Video 4 - Interest Rates: Effective Period Rates

The following is a supplementary transcript for tutorial videos from

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Welcome back, everyone! In our last video, we went over how to calculate effective annual rates. But, what if we want to calculate the effective rate for a period other than 1 year? First, we will look at how we can calculate the effective rate for these non-annual periods. Next, I will explain how we can convert effective annual rates (EAR) into periodic rates (EPR). And lastly, I will explain how this formula can apply in all situations, including those who are solving for the effective annual rate.

Video at 00:32

Let's start with a simple example. Suppose your boyfriend's cousin's uncle's godson's ex-wife lends you $500 at 9% interest, compounded quarterly. What is the effective interest rate over the life of the loan, if you pay the loan back after 2 months. Obviously, an annual rate wouldn't be very useful. We want to know *effectively* how much interest was paid for those 2 months, expressed as a percentage of the total investment. To calculate this, we have to figure out what 2-month effective rate would give us the same return as 9% compounded quarterly over those 2 months.

Video at 01:05

We can start this problem in a similar way to how we would calculate the effective annual rate. If we were calculating EAR, we would think of 1 year as 1 compounding period, and calculate the interest rate that would earn us the same return compounded only once per year, as we earn on our interest compounded each quarter. Except here, we want to find the effective return for a 2-month period, instead. If interest were compounded only once in a 2-month period, then we would have 6 periods in a year. We want to know what rate compounded 6 times in a year is equivalent to the return we earn in 1 year, at 9% interest, compounded quarterly. We can display this as:

$FV_1 = \$500 \times (1 + r_{quarterly})^4 = \$500 \times (1 + \frac{0.09}{4})^4$ should be the same as

$FV_1 = \$500 \times (1 + r_{2-month})^6$

Note that $\frac{0.09}{4}$ is our APR (9%) divided by the number of periods in a year (4 quarters) to get the rate that we earn for one quarter ($r_{quarterly}$), which would be compounded 4 times in a year.

Video at 01:59

Here, we are expressing both of these periodic returns on an annual basis. We do this so it is easier to compare the two situations. We know that there are 4 quarters in a year and six 2-month periods in a year. It is sort of like finding the lowest common denominator of two fractions, or putting both rates on the same playing field, annual terms.

Video at 02:18

Let's set these returns equal and isolate for the 2-month rate. Isolating for $r_{2-month}$, the rate of the 2-month period, gives us:

$$r_{2-month} = \sqrt[6]{(1 + \frac{0.09}{4})^4} - 1 = (1 + \frac{0.09}{4})^{4/6} - 1 = 1.49\%$$

Remember that a sixth root is equivalent to a 1/6 fractional exponent. So, our effective 2-month rate is 1.49%. We can express this logic as a general formula:

$$(1 + \frac{APR}{m})^m = (1 + r_{periodic})^k$$

where “k” is the number of effective periods in 1 year.

Video at 02:54

The right hand side of this formula tells us how much we earn on our investment at our given interest rate and compounding schedule. In our previous example, the APR=9% (sometimes “i” is used to represent the APR), and m=4, since we compounded our investment every quarter. Essentially, the left hand side of the equation is our return for one quarter, $\frac{APR}{m}$, compounded 4 times to get an annual return. The left hand side in this formula sets the return equal by adjusting the period rate. The exponent “k” is the number of effective periods that could fit in 1 year. It helps to express our period returns in annual terms by compounding the number of periods we have in a year. In our previous example, the period was 2 months, so we could compound our effective period rate k=6 times during 1 year.

Video at 03:37

Isolating for $r_{periodic}$, we get:

$$r_{periodic} = (1 + \frac{i}{m})^{m/k} - 1$$

We can plug in the figures from our previous example:

$$r_{2-month} = (1 + \frac{0.09}{4})^{4/6} - 1 = 1.49\% \text{ again}$$

Our fractional exponent essentially tells us how many compounding periods ("m") there are in one of our effective periods ("k"). Here, we have $\frac{4}{6}$. Thus, there is ⅔ of one compounding period (1 quarter = 3 months) in our 2-month effective period.

Video at 04:10

From the timeline, you can see that, because there are 3 months in a quarter, then ⅔ of a quarter is equivalent to a 2-month period. As the number of compounding periods ("m") in 1 year increases, the effective period rate increases. This is because more frequent compounding leads to a higher effective return. In other words, you are earning interest on your interest at a faster rate. You can also see that, as "k" (the number of effective periods in 1 year) increases, the effective period rate decreases. This is because the more effective periods you can fit into 1 year, the shorter these periods must be. The shorter the period, the lower the effective return, because you will be taking the same annualized rate and chopping it into smaller effective rates.

Video at 04:51

Let's try one more example and use our results to explain how our formula might work in cases where the effective period is the same length as the compounding period, or our effective period is 1 year. Please pause the video and try this problem on your own.

You decide to pay your $10,000 September 1st credit card statement on March 1st. Luckily, your credit card company only charges 15% interest, compounded every 4 months. What is the effective interest rate for the period in which the balance is outstanding?

Video at 05:06

Now, let's try it together. Our stated interest rate is 15%, compounded every 4 months, so 3 times in 1 year: m=3. There are 6 months between September and March, so k=2, since there are two 6-month periods in a year. Essentially, we are looking for the effective semi-annual rate. Thus, our effective semi-annual rate is

$$r_{semi-annual} = (1 + \frac{0.15}{3})^{3/2} - 1 = 7.6\%$$

Video at 05:36

Notice that if "k" (the period for which we are calculating the period rate) is the same length as "m" (our actual compounding periods), then the effective period rate will just be $\frac{APR}{m}$, the same effective rate we have been using in our time value of money formulas. Since this is aligned with the way we actually compound our investment, there will be no other compounding effects to consider. Our exponent would simply be 1 (because m=k), causing

$$r_{periodic} = (1 + \frac{APR}{m})^1 - 1 = 1 + \frac{APR}{m} - 1 = \frac{APR}{m}$$

For example, if you paid back your loan after 4 months instead of 6, then m=k=3, and thus, the effective period rate is simply

$$r_{periodic} = (1 + \frac{APR}{m})^{m/k} - 1 = (1 + \frac{0.15}{3})^{3/3} - 1 = \frac{0.15}{3} = 5\% = r_{4-month\ rate}$$

Video at 06:19

Notice that this formula works in all cases, including cases where we are trying to calculate the effective annual rate. In this case, k=1, and we would be left with the formula we discussed for calculating EAR:

$$r_{periodic} = (1 + \frac{APR}{m})^{m/k} - 1 = (1 + \frac{APR}{m})^{m/1} - 1 = (1 + \frac{APR}{m})^m - 1 = r_{annual} = EAR$$

So you only need to remember one formula.

Video at 06:36

Today, we learned how to solve for the effective rate of a period other than 1 year. We can apply this to more complex investing and borrowing problems, where the length of our investment cannot be expressed in whole compounding periods. In these problems, we can use a fraction to represent the total number of compounding periods that occur, where "m" is the number of compounding periods that can fit in 1 year, and "k" is the number of effective periods that can fit in 1 year. Thanks for watching!