Video 26 - Capital Market Line

The following is a supplementary transcript for tutorial videos from

https://blogs.ubc.ca/financefundamentals/

Hi everyone! Today, we will be learning about how investors can achieve maximum returns for a given level of risk by selecting a portfolio along the efficient frontier. We will also learn how investors can further adjust their risk by including risk-free assets to achieve a point on the capital market line. By the end of this video, you will learn how we build the efficient frontier, the significance of the minimum variance portfolio, and how to interpret the capital allocation line and capital market line.

Video at 00:39

So, what is the efficient frontier? Let me explain it through an example. Suppose there are only two risky assets available in the market, oranges and bananas. Oranges have an expected return of 8% and a standard deviation of 15%, and bananas have an expected return of 5% and standard deviation of 10%. Let's create a table to record the expected return and variance of a portfolio with different weights in oranges and bananas.

Video at 01:07

As investors, we would like to know how much of our funds to invest in oranges, and how much of our funds to invest in bananas. Since most investors are risk averse, we would like to find a portfolio that will maximize our expected returns, while minimizing the risk we take on. The combination of oranges and bananas is known as our investment portfolio.

Video at 01:25

The formula for calculating expected return is

$$expected\ return\ on\ the\ portfolio\ = E(r_p) = [w_O \times E(r_O)]\ +\ [w_B \times E(r_B)]$$

where w_O is the weight of oranges, which is the proportion of the portfolio made up of oranges; $E(r_O)$ is the expected return of oranges; w_B is the weight or proportion of the portfolio made up of bananas; and $E(r_B)$ is the expected return of bananas. The calculated expected returns are as below (Figure 1).

Video at 01:45

Now, suppose that these two risky assets are negatively correlated with a correlation of ($\rho_{O,B}$) -0.85. Recall that the correlation represents how two assets move together, expressed between -1 and 1, so a correlation of -0.85 means that, when one asset increases in returns, the other asset tends to decrease in returns. Now, let's calculate the new standard deviations of the portfolio at different weights (Figure 1)

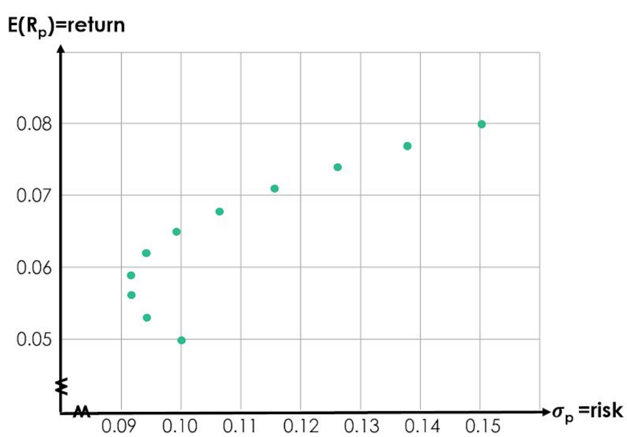
$$variance\ of\ the\ portfolio\ =\ \sigma_p^2 = (w_O \times \sigma_O^2)\ +\ (w_B \times \sigma_B^2)\ +\ (2 \times \rho_{O,B} \times w_O \times w_B \times \sigma_O \times \sigma_B)$$

$$standard\ deviation\ of\ the\ portfolio\ =\ \sigma_p = \sqrt{\sigma_p^2}$$

where σ_O^2 is the variance of oranges (standard deviation, σ_O , squared); σ_B^2 is the variance of bananas (standard deviation, σ_B , squared); and $\rho_{O,B}$ is the correlation (-0.85) between oranges and bananas.

Video at 2:13

Our next step is to plot the expected return and standard deviation of all these portfolios so that we can easily visualize the trade-off between risk (the standard deviation) and expected return for portfolios of different weights of oranges and bananas. The expected return is on the y-axis, and the standard deviation is on the x-axis.



w_O	w_B	$E(R_p)$	σ^2_p	σ_p
0.0	1.0	**5.0%**	0.01000	**10.0%**
0.1	0.9	**5.3%**	0.00896	**9.46%**
0.2	0.8	**5.6%**	0.00842	**9.18%**
0.3	0.7	**5.9%**	0.00840	**9.16%**
0.4	0.6	**6.2%**	0.00888	**9.42%**
0.5	0.5	**6.5%**	0.00988	**9.94%**
0.6	0.4	**6.8%**	0.01138	**10.7%**
0.7	0.3	**7.1%**	0.01340	**11.6%**
0.8	0.2	**7.4%**	0.01592	**12.6%**
0.9	0.1	**7.7&**	0.01896	**13.8%**
1.0	0.0	**8.0%**	0.02250	**15.0%**

Figure 1

Video at 02:32

In reality, where there are more than two risky assets, there will be countless portfolios with different combinations of weights in different assets. The scatter plot for a realistic market with practically infinitely many portfolios should look something like this.

