Video 8 - Amortization of Loans: Fixed Payment

The following is a supplementary transcript for tutorial videos from

https://blogs.ubc.ca/financefundamentals/

Hi everyone, welcome back. Today, we will continue learning about amortized loans. In this video, we will be talking about the fixed payment amortized loan. In this lesson, we will learn: 1) how to calculate the fixed payment required for the loan; 2) how to determine which portion of the payment is towards the interest and which portion is towards the principal; and 3) demonstrate this concept in a practice problem.

Video at 00:30

In the previous video, we discussed the three types of loans: the pure discount loan, the interest only loan, and the amortized loan. The amortized loan was broken down into two types, fixed principal and fixed payments. We talked about the fixed principal amortized loan, where the payment of the principal amount is set in the loan agreement and the interest payment decreases over time as the principal balance amortizes.

Video at 00:51

In this video, we will discuss the fixed payment amortized loan. With the fixed payment amortized loan, the periodic payment amount is inclusive of the interest and principal payments. Since this type of loan has equal payments per period, they are annuities. Mortgages are a common example of an amortized loan, but how do we know how much of the fixed payment goes towards paying off the principal and how much of the fixed payment is paid towards the interest each period?

Video at 01:15

Let's take a loan of $500,000 with an annual interest rate of 14%, compounded monthly, over 2 years. This time, we want to determine a fixed payment to be paid monthly, inclusive of interest and principal payments. To calculate this, we will use the annuity formula to determine the fixed payment. The formula is as follows:

$$PV\ annuity = A\left[\frac{1-(1+r)^{-n}}{r}\right]$$

Video at 01:36

We plug in our values, where the effective monthly rate is 14% / 12 months per year = 1.17%, and n = 2 years * 12 months per year = 24 months in total (ie 24 monthly loan payments).

$$\$500,000 = A\left[\frac{1-(1.0117)^{-24}}{0.0117}\right]$$

$$A = \$500,000 \div \left[\frac{1-(1.0117)^{-24}}{0.0117}\right] = \$24,006.44$$

We calculate a fixed payment of $24,006.44 per month.

Tip: for the most accurate calculation, use 0.14/12 instead of 0.0117 in the annuity formula. This will make a difference in your ending calculation, since we are working with large numbers like $500,000.)

Video at 01:51

Note that, in reality, loans and mortgages will rarely be paid off so quickly; however, for the purposes of this example, we will work with the 24-month mortgage, so that we can see what happens to the loan over the entire life of the loan. Although our 24-month mortgage isn't quite realistic, the mechanics will still work the same as mortgages in real life.

Video at 02:10

We can display the monthly payments of this mortgage in an amortization schedule to break down the mortgage payments. This is the complete amortization schedule for the loan over the entire 24 months (Figure 1).

Video at 02:20

We can see the fixed payments broken down into the portions of interest payment and principal payment. Notice that, as time goes on, the portion of the interest in the monthly payment decreases. This is because the interest payment is the rate (1.17%) multiplied by the beginning balance of the loan (column B). As the periods go on, the loan gets paid off, and thus, the interest amount decreases each period.

