Video 09 - Relating All the Factors of Present Value

The following is a supplementary transcript for tutorial videos from
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Hello! So far in this video series, we have talked a lot about the time value of money. We discussed concepts such as discounting and compounding cash flows, annuities, and perpetuities, in order to calculate their present values. Thinking back to these concepts, the three primary variables that affect present value are time, discount rate, and the desired future value. Now that you are comfortable using these formulas to calculate present value, today we will be stepping back to ensure that we really understand how changing these variables in the PV formula, like pulling on different levers, affect the PV calculated. By the end of this video, you should be able to understand how interest rates affect PV, how the length of the time we hold an investment for affects PV, and how the value of the future fixed payment or payments affects PV.

Video at 00:55
Let's begin by thinking about something that you want to buy in the future. What about the new iPhone 20 that will come out 10 years from today? Imagine that in 10 years, that phone will cost $4,000. Now, being conscientious, you have convinced yourself to start saving money now in order to ensure that you have enough money to buy the phone in the future. How much should you invest if you want to make a one-time investment today, that would amount to $4,000 in the future? As we have learned in previous videos, we would discount $4,000 by the effective annual rate over 10 years to find how much the $4,000 iPhone 20 would cost in today's dollars (i.e the present value today). We will use this iPhone scenario to explore how changing the effective annual rate, time and payment amounts, will change the calculated present value.

Video at 01:49
First things first: how does the effective annual rate impact the present value calculated of the iPhone 20? Note that when we are saving money, we are more likely to use effective rates (EAR) compared to when we are looking at loans, which are quoted as APR rates. Now what would happen if the rate was 5%, 10%, or 15%? Pause here and calculate the three different PV's today of the $4,000 iPhone payment in ten years, with effective annual rates of 5%, 10% or 15%.

Video at 02:26

As a reminder, finance often uses different terms to describe the same concept, which can get confusing. Here, the effective annual rate, as denoted by “r”, is also referred to as a discount rate, rate of return, interest rate, or cost of capital. There are nuanced differences on when one of the terms is preferred over another, which depends on the context. Here, since we will be using “r” to discount the future value of $4,000 back to today, we will be calling “r” the discount rate.

Video at 02:59

Plugging the discount rates given in the problem into the PV formula, you should have calculated:

when r = 0.05: $PV = \frac{\$4,000}{(1+0.05)^{10}} = \$2,455.65$

when r = 0.10: $PV = \frac{\$4,000}{(1+0.10)^{10}} = \$1,542.17$

when r = 0.15: $PV = \frac{\$4,000}{(1+0.15)^{10}} = \$988.74$

From this example, we can see that the PV of the $4000 future value of the iPhone 20 decreases as the effective annual rate increases.

Video at 03:40

Let's think about this intuitively. Recall that the discount rate “r” reflects the time value of money, inflation, and risk of uncertainty. A higher discount rate means that, for these three reasons, $4,000 in ten years is less valuable in today's dollars. An example of this is thinking about a start-up that contains inherent risk and uncertainty in its future cash flows. Because of the higher uncertainty in the startup, there is a higher discount rate, and in turn an investor would be willing to pay less for the stock compared to a loan with little to no uncertainty of future cash flows. At the same time, a small discount rate means that $1 in 10 years will be more similar in value to $1 today. Another way you can think of this is that at a higher rate, a small amount of money (such as PV=$988.74 at a rate of 15%) needs to be invested today to result in $4,000 in the future.

Video at 04:32

Let's see a more explicit example of this. Imagine you have $100 today you have worked hard to save. You are looking now to invest this money into an investment vehicle to grow your money for the next three years. You are looking at short term investments that offer a 2%, 4%, and 7% effective annual rate. Which one should you invest your money into? Pause here and calculate the FV of your $100 investment 3 years from now.

Video at 05:03

Here, "r" might be referred to as the interest rate, because it is telling us how much interest to be earned in growing our money. However the interest rate here has the same mechanics as a "discount rate" in the previous example - they are both "r". The future value 3 years from now of your $100 when investing into the investment vehicles with varying effective rates are

when r = 0.02: $FV = \$100 \times (1 + 0.02)^3 = \$106.12$

when r = 0.04: $FV = \$100 \times (1 + 0.04)^3 = \$112.48$

when r = 0.07: $FV = \$100 \times (1 + 0.07)^3 = \$122.50$

As expected and mentioned before, the higher the rate, the larger the initial value grows within the same amount of time. With the two previous examples we have identified that discount rate and present value have an *inverse* relationship: as rate increases, present value decreases.

Video at 05:56

Now that we have covered how rates affect present value calculations, let's think about time. How does the amount of time affect present value? Let's circle back to our iPhone 20 example. Recall that, when setting the effective annual discount rate at 10%, the present value was $1,542.17. This means that our initial investment of $1,542.17 would be worth $4,000 in 10 years from now. But, what if we invested it for 5 years to buy the iPhone 15, or invested it for 15 years to buy the iPhone 25, instead of 10 years? Pause here and calculate the exact future value of the initial $1,542.17 investment in 5, 10, and 15 years from now.