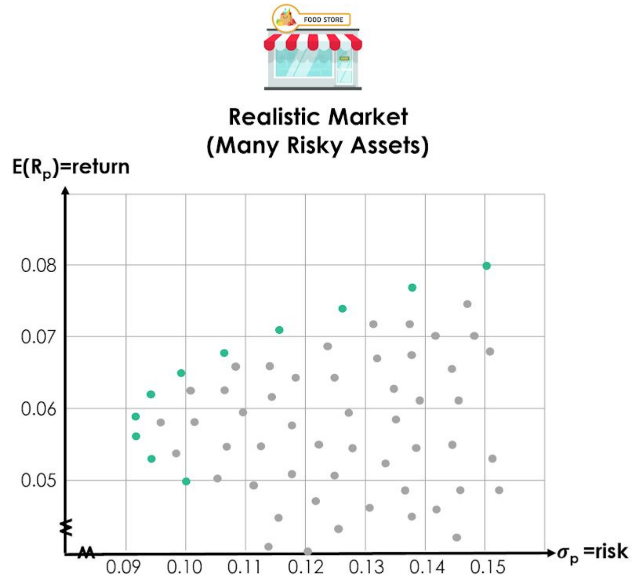


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Figure 2

Video at 02:46

We can connect the dots that are at the leftmost of the graph to create an arc-shaped line. Notice how, as we move to the right, our risk increases without increasing our expected return; thus, the portfolios along this line represent the lowest risk (minimum variance) that we can achieve at our desired expected return. This arc is called the "minimum variance frontier". The upper part of the arc is called the "efficient frontier" (Figure 3).

Video at 03:11

The lower part of the arc is not efficient. For each of these points, there is a portfolio on the efficient frontier with the same level of risk that yields a better expected return. Take this portfolio (red star), for example (Figure 4). It has a return of around 6% and a standard deviation of around 11.75%. Compare it (red star) to this portfolio that's on the efficient frontier (green star). This portfolio (green star) has the same standard deviation but a return of 7%. Thus, the portfolio earning only 6% (red star) is not efficient.

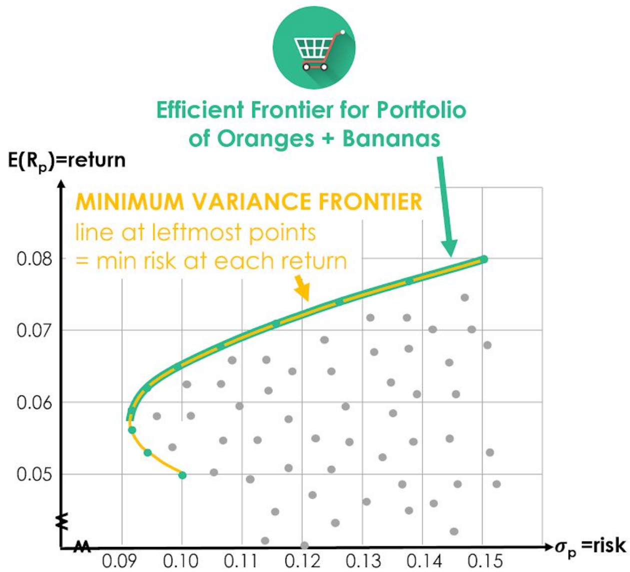

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Figure 3

Video at 03:41

As a risk averse investor, we will always prefer the portfolio that gives us the highest expected return possible for the lowest risk possible (measured as the standard deviation). In other words, if an investor has to endure a higher risk of 11.75% (red star), then she would expect to be rewarded with the highest return possible, which is 7% (green star). Or, if the investor wishes to earn a return of 7% (pink star), she would choose a portfolio that earned that return at the lowest possible risk, which is 11.75% (standard deviation).