	A	B	C	D	E	F	G
1		loan =	$500,000.00				
2		interest rate =	1.17%				
3		fixed monthly payment =	$24,006.44				
4							
5				=(B*1.17%)	=(C-D)	=(B-E)	
6	Month	Beginning Loan Balance	Fixed Monthly Payment	Interest Payment	Principal Payment	Ending Loan Balance	% monthly payment towards principal
7	1	$500,000.00	$24,006.44	$5,833.33	$18,173.11	$481,826.89	75.70%
8	2	$481,826.89	$24,006.44	$5,621.31	$18,385.13	$463,441.76	76.58%
9	3	$463,441.76	$24,006.44	$5,406.82	$18,599.62	$444,842.14	77.48%
10	4	$444,842.14	$24,006.44	$5,189.82	$18,816.62	$426,025.53	78.38%
11	5	$426,025.53	$24,006.44	$4,970.30	$19,036.14	$406,989.38	79.30%
12	6	$406,989.38	$24,006.44	$4,748.21	$19,258.23	$387,731.15	80.22%
13	7	$387,731.15	$24,006.44	$4,523.53	$19,482.91	$368,248.24	81.16%
14	8	$368,248.24	$24,006.44	$4,296.23	$19,710.21	$348,538.03	82.10%
15	9	$348,538.03	$24,006.44	$4,066.28	$19,940.16	$328,597.86	83.06%
16	10	$328,597.86	$24,006.44	$3,833.64	$20,172.80	$308,425.06	84.03%
17	11	$308,425.06	$24,006.44	$3,598.29	$20,408.15	$288,016.91	85.01%
18	12	$288,016.91	$24,006.44	$3,360.20	$20,646.24	$267,370.67	86.00%
19	13	$267,370.67	$24,006.44	$3,119.32	$20,887.12	$246,483.55	87.01%
20	14	$246,483.55	$24,006.44	$2,875.64	$21,130.80	$225,352.75	88.02%
21	15	$225,352.75	$24,006.44	$2,629.12	$21,377.33	$203,975.42	89.05%
22	16	$203,975.42	$24,006.44	$2,379.71	$21,626.73	$182,348.70	90.09%
23	17	$182,348.70	$24,006.44	$2,127.40	$21,879.04	$160,469.66	91.14%
24	18	$160,469.66	$24,006.44	$1,872.15	$22,134.30	$138,335.36	92.20%
25	19	$138,335.36	$24,006.44	$1,613.91	$22,392.53	$115,942.83	93.28%
26	20	$115,942.83	$24,006.44	$1,352.67	$22,653.78	$93,289.06	94.37%
27	21	$93,289.06	$24,006.44	$1,088.37	$22,918.07	$70,370.99	95.47%
28	22	$70,370.99	$24,006.44	$820.99	$23,185.45	$47,185.54	96.58%
29	23	$47,185.54	$24,006.44	$550.50	$23,455.94	$23,729.60	97.71%
30	24	$23,729.60	$24,006.44	$276.85	$23,729.60	$0.00	98.85%

Figure 1

Video at 02:40

This is quite overwhelming at first glance, so let's walk through this together. You can see that the process will repeat itself every month until we reach the end of the mortgage, when there will be $0 left with what we owe (cell F30); so, instead of typing out the formula manually for each cell for the entire term of our mortgage, we can use the power of Excel to write formulas to calculate everything for us.

Video at 03:00

Let's open a fresh Excel sheet. On the blank Excel sheet, input the loan interest rate and fixed payment in cells B1:C3 in cells B1:C3, like you see in our Excel sheet.

Note: we calculated these in Excel as formulas, so Excel keeps ALL the decimals, to be as accurate as possible.

	A	B	C
1		loan =	500000
2		interest rate =	=0.14/12
3		fixed monthly payment =	=$C$1/((1-((1+$C$2)^(-$A$30)))/$C$2)

Figure 2

Video at 03:10

In the first column, we will set the periods (1 to 24 from cells A7:A30). In our case, it is two years of monthly payments for 24 periods total. In column B, we will set the beginning balance of the loan to $500,000 in cell B7 (you can reference cell C1).

B7 =C1

	A	B	C	D	E	F	G
1		loan =	$500,000.00				
2		interest rate =	1.17%				
3		fixed monthly payment =	$24,006.44				
4							
5							
6	Month	Beginning Loan Balance					
7	1	$500,000.00					
8	2						
9	3						
10	4						
11	5						
12	6						
13	7						
14	8						
15	9						
16	10						
17	11						
18	12						
19	13						
20	14						
21	15						
22	16						
23	17						
24	18						
25	19						
26	20						
27	21						
28	22						
29	23						
30	24						

Figure 3

Video at 03:22

In column C, we will include the monthly fixed payment. Since, each month, we paid the same amount, this column will remain the same value all the way through our schedule. In cell C7, type the formula: =$C$3 then click and drag the formula down to cell C30 (so cell C30 still contains =$C$3). The dollar signs on the $C$3 cell reference is called "absolute referencing".

This makes sure that, when we drag the formula down, the cells will always return the number in cell C3 (our fixed payment), no matter where we drag the cursor.

