, also known as the market portfolio, the unsystematic (or diversifiable or idiosyncratic) risk will be completely eliminated. Why? Because every security has different unsystematic risks.

Video at 04:14

We can illustrate this using a portfolio with only two stocks: one in a tourism company, and the other in a farming company. In rainy years, the farmer's crops flourish and his stock price goes up. Meanwhile, the tourism company's stock price suffers as nobody wants to tour in the rain. In dry years, the farmer's crops suffer, but the tourism business is booming. If you held only farm stocks, you would get an especially high return in the rainy years, and an especially low return in the dry years. (If you held only tourism stocks, you would get an especially low return in the rainy years, and an especially high return in the dry years). These two securities are negatively correlated. So as one goes up, the other goes down. One's bad performance will be balanced out by the other's good performance (their unsystematic risks “cancel out”), leaving the portfolio of the two stocks rather stable. This leads to less variance in your *total* returns in the portfolio, and therefore lower risk to you as an investor.

Video at 05:01

When assets are correlated, it means that changes in their returns are connected. If the farm and tourism stocks always move in opposite directions, we say that they have perfect negative correlation, which has a value of -1, the lowest possible value for correlation. If two stocks always move together, whether up or down, then we say that they have perfect correlation, which has a value of 1, the highest possible value for correlation. Usually, these increases or decreases will have a common cause, like the weather in our farming example. Lastly, if the movement of one stock had no effect on the returns of the other stock -- meaning that when one moves up, we cannot tell whether the other will move up or down, or perhaps remain unchanged -- then we say that these stocks have no correlation; that is, their correlation is 0.

perfect negative correlation: $\rho = -1$

perfect positive correlation: $\rho = +1$

no correlation: $\rho = 0$

Video at 05:50

When it comes to correlation, the lower, the better. The correlation of the two stocks is the covariance of the stocks divided by the standard deviation of the farm stock times the standard deviation of the tourism stock.

$$correlation\ of\ two\ stocks\ = \rho_{A,B} = \frac{\sigma_{A,B}}{\sigma_A \times \sigma_B} = \frac{covariance\ of\ stocks\ A\ and\ B}{stdev\ of\ stock\ A \times stdev\ of\ stock\ B}$$

Since standard deviation is always positive (denominator is always positive), then the direction of correlation (positive, zero, or negative) is determined by the covariance. But how do we calculate covariance?

The covariance of the two stocks is a measure of their joint variability. For any possible outcome ($i = 1 \ldots n$), we will take the return of asset A (R_A) minus the expected return of asset A ($E(R_A)$), times the return of asset B (R_B) minus the expected return of asset B ($E(R_B)$), times the probability of this outcome occurring (p), and will sum it for every possible outcome ($\sum_{i=1}^{n}$).

$$covariance\ of\ two\ stocks\ = \sigma_{A,B} = \sum_{i=1}^{n} p\left[R_A - E(R_A)\right]\left[R_B - E(R_B)\right]$$

That is, for each possible return for farm stock and tourism stock, how would they differ from their expected return?

Video at 6:38

As we saw in our previous example, when the returns for farm stock get higher as we get more rain, the returns for the tourism stock get lower. As the weather gets drier, the return on the farm stock is lower than its expected (average) value, and the return on the tourism stock is higher than its expected value. The return on one asset is always higher than its expected value, while the other is lower, so we will be multiplying a positive and a negative for every possible pair of returns to get a negative sum for the covariance. If the correlation is positive, it means that as one term goes up the other term also goes up. The returns are always either both above their expected returns, or below -- so we are always multiplying two negatives or two positives, resulting in a positive sum for covariance. And for assets with no correlation, returns are completely random and independent: sometimes both will go up, sometimes both will go down, and sometimes one will go up while the other goes down. When we add all of these random values together, we can expect them to cancel out and leave us with a covariance of 0.

Video at 07:45

When we talk about the relationship between two securities, we prefer to use correlation over covariance. Correlation must always be between -1 and 1, making it easier to interpret. If we have a covariance of, say, -10,000, all we know is that the securities tend to move in opposite directions. But if we know that the correlation is -0.2, we can also see that the strength of this relationship is low. But assets don't need to be negatively correlated in order to have risk reduction benefits. In fact, adding any security to your portfolio will lower the risk, if it has a correlation of less than 1 with your existing securities. An asset is always perfectly correlated with itself; for example, all shares in a company sell at the same price in the market at all times, so adding an imperfectly correlated asset means that we will not be as affected when one asset is performing poorly, because the performance of our other assets are not perfectly following the same pattern.

Video at 08:41

Holding onto a portfolio of assets that do not behave identically is called diversification. Having some variability between our stocks helps us to diversify, and thus reduce our risk. This is the financial equivalent of saying "don't put all your eggs in one basket." We can see this in our formula for the variance of a portfolio comprised of two assets:

$$variance\ of\ portfolio\ =\ \sigma_p^2\ =(w_B\sigma_B)^2\ +\ (w_S\sigma_S)^2\ +\ 2\,(w_B\sigma_B)\,(w_S\sigma_S)\,\rho_{B,S}$$

where subscript “p” refers to the portfolio, subscript “B” refers to bonds, and subscript “S” refers to stocks. The variance of the two securities is affected by the variance of each stock individually, as well as the covariance ($\rho_{B,S}$). Notice that because of this covariance term, the higher the correlation is ($\uparrow \rho_{B,S}$), the higher the portfolio variance becomes ($\uparrow \sigma_p^2$). In a portfolio with perfect positive correlation, a correlation of +1, the standard deviation of the portfolio is simply the weighted average of the two assets' standard deviations. That's because when securities move together in the same direction, the variances of each are essentially amplified -- the higher the correlation, the greater the amplification effect. But the lower the correlation is ($\downarrow \rho_{B,S}$), the smaller the portfolio variance becomes ($\downarrow \sigma_p^2$). When securities move somewhat differently from each other, the variations are tempered -- or, in cases of perfect negative correlation, cancelled out.