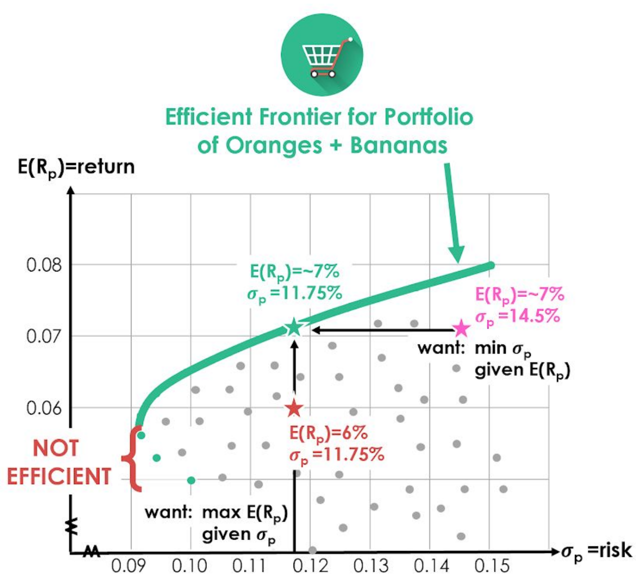

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Figure 4

Video at 04:10

Since we assumed, in this fictional universe of only two fruits, that we have included all combinations of all risky assets possible when creating our portfolios, this point "A", for example, is unachievable, because there are no risk assets combination of risky assets that can give a 6% return for an 8.75% risk. In other words, the best portfolio we can achieve, as a risk averse investor, is a portfolio on the efficient frontier.

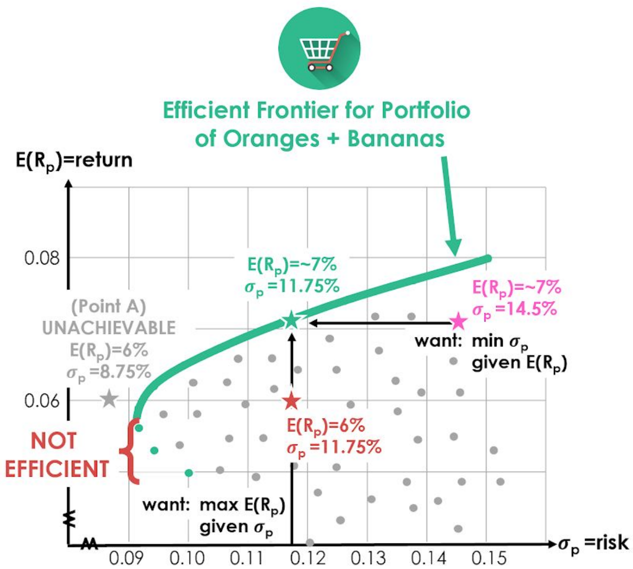

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Figure 5

Video at 04:36

There is a special portfolio on the efficient frontier called the "minimum variance portfolio". Because it is at the leftmost tip of the arc, it is the portfolio with the minimum amount of risk (or variance), hence the name, "minimum variance portfolio" (purple star). There are pros and cons to holding the minimum variance portfolio. The most obvious pro is that it has very minimal risk due to choosing risk assets that have low risk themselves and low correlation with each other, which is known as diversification. The con of the minimum variance portfolio is that it often has a lower return than we could achieve if we were willing to accept more risk. However, the minimum variance portfolio has a special significance in portfolio management: it is often used by portfolio managers in combination with other risk assets and portfolios. By adding the minimum variance portfolio to their funds, they can lower the risk of the entire fund.