C7 | f_x | =$C$3

	A	B	C	D	E	F	G
1		loan =	$500,000.00				
2		interest rate =	1.17%				
3		fixed monthly payment =	$24,006.44				
4							
5							
6	**Month**	**Beginning Loan Balance**	**Fixed Monthly Payment**				
7	1	$500,000.00	$24,006.44				
8	2		$24,006.44				
9	3		$24,006.44				
10	4		$24,006.44				
11	5		$24,006.44				
12	6		$24,006.44				
13	7		$24,006.44				
14	8		$24,006.44				
15	9		$24,006.44				
16	10		$24,006.44				
17	11		$24,006.44				
18	12		$24,006.44				
19	13		$24,006.44				
20	14		$24,006.44				
21	15		$24,006.44				
22	16		$24,006.44				
23	17		$24,006.44				
24	18		$24,006.44				
25	19		$24,006.44				
26	20		$24,006.44				
27	21		$24,006.44				
28	22		$24,006.44				
29	23		$24,006.44				
30	24		$24,006.44				

Figure 4

Video at 03:50

In column D, we will use Excel to calculate the portion of the payment that is interest. We now know that the interest payment is the interest rate multiplied by the beginning balance of the loan. So, in cell D7, we will enter the formula: =$C$2*B7. Similar to in column C, we want to absolute reference the interest rate cell ($C$2), so that it always returns 1.17%.

Note: we do not want to absolute reference cell B7, so that the formula in column D will always refer to the cell in column B of the same row, which is the beginning balance of *that* period.

Now, we can click and drag this formula down to D30 to carry through the formula (so cell D30 contains =$C$2*D30). Now, the cells in column D will automatically multiply the values from column B (the beginning loan balance) by 1.17% (cell $C$2) to calculate our interest payment each period.

D7 =B7*$C$2

	A	B	C	D	E	F	G
1		loan =	$500,000.00				
2		interest rate =	1.17%				
3		fixed monthly payment =	$24,006.44				
4							
5							
6	Month	Beginning Loan Balance	Fixed Monthly Payment	Interest Payment			
7	1	$500,000.00	$24,006.44	$5,833.33			

Figure 5

Video at 04:29

In column E, we calculate the portion of the loan which is payment towards the principal value. We just learned that the principal payment = the monthly payment - the interest payment. So in cell E7, we will enter the formula: =C7-D7. Now, we will drag the formula down to cell E30 (so cell E30 contains =C30-D30). This carries the formula all the way to the end of our period.

E7 =C7-D7

	A	B	C	D	E	F	G
1		loan =	$500,000.00				
2		interest rate =	1.17%				
3		fixed monthly payment =	$24,006.44				
4							
5							
6	Month	Beginning Loan Balance	Fixed Monthly Payment	Interest Payment	Principal Payment		
7	1	$500,000.00	$24,006.44	$5,833.33	$18,173.11		

Figure 6

Video at 04:48

In column F, we will calculate the ending loan balance. We now know that the ending loan balance = the beginning loan balance - the principal payment. In cell F7, we type =B7-E7 Now, let's drag this formula down to cell F30 (so cell F30 contains =B30-E30).

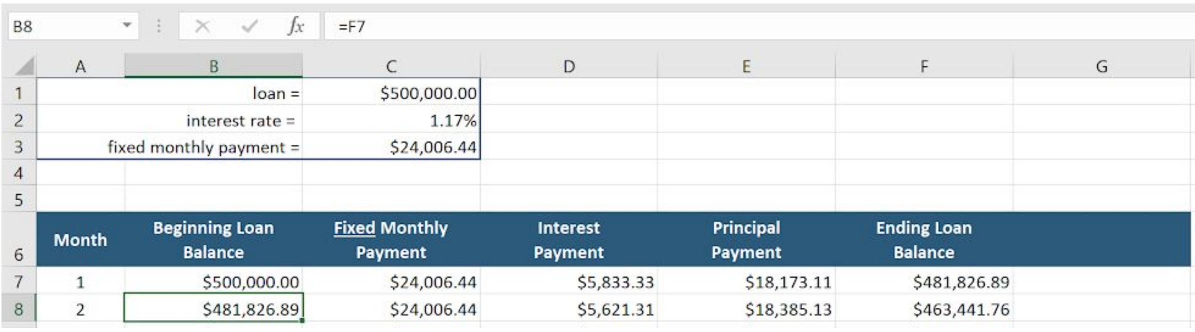
F7 =B7-E7

	A	B	C	D	E	F	G
1		loan =	$500,000.00				
2		interest rate =	1.17%				
3		fixed monthly payment =	$24,006.44				
4							
5							
6	Month	Beginning Loan Balance	Fixed Monthly Payment	Interest Payment	Principal Payment	Ending Loan Balance	
7	1	$500,000.00	$24,006.44	$5,833.33	$18,173.11	$481,826.89	

Figure 7

Video at 05:02

Finally, let's return to column B; I haven't forgotten about it! In cell B8, we want to input the formula: =F7. Naturally, the ending balance of the loan in period 1 (cell F7) is equal to the beginning balance of the loan in period 2 (cell B8). Let's click and drag this formula all the way down to B30 (so cell B30 contains =F29).