Video at 06:45

When investing the initial $1,542.17 at an effective annual rate of 10%

when t = 5: $FV_5 = \$1,542.17 \times (1 + 0.10)^5 = \$2,482.68$

when t = 10: $FV_{10} = \$1,542.17 \times (1 + 0.10)^{10} = \$4,000$

when t = 15: $FV_{15} = \$1,542.17 \times (1 + 0.10)^{15} = \$6,442.03$

In simplistic terms, this example highlights exactly what your mom told you ever since you were a kid: " by saving your money longer, the more money you will have in the future." On the flip side, discounting cash flows back in time over a longer time horizon means that you are making today's dollar less and less valuable. Imagine that, for every period that you discount your money, the time is eating away at its value. Therefore, the time period and present value have an *inverse* relationship: for the same discount rate and same amount of money, as you discount your money further and further back in time, the less and less valuable your money will be.

Video at 07:45

Lastly, let's think about how the payment amount, which is “A” in the PV of an annuity formula, affects the PV calculation. Let's imagine that Apple has decided to release a promotion to help you fund your iPhone 20. They are willing to give you $100 every year for the next three years starting one year from now at an effective annual rate of 10%. What if they gave you $50 instead? Which cash flow would result in a higher present value? In other words, which promotion deal would you prefer? Pause here and calculate the present value of these annuities.

Video at 08:26

After plugging the numbers into the formula, three annual cash flows at an effective annual discount rate of 10% will have

when A = \$50: $PV\,annuity = \$50 \times \left[\frac{1-(1.10)^{3}}{0.10}\right] = \$124.34$

when A = \$100: $PV\,annuity = \$100 \times \left[\frac{1-(1.10)^{3}}{0.10}\right] = \$248.68$

Intuitively, this makes sense. If you are receiving larger contributions and discounting these cash flows back, it should result in a larger present value today. Or, in other words, between someone offering you three $50 payments or three $100 payments, of course the three $100 payments are more valuable to you, while the three $50 payments are less attractive. Here, we can define the *positive* relationship that exists between A and PV: as A increases PV also increases. Maybe your dad was right when he told you that you should not only start saving early, but also to save a large sum of money each time!

Video at 09:26

Up until this point, we isolated each variable to determine its relationship with PV. But what happens when you change multiple variables? Let's unpack this further. Say the iPhone 20 still costs $4,000 and you are willing to invest over 6 years in order to buy it. If the effective annual rate was 13%, how much would you need to invest each year? Pause here and calculate the value of “A” in this example. Note that you will need to use the FV of a standard annuity here.

Video at 10:00

In order to have enough money to buy the iPhone 20

$$FV\ annuity\ =\ \$A \times \left[\frac{(1+r)^n - 1}{r}\right]$$

$$\$4,000\ =\ \$A \times \left[\frac{(1+0.13)^6 - 1}{0.13}\right]$$

$$\$A\ =\ \$4,000\ \div \left[\frac{(1+0.13)^6 - 1}{0.13}\right] = \$480.61$$

would need to be invested each year over the six year time period. Now, what if the effective annual rate was 8%? This is 5% less than the rate in the previous example. Let's see what would happen if you were willing to pay 5% more annually over six years to compensate for the difference in the annual rate (i.e pay \$505 because \$480 x 1.05 = \$505). Would you have enough money to buy the iPhone? Pause here and see for yourself.

Video at 10:39

We can expect that, when the effective annual rate falls, the future value will fall, because our money is not compounding or growing as quickly. Yet, with bigger fixed payments, we can expect that the future value will rise, because we are contributing more money into our savings to buy that iPhone. So what will the net effect be? We must use the FV of an annuity formula: using A = \$505, r = 0.08 and t = 6, the FV of the cash flows would be

$$FV\ annuity\ =\ \$505 \times \left[\frac{(1+0.08)^6 - 1}{0.08}\right] = \$3,704.64\ <\ \$4,000$$

which is not enough to buy the iPhone 20.

Video at 11:18

This shows that the 5% increase in annual payment amount does not compensate for the 5% decrease in the effective annual rate. Intuitively, this makes sense given that, in the FV of an annuity formula, the effective annual rate is compounded by the number of periods (the effect of the 5% decrease in “r” is compounded 6 times). From the past two examples, we have seen that when you change multiple values, the variables can have counterbalancing effects, so we cannot always say what will happen when more than one thing is changing, unless we calculate it.

Video at 11:50

In this video, we unpack the levers that contribute to calculating present value. In doing so we identified that:

1. There is an *inverse* relationship between “r” (the effective rate) and PV, such that as rate increases, present value decreases. This is because, at higher rates, you need less money today to save up for that $4000 iPhone 20 in ten years.
2. There is an *inverse* relationship between “t” (the time horizon of the investment) and PV; as “t” increases, present value decreases. This is because, the longer you invest your money, the more time it will have to grow, and thus, the less money you will have to invest today to save up for that $4000 iPhone.
3. There is a *positive* relationship between “A” (the fixed payment amount) and PV, such that the bigger the payments you make, the greater the present value of those cash flows are today. Intuitively, this is because a stream of cash flows with larger fixed payments will be more valuable today than a stream of cash flows with smaller fixed payments.
4. We also learned that changing multiple variables can have counterbalancing effects, so it is difficult to determine the present or future value calculated.

I hope that through this video you were able to gain some intuition on the factors surrounding present value.