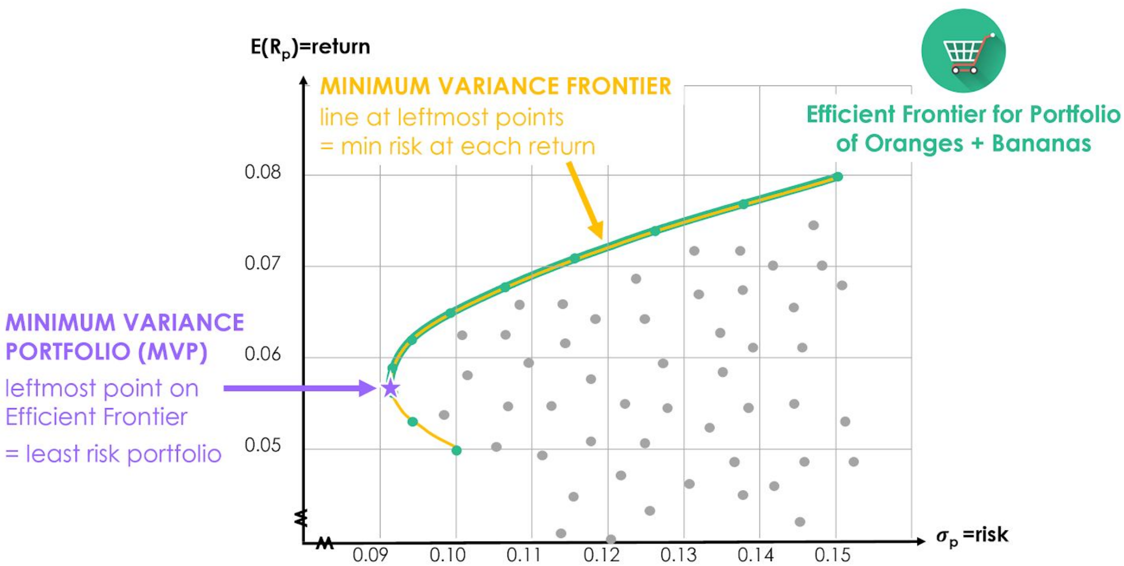

Figure 6

Video at 05:31

Remember earlier when I gave two examples of points that are unachievable under the assumption of the efficient frontier? The efficient frontier focuses on creating portfolios using risky assets, while the capital market line introduces the concept of risk-free assets. The capital market theory states that, when including risk-free assets in the portfolio, some points that are on the left side of the efficient frontier (in other words, that used to be unachievable) can now be achieved.

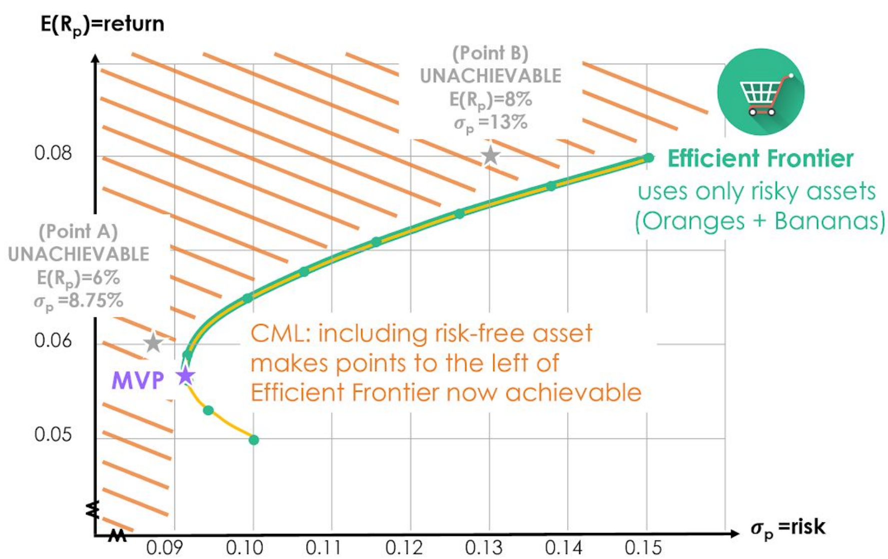

Figure 7

Video at 05:58

To illustrate the capital market line, we want to take our efficient frontier and zoom out a bit (Figure 8). Suppose the risk-free return is 2%. Assets that pay a risk-free return essentially compensate investors for the time value of money, which is why the risk-free rate is usually quite low. This is because, as a risk-free asset, such as short term government bonds, the returns are almost entirely guaranteed. For example, the Government of Canada has not yet ever failed to repay investors on its government bonds. Thus, with such a safe and reliable asset, investors do not need to be compensated for any additional risk, like default risk.

Video at 06:32

We want to start at 2% (the risk-free rate) and draw a line that is tangent to the efficient frontier. This is the "optimal capital allocation line". This point that is tangent to the efficient frontier is called the "optimal risky asset portfolio" (orange star). This is the point on the efficient frontier with the highest slope, which means that it maximizes the amount of return for every unit of risk (standard deviation) taken.

