Video 06 - Annuities Due and Moving Annuities

The following is a supplementary transcript for tutorial videos from

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In this video we will discuss how to use PV and FV of an annuity formula to calculate moving annuities and annuities due. Before diving into these concepts, let's first recall what an annuity is. An annuity is a series of fixed payments that occur at fixed intervals. Examples include regular deposits into a savings account or monthly insurance payments. Annuities all display certain characteristics. They have consistent cash flows that have a fixed value, cash flows that occur at regular intervals, cash flows that occur for a fixed amount of time (i.e. the cash flow ends), and for a standard annuity, the cash flows start one period from now.

Video at 00:45

Now, let's recall what the formula is for calculating the present value of a standard annuity.

$$PV\ annuity\ =\ \$A \times \left[\frac{1-(1+r)^{-n}}{r}\right]$$

To use the present value or future value annuity formula, you need to know: what amount is being paid each time, as represented by “A”; the effective rate of interest period, represented by “r”, where the period is how much time there is between payments; and how many payments are in the annuity, represented by “n”. The key here to remember is to always match the period of the interest rate “r” with the period of the fixed payments, “A”. For example, if you were making *monthly* insurance payments, then be sure to use the periodic *monthly* interest rate. Always assume that the payments occur at the end of each time period, unless otherwise specified.

Video at 01:32

Now, you may be asking yourself, how do you actually use these formulas? The first thing we can use the formulas for is to calculate moving annuities. A moving annuity is calculating the present value or future value of an annuity of different time periods.

Video at 01:50

Let's imagine that you won the lottery. Congrats! Now, this lottery has promised to give you $5,000; a small pot but that's okay - you finally won something! In the terms and conditions of

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winning this lottery, they have said that you will receive the cash prize starting four years from now, or at the beginning of the fifth year, in $1,000 increments over five years where the effective rate of interest for the year is 10%. Using the PV of the annuity formula, we can calculate the present value of the annuity and then discount the value back to see how much the annuity is worth today (i.e. the present value over the cash payments that occur over the five years at PV_0, which is today). It is important to remember that the PV of the annuity formula calculates the PV of the cash flows one time period *before* the first cash flow begins (at t=5). Pause the video here and try to calculate PV_4 for yourself.

Video at 02:47

Mapping the values identified in the problems with the variables in the formula, A = $1,000, as $1,000 is the amount paid at the end of each year; r = 0.10, the effective annual interest rate represented as a decimal, matching the $1,000 annual cash flows; and n = 5, as the payments occur over five years (there are 5 payments in total). Plugging this into the PV formula,

$$PV_4 = \$1,000 \times \left[\frac{1-(1+0.10)^{-5}}{0.10}\right] = \$3,790.79$$

This means that, if you were to calculate what the value four years from now of getting $1,000 payments over the five years is, it is worth $3790.79. However, this is not super helpful, as this is the value four years from today (PV_4). We need to discount this value by four years to calculate the value in today's dollars (PV_0). Pause the video here and calculate PV_0.

Video at 03:42

After discounting $3790.79 by four years, the value of the annuity in today's dollars is

$$PV_0 = \frac{PV_t}{(1+r)^t} = \frac{\$3,790.79}{(1+0.10)^4} = \$2,589.16$$

Notice how, although you technically receive 5 x $1000, the actual value of these cash flows is worth only $2589.16, due to the time value of money.

Video at 04:07

Another way that we could have tackled the same problem is to calculate what the future value of the payments were at the nine year mark (FV_9 after we receive the last $1,000 payment), using the future value of an annuity formula.

$$FV\ annuity = \$A \times \left[\frac{(1+r)^n - 1}{r}\right]$$

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Here, it's important to remember that the FV of an annuity formula calculates the FV on the date of the *last* payment. Pause here and calculate the FV at time 9 for yourself.

Video at 04:26

Using the same values to be identified for “A” ($1,000), “r” (0.10), and “n” (5) as in the PV calculation, the future value at time 9

$$FV_9 = \$1,000 \times \left[\frac{(1+0.10)^5 - 1}{0.10}\right] = \$6,105.10$$

This means that, after receiving the five $1000 payments during the five years, taking into account the time value of money, the value of the payments is $6,105.10 in terms of year nine dollars. Following this, we would need to discount the value at FV_9 back to time zero (PV_0). Pause here and do this now.

Video at 05:01

Just like when we discounted the value of PV_4 back to PV_0, when discounting FV_9 ($6,105.10) back 9 years to PV_0, the value of the annuity in today's dollars is

$$PV_0 = \frac{PV_t}{(1+r)^t} = \frac{\$6,105.10}{(1+0.10)^9} = \$2,589.16$$

In this example, we have learned that we can use the PV and FV formulas to calculate the value of the cash flow at different time periods. Then, through using the concept of compounding and or discounting, discounting in this example, we can essentially "move" the annuities, which are valuing the exact same stream of cash flows to calculate its value at different times. Because these calculations involve moving to different times, it's very important to use subscripts (i.e. PV_4 and FV_9) to denote what time period the value you are calculating is in.

Video at 05:50

Now that we have practiced how to use the annuity formulas and know how to "move" the annuities, the last concept we will cover is annuities due. Annuities due are annuities where you would receive your first payment at time=0 (today). Recalling the previous example, imagine what you are now receiving the five $1,000 payments with the first one starting today. How can we calculate the value of these cash flows today (PV_0)? Pause the video here and think about how we can solve this problem. The key thing that we need to remember is that the PV of an annuity formula calculates the value of the cash flows one period *before* the first cash flow.

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There are two ways that we can manipulate the formula to calculate the value of the cash flows at time 0.

Video at 06:32

The first method is to calculate the value of the four subsequent cash flows from t=1 to t=4 separate from the first $1,000 given in t=0. (Note: for this four-payment annuity starting in t=1, the annuity formula will give us the PV one period before t=1, which is PV_0).Through using the PV of an annuity formula, the value of a four-payment annuity is

$$PV_0 = \$1,000 \times \left[\frac{1-(1+0.10)^{-4}}{0.10}\right] = \$3,169.87 ,$$

so by adding $1,000 (the first payment, which occurs today) we get

$$PV_0 = \$1,000\, first\ cash\, flow\ today\ +\ \$3,169.75\, four\ subsequent\ cash\, flows\ =\ \$4,169.87$$

Video at 07:01

The second method we can do is to use the PV of a 5 payment annuity, giving PV_{-1} (because the first cash flow arrives today, t=0), and then compound up one period to get PV_0 . Since we calculated the PV of a 5 payment annuity in the first example ($3,790.79), we just compound the value up one year (from PV_{-1} to PV_0) and get

$$PV_{-1} = \$1,000 \times \left[\frac{1-(1+0.10)^{-5}}{0.10}\right] = \$3,790.79$$

$$PV_0 = PV_{-1} \times (1+r)^1 = \$3,790.79 \times (1+0.10)^1 = \$4,169.87 ,$$

which gives the same answer as the first method.

Video at 07:25

Congratulations! After watching this video you should now be able to use the PV and FV of an annuity formula to calculate moving annuities and annuities due. Until next time!

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