Video at 09:46

Why do we care so much about the covariance? Why not put more emphasis on the variance of the assets? Covariance has a larger impact on the portfolio's total variance than the variance of individual assets. As you add assets to a portfolio, you introduce only one variance term, but several covariance terms: the new asset has a covariance term with each of the existing assets in the portfolio. Look at how the ratio of covariance terms to variance terms increases as we add assets to the portfolio (Figure 1). In this way, unsystematic risk can be diversified away by carrying a set of assets that have a correlation of less than 1.

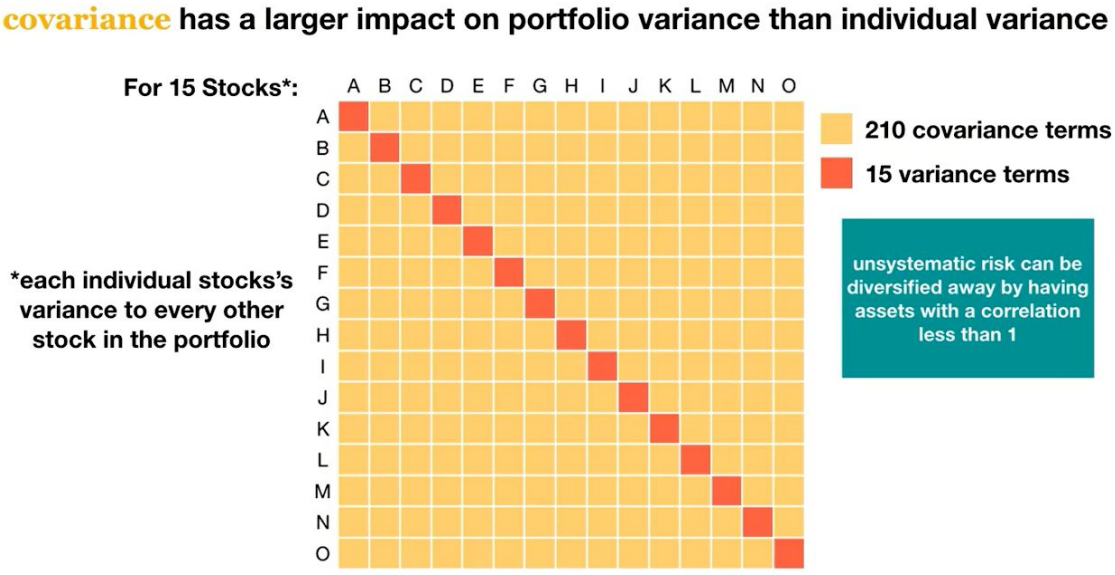

Figure 1

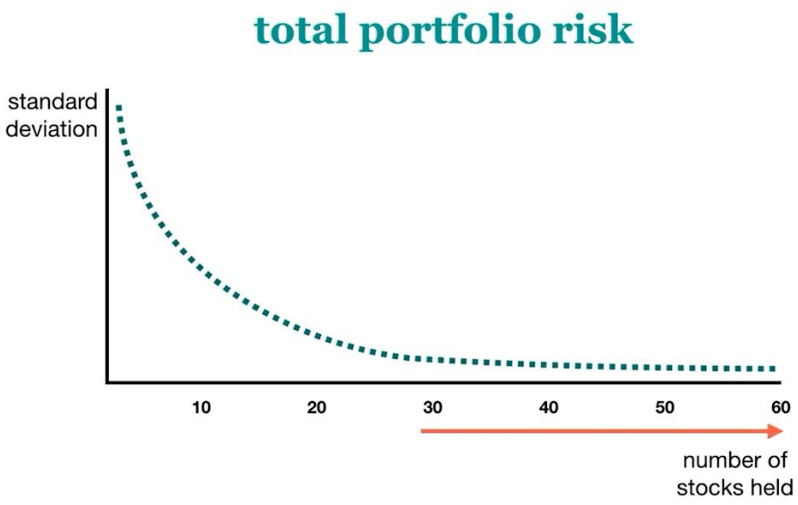

Figure 2

Video at 10:20

Because smart investors can so easily do away with unsystematic risk by diversifying their portfolios, investors do not even expect to be compensated for it. In the graph here (Figure 2), we can see that the risk reduction effect is most prominent as we add the first 30 assets to our portfolio. In theory, adding another asset that has low correlation with other assets in the portfolio can decrease the portfolio variance, but in reality after 30 assets there is not much of an additional effect. The only risk that affects the security's return in the market is systemic risk.

Video at 10:50

Systemic risk, also known as market risk or portfolio risk, is the risk that remains after diversification. It is also referred to as undiversifiable risk. Because it is caused by macroeconomic factors that affect the entire market, not just a group of securities. Examples include changes in interest rates, inflation, wars, and economic recessions. These events impact the market portfolio, which theoretically contains all the assets in the market, in the same way (positively or negatively) so the returns on the entire market portfolio move together, increasing total return variance and introducing risk. This risk cannot be diversified away, and so investors will expect to earn an additional return to make up for that risk.

Video at 11:37

Today, we learned about the trade-off between risk and return. Then, we learned how to use the variance or standard deviation of returns to measure the risk of an asset. Later, we looked at the difference between systematic and unsystematic risk. Remember that unsystematic risk can be diversified away by holding a portfolio of assets with low correlation, so investors don't expect to be compensated for it. Thanks for watching!