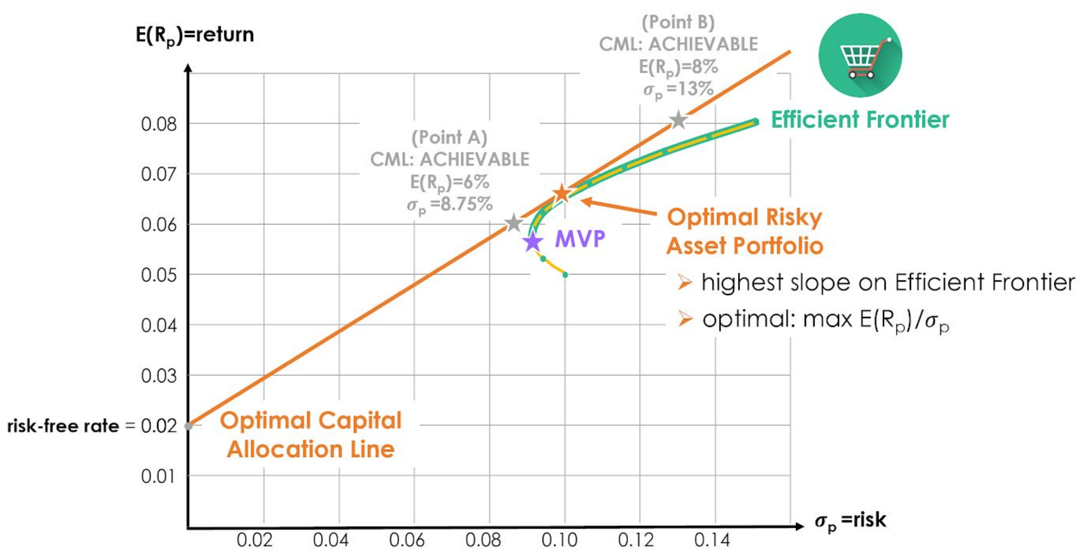

Figure 8

Video at 06:53

Assuming that all investors in the market are rational, risk-averse, and have the same expectations of risk and return, then the optimal capital allocation line becomes the "capital market line", and the optimal risky asset portfolio becomes the "market portfolio" (Figure 9). Any point on this capital market line can be achieved through investing in a mix of risk-free assets and the market portfolio. Because the variance of the risk-free asset is zero, when we combine it with the risky asset, our risk decreases proportionally.

Video at 07:22

Take point "A", for example (Figure 9). It can be achieved through investing approximately 90% in the market portfolio and 10% in the risk-free asset. This yields a standard deviation

$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{(w_{MP} \times \sigma_{MP}^2) + (w_{RF} \times \sigma_{RF}^2) + (2 \times \rho_{MP,RF} \times w_{MP} \times w_{RF} \times \sigma_{MP} \times \sigma_{RF})}$$

$$= \sqrt{(90\% \times 0.1^2) + (10\% \times 0) + (2 \times 0 \times 90\% \times 10\% \times 0.1 \times 0)} \approx 0.09 = 9\%$$

This is because the risk-free rate, by definition, has a variance of zero. Point “A” has an expected return of 6% ($E(r_p) = (90\% \times 0.065) + (10\% \times 0.02) \approx 6\%$), which is the same we could expect to earn at Point “C” (green star) on the efficient frontier, but with lower risk.

Video at 7:45

As for point "B", we will be investing 133% of our funds into the market portfolio. But how can we invest more than 100% of the money we have? We would have to borrow approximately 33% of our portfolio at the risk-free rate, and invest all 133% into the market portfolio. This is favorable for investors who are willing to take on more risk, because they are borrowing at a very low rate, like 2%, in order to achieve higher returns in the market portfolio, like 10%.

