Video 3 - Interest Rates: Effective Annual Rate vs Annual Percentage Rate

The following is a supplementary transcript for tutorial videos from

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Hey everybody! Today, we will be discussing the different ways we can express or quote an interest rate. Each method provides different information to an investor. We will start by comparing the difference between the stated annual percentage rate (APR) of an investment to its effective annual rate (EAR). Then, we will look at how APR and EAR can help us evaluate whether a loan is attractive, or even legal.

Video at 00:31

Interest rates are quoted annually. This way, it is easy for investors to compare between investments, as they are always dealing in annual terms. But often, interest is not compounded annually. That is why, up to this point, we've had to divide the annual percentage rate (APR) by the number of compounding periods in one year to determine the interest rate (effective periodic rate) we should use in our time value of money formulas

$$effective\ periodic\ rate\ =\ \frac{APR}{number\ of\ compounding\ periods\ in\ a\ year} = \frac{i}{m}$$

Video at 00:55

For example, I might earn 12% on an investment that is compounded monthly. I would take

$$effective\ periodic\ rate\ = r_{monthly}\ =\ \frac{12\%\ APR}{12\ months\ per\ year} = 1\%$$

to determine the interest I earn for each monthly compounding period, which is 1% per month. But sometimes, instead of APR, we want to know the effective annual rate of interest. That is, an investment's EAR.

Video at 01:17

The effective annual rate tells us what the effective rate of return would have been, if interests were compounded only <u>once</u> in the year. We already know that the more frequently interest is compounded, the more quickly an investment grows. So, if an investment is compounded more than once per year, the effective annual rate would be higher than its APR (stated interest rate). EAR tells us what rate, compounded once for the year, would increase the size of our investment by the same amount as if we had compounded our APR at the given compounding frequency.

Video at 01:48

For example, let's say we earn an 8% return on our $1,000 investment, compounded quarterly. If our investment is compounded quarterly, then our effective quarterly rate

$$effective\ periodic\ rate = r_{quarterly} = \frac{8\%\ APR}{4\ periods\ per\ year} = 2\%$$

At the end of the year, our investment would be worth

$$FV_t = PV \times (1 + r)^t$$

$$FV_1 = \$1,000 \times (1 + 0.02)^4 = \$1,082.43$$

Video at 02:13

What effective annual rate would have given us this same return? We want our future value in a year (FV_1) to be $1,082.43. Therefore, we can set

$$FV_1 = \$1,082.43 = \$1,000 \times (1 + r_{annual})^1$$

and isolate for r_{annual}. This tells us what interest rate will cause our $1,000 today, when compounded only <u>once</u> for the year, to have the same future value as if we compounded it quarterly. Dividing by the initial value of $1,000 and subtracting 1 allows us to isolate for r_{annual}, which is

$$r_{annual} = \left(\frac{FV_1}{PV}\right)^{1/1} - 1 = \left(\frac{\$1,082.43}{\$1,000}\right) - 1 = 8.243\%$$

Video at 02:47

This makes sense since we earned a return of $82.43 on $1,000 (interest = $\frac{\$82.43}{\$1,000}$ = 8.243%).

Notice that the APR simply takes our 2% effective quarterly rate and multiplies it by 4, which is the number of compounding periods in a year.

$$APR = effective\ periodic\ rate \times number\ of\ compounding\ periods\ in\ a\ year$$

$$APR = r_{quarterly} \times m = 2\% \times 4 = 8\%$$

This 8% APR is smaller than the EAR of 8.243%, because the APR is not capturing the effect of compounding: the more frequently our investment is compounded, the more we put our money to work, and the more interest on interest we make.When expressed in annual terms, the EAR captures this compounding effect, while the APR does not.

Video at 03:22

You may wonder why we use APR at all, if it doesn't show us the effective return on our investment. APR is easier for investors to understand and compare between investments, since it doesn't change when we change the frequency of compounding periods, but EAR tells us our actual effective return.