B8 =F7

	A	B	C	D	E	F	G
1		loan =	$500,000.00				
2		interest rate =	1.17%				
3		fixed monthly payment =	$24,006.44				
4							
5							
6	Month	Beginning Loan Balance	Fixed Monthly Payment	Interest Payment	Principal Payment	Ending Loan Balance	
7	1	$500,000.00	$24,006.44	$5,833.33	$18,173.11	$481,826.89	
8	2	$481,826.89	$24,006.44	$5,621.31	$18,385.13	$463,441.76	

Figure 8

Video at 05:19

Great! Our amortization schedule is done. In the last cell of the schedule, cell F30, we should see a value of $0 (Figure 9). As you read the schedule across, we can see how the payments slowly chip away at both the interest and the principal amounts, so that, by the end of the 24 months, we can see that we have paid off all of the payments on this loan.

Video at 05:38

Notice how, while the total payment is the same each month at $24,006.44, the amount paid towards interest versus the principal owed changes each month. This becomes clear when we calculate the percentage of the monthly payment that is paying off the principal owed (column G in Figure 9.

Video at 05:51

We can also see how that, as we pay down more of the loan principal amount, our interest payments also decrease. This is because our interest payments are calculated off of the principal amount. When our principal amount is low, then our interest payments are also lower. This is why it is beneficial to try and pay down as much of the principal as possible.

loan =	$500,000.00
interest rate =	1.17%
fixed monthly payment =	$24,006.44

Month	Beginning Loan Balance	Fixed Monthly Payment	Interest Payment	Principal Payment	Ending Loan Balance	% monthly payment towards principal
1	$500,000.00	$24,006.44	$5,833.33	$18,173.11	$481,826.89	75.70%
2	$481,826.89	$24,006.44	$5,621.31	$18,385.13	$463,441.76	76.58%
3	$463,441.76	$24,006.44	$5,406.82	$18,599.62	$444,842.14	77.48%
4	$444,842.14	$24,006.44	$5,189.82	$18,816.62	$426,025.53	78.38%
5	$426,025.53	$24,006.44	$4,970.30	$19,036.14	$406,989.38	79.30%
6	$406,989.38	$24,006.44	$4,748.21	$19,258.23	$387,731.15	80.22%
7	$387,731.15	$24,006.44	$4,523.53	$19,482.91	$368,248.24	81.16%
8	$368,248.24	$24,006.44	$4,296.23	$19,710.21	$348,538.03	82.10%
9	$348,538.03	$24,006.44	$4,066.28	$19,940.16	$328,597.86	83.06%
10	$328,597.86	$24,006.44	$3,833.64	$20,172.80	$308,425.06	84.03%
11	$308,425.06	$24,006.44	$3,598.29	$20,408.15	$288,016.91	85.01%
12	$288,016.91	$24,006.44	$3,360.20	$20,646.24	$267,370.67	86.00%
13	$267,370.67	$24,006.44	$3,119.32	$20,887.12	$246,483.55	87.01%
14	$246,483.55	$24,006.44	$2,875.64	$21,130.80	$225,352.75	88.02%
15	$225,352.75	$24,006.44	$2,629.12	$21,377.33	$203,975.42	89.05%
16	$203,975.42	$24,006.44	$2,379.71	$21,626.73	$182,348.70	90.09%
17	$182,348.70	$24,006.44	$2,127.40	$21,879.04	$160,469.66	91.14%
18	$160,469.66	$24,006.44	$1,872.15	$22,134.30	$138,335.36	92.20%
19	$138,335.36	$24,006.44	$1,613.91	$22,392.53	$115,942.83	93.28%
20	$115,942.83	$24,006.44	$1,352.67	$22,653.78	$93,289.06	94.37%
21	$93,289.06	$24,006.44	$1,088.37	$22,918.07	$70,370.99	95.47%
22	$70,370.99	$24,006.44	$820.99	$23,185.45	$47,185.54	96.58%
23	$47,185.54	$24,006.44	$550.50	$23,455.94	$23,729.60	97.71%
24	$23,729.60	$24,006.44	$276.85	$23,729.60	$0.00	98.85%

Figure 9

Video at 06:11

To summarize, we pay the same $24,006.44 each month for two years and, over the two years, we will be paying off the entire $500,000 mortgage, as well as the interest owed. However, over time, we will be paying less and less interest, as we pay off more and more of the principal owed. So now you will understand exactly what is going on when you have a big scary mortgage on your future home.