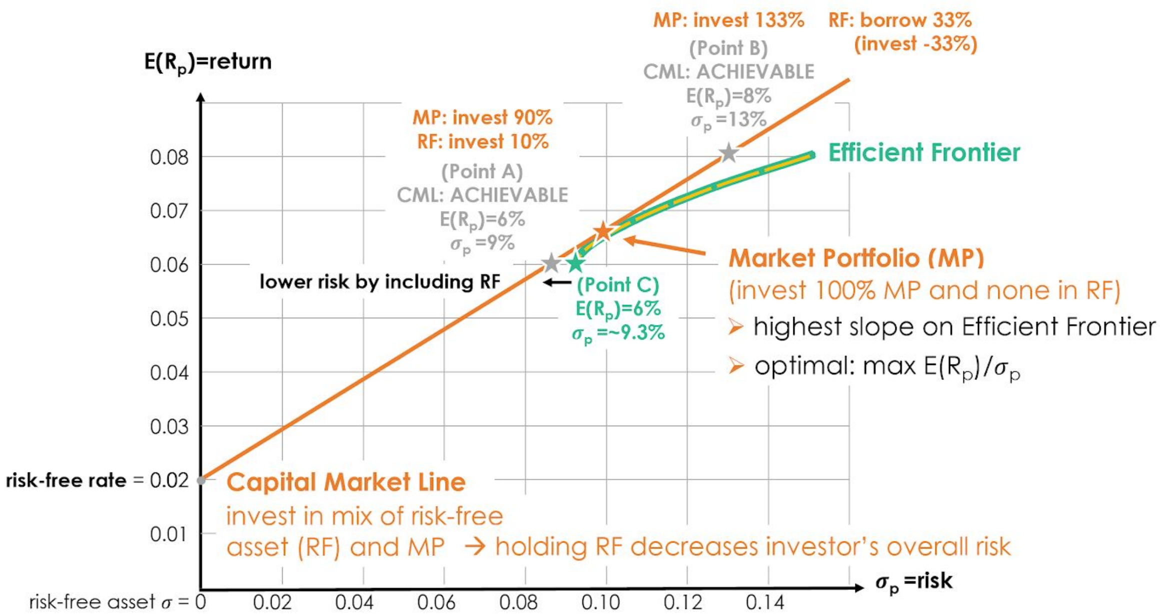

Figure 9

Video at 08:14

Where an investor is along the capital market line depends on how much they invest in the risk-free asset versus the market portfolio, which reflects how much risk they are willing to take on. To have a capital market line that continues straight for infinity, we must assume that the investor could invest and borrow unlimited funds at the risk-free rate. But in reality, the borrowing rate would be much higher because of the risk that the investor would default on the loan, meaning that the more an investor borrows, the less likely it will be that the investor can meet all the interest payments on the loans, and the more risky that the investor becomes. Therefore, the half of the capital market line beyond the market portfolio would have a flatter slope, because of your increased borrowing cost. The more you borrow at the risk-free rate, the less return you can earn for a given increase in risk.

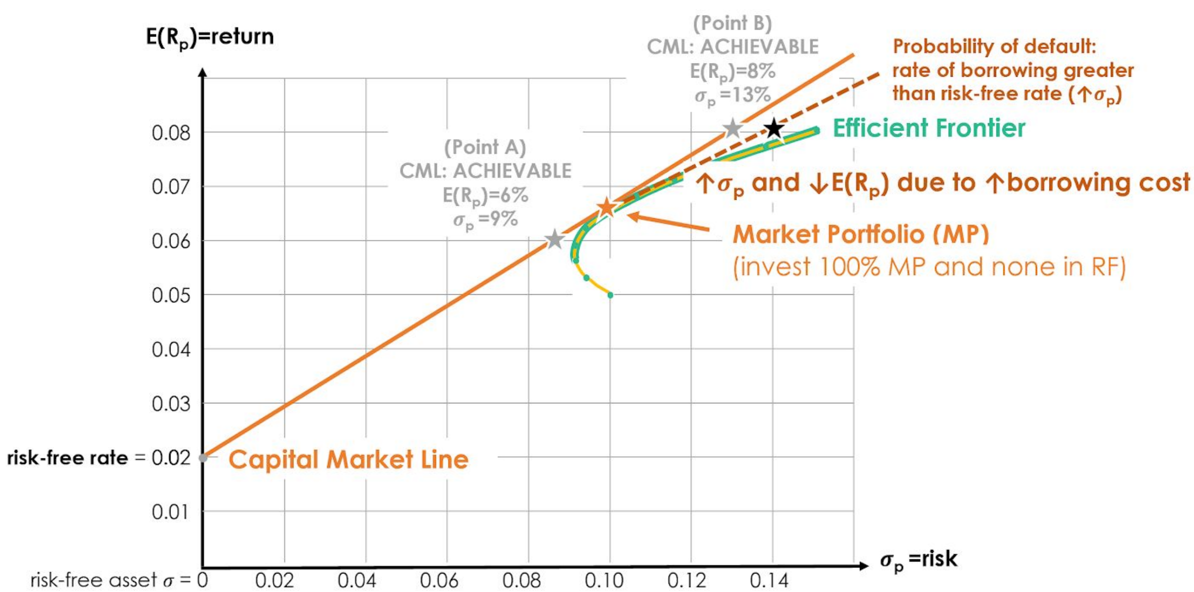

Figure 10

Video at 09:03

Today, we have learned that all investors in the market, being rational, will invest in a portfolio on the capital market line. That is, they will hold a combination of the risk-free asset and the minimum variance portfolio, which is the market portfolio. Thanks for watching!