Video at 03:39

A general formula for this situation can be derived by isolating for r_{annual}:

$$PV \times [1 + \tfrac{i}{m}]^m = PV \times [1 + r_{annual}]^1$$

On the left, we have $\frac{i}{m}$, where "i" is the APR, and "m" is the number of compounding periods in a year. The left side of the equal sign expresses the value of our investment after one year's worth of compounding. The right hand side represents a situation where we only compound our investment annually (hence, to the power of 1) to find the EAR (r_{annual}).

Video at 04:13

In order for these two sides to be equal, our effective interest rate (EAR = r_{annual}) must be larger than the APR, since investments which are compounded more often grow faster than investments compounded less frequently. at the same interest rate. Isolating for r_{annual}, we get:

$$\text{EAR} = r_{annual} = [1 + \tfrac{i}{m}]^m - 1$$

Notice that the initial or present value (PV) of our investment isn't relevant here, since it's the same for APR and EAR, and we can eliminate it from our equation.

Video at 04:40

This formula reflects the same logic as the steps we just used to calculate EAR, but it saves us some time. Applying it to the example we just discussed, we get

$$\text{EAR} = r_{annual} = [1 + \tfrac{i}{m}]^m - 1 = [1 + \tfrac{0.08}{4}]^4 - 1 = 8.243\%,$$

the same answer we just calculated.

Video at 04:59

It is important to note that the effective annual rate of an investment that is compounded annually, would be the same as the stated rate (APR), since it's already expressed in annual terms. There are no compounding effects that would change the effective return. In such a situation, m=1, so

EAR = $r_{annual} = \left[1 + \frac{i}{m}\right]^m - 1 = \left[1 + \frac{i}{1}\right]^1 - 1 = 1 + i - 1 = i$ = APR

Video at 05:23

When might we use the EAR over the APR? Each rate can be useful to help us evaluate whether an interest rate is reasonable, or even legal. For example, in Canada, lenders cannot charge more than 60% APR. One exception is payday loans, which are loans less than $1,500 with terms between 2 weeks - 2 months. These loans carry period rates of up to 17%. For a 2-week loan, this would make the maximum

$APR = r_{2week} \times m = 17\% \times 26$ = 443%

where “m” = 26 = the number of bi-weekly periods in a year.

Video at 05:57

An effective annual rate is an even more useful indicator to an investor of how much a loan truly costs, as it factors in the increased effective costs as a result of compounding interest. Suppose your friend Linda needs money fast, so she gets an $800 two-week loan from a sketchy looking man named Sylvester Sta-loan. She agrees to pay him $50 as interest. Try and pause the video to figure out, first the APR, and then the EAR of this investment. See if you can decide whether or not this loan would be legal in Canada.

Video at 06:29

Let's go over this together. First, what is the APR? Linda's interest for the period is

interest = $\frac{\$50}{\$800}$ = 6.25% (over two weeks).

Since there's m = $\frac{365 \textit{ days in a year}}{14 \textit{ days per two-week period}}$ = 26 (two-week) periods in a year, the APR on this loan is

$APR = r_{2week} \times m = 6.25\% \times 26$ = 162.5%

Video at 06:51

What about the EAR? Applying the formula we used earlier, we will take

EAR = $r_{annual} = \left[1 + \frac{162.5\%}{26}\right]^{26} - 1$ = 383.7%

to figure out the effective annualized rate. This is an effective return of 383.7%. ¡Ay, caramba!

Video at 07:08

Notice how much higher the effective rate (EAR) is than the APR. This is due to both the high compounding frequency and the large APR of this investment. This loan falls within the legal limit of 443% APR, although it still doesn't sound like a great deal. I would ask Linda if she can avoid taking out a loan to cover her until payday.

Video at 07:28

If you have made it this far, you have learned that it is sometimes useful to quote interest rates in effective terms, as an EAR (effective annual rates), rather than as an APR (annual percentage rates). We learned how to convert an APR into an EAR, and you can use the same formula to convert an EAR into an APR, although you'll see this situation less often. Make sure to tune in to our next video, where we discuss how to modify this formula slightly to calculate the effective return over a period other than one year. See you next time!