Video at 06:34

We can always replicate the outstanding principal amount of the loan using the PV of annuity formula. For example, let's imagine you inherit a large sum of money at the end of month 10, after making the 10th payment. You want to use this inheritance to pay off the remainder of the loan. In order to calculate the remaining balance of the loan at a given point in time, we will use the PV annuity formula and calculate the value by hand.

Video at 06:55

Let's recall: thinking back to the loan mortgage that we calculated the amortization schedule for earlier, the loan balance is $500,000, the interest rate is 1.17%, and the fixed payment we calculated is $24,006.44 a month. We want to know how much of the loan needs to be paid at the end of the 10th month.

Video at 07:15

Pause the video, and input these numbers into the PV annuity formula we just talked about. When you have calculated the final amount, resume the video to see how we did it.

Video at 07:24

First our "C" value (for the fixed cash flow in an annuity, which is sometimes also denoted by "A") is $24,006.44. The interest rate is 1.17% (r = effective monthly rate), and "n" is 14 (24 periods total - 10 periods that have already been paid). By plugging these values into the first half of the formula, we calculate:

$$PV\ of\ loan = \$24,006.44\left[\frac{1-(1.0117)^{-14}}{0.0117}\right] = \$308,425.06$$

The value of the total loan after 10 months = $308,425.06 remaining.

Video at 07:47

I will remind you that the ending balance of one period is the beginning balance of the next period. When we look back to the amortization schedule we created before (Figure 10), we can confirm that the balance of the loan at the beginning of month 11 (ending balance of the loan of month 10) is indeed $308,425.06.

Video at 08:03

Great! Now, we know how to calculate the remaining balance of our mortgages at any given point in time. This is useful as, sometimes, the numbers and values can get so big, it's easy to get mixed up on our payments.

B17 =F16

	loan =	$500,000.00
	interest rate =	1.17%
	fixed monthly payment =	$24,006.44

Month	Beginning Loan Balance	Fixed Monthly Payment	Interest Payment	Principal Payment	Ending Loan Balance	% monthly payment towards principal
1	$500,000.00	$24,006.44	$5,833.33	$18,173.11	$481,826.89	75.70%
2	$481,826.89	$24,006.44	$5,621.31	$18,385.13	$463,441.76	76.58%
3	$463,441.76	$24,006.44	$5,406.82	$18,599.62	$444,842.14	77.48%
4	$444,842.14	$24,006.44	$5,189.82	$18,816.62	$426,025.53	78.38%
5	$426,025.53	$24,006.44	$4,970.30	$19,036.14	$406,989.38	79.30%
6	$406,989.38	$24,006.44	$4,748.21	$19,258.23	$387,731.15	80.22%
7	$387,731.15	$24,006.44	$4,523.53	$19,482.91	$368,248.24	81.16%
8	$368,248.24	$24,006.44	$4,296.23	$19,710.21	$348,538.03	82.10%
9	$348,538.03	$24,006.44	$4,066.28	$19,940.16	$328,597.86	83.06%
10	$328,597.86	$24,006.44	$3,833.64	$20,172.80	$308,425.06	84.03%
11	$308,425.06	$24,006.44	$3,598.29	$20,408.15	$288,016.91	85.01%

Figure 10

Video at 08:14

Well done! Let's recap what we learned in this video. First, we reviewed that amortized loan payments contribute to both the interest amount and the principal amount throughout the lifetime of its loan. Then, we learned about the second type of amortized loan, the fixed payment loan. This means that the periodic payment remains the same, but the portion that contributes to interest amount and the portion that contributes to the principal amount varies each period.

Video at 08:41

We also learned that we can use the annuity formula to calculate the fixed payment of a loan, and we can use it to solve for the outstanding balance of a loan at a given point in time. We can also use the annuity formula to create an amortization schedule manually. We can use Excel to create amortization schedules, however, usually for loans that are much longer and stretch over many periods. Thanks for watching, and we'll